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Faculty of sciences
Field : Sciences of matter (SM)
1st year LMD Semester 02.

جامعـة محمد بوضياف ـالمسيلـة
كليـة العلـوم
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Physics 02: Electricity and magnetism

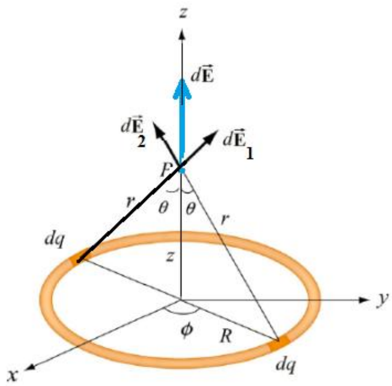
University Year 2023-2024

Series N° 01: ELECTROSTATICS

Part 2: Continuous distribution and theorem of Gauss

EXERCISE 01

1- The electric field at a point P, located at a distance z from the center of the ring along its axis of symmetry.



$$\boldsymbol{dQ = \lambda\,.\,dl = \lambda\, R d\varphi}$$

$$\boldsymbol{d\vec{E} = d\overrightarrow{E_1} + d\overrightarrow{E_2} \Rightarrow \vec{E} = \overrightarrow{E_1} + \overrightarrow{E_2}}$$

$$\boldsymbol{\overrightarrow{E_1} = \int d\overrightarrow{E_1} = k\int \frac{\lambda\, R d\varphi}{r^2}\,\overrightarrow{u_1}}$$

$$\boldsymbol{\overrightarrow{E_2} = \int d\overrightarrow{E_2} = k\int \frac{\lambda\, R d\varphi}{r^2}\,\overrightarrow{u_2}}$$

$$\boldsymbol{r^2 = R^2 + z^2}$$

$$\boldsymbol{\overrightarrow{u_1} = sin\theta\ \overrightarrow{u_\rho} + cos\theta\,\vec{k}}$$

$$\boldsymbol{\overrightarrow{u_2} = -sin\theta\ \overrightarrow{u_\rho} + cos\theta\,\vec{k}}$$

$$\overrightarrow{E_1} = k\int \frac{\lambda\, R d\varphi}{R^2+z^2}\left(sin\theta\ \overrightarrow{u_\rho} + cos\theta\,\vec{k}\right) \quad , \quad \overrightarrow{E_2} = k\int \frac{\lambda\, R d\varphi}{R^2+z^2}\left(-sin\theta\ \overrightarrow{u_\rho} + cos\theta\,\vec{k}\right)$$

$$\vec{E} = \overrightarrow{E_1} + \overrightarrow{E_2} = k\int \frac{\lambda\, R d\varphi}{R^2+z^2}\left(2\,cos\theta\,\vec{k}\right) \quad , \quad cos\theta = \frac{z}{r} = \frac{z}{\sqrt{R^2+z^2}}$$

$$\vec{E} = \overrightarrow{E_1} + \overrightarrow{E_2} = k\int \frac{\lambda\, R d\varphi}{R^2+z^2}\left(2\,\frac{z}{\sqrt{R^2+z^2}}\,\vec{k}\right)$$

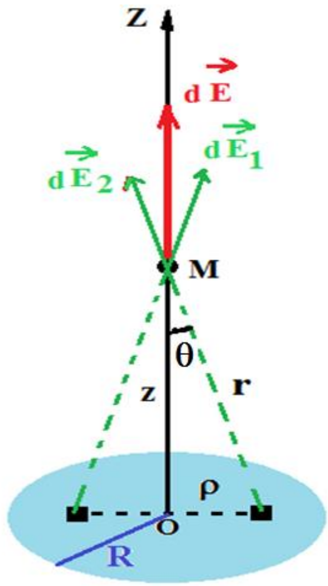
$$\vec{E} = \overrightarrow{E_1} + \overrightarrow{E_2} = \frac{k\lambda\, 2Rz}{(R^2+z^2)^{3/2}}\int_0^{\pi} d\varphi\,\vec{k}$$

$$\boldsymbol{\vec{E} = \frac{k(\lambda 2\pi R)z}{(R^2+z^2)^{3/2}}\,\vec{k}}$$

$$\boldsymbol{Q = \lambda 2\pi R \Rightarrow \vec{E} = k\,\frac{Qz}{(R^2+z^2)^{3/2}}\,\vec{k}}$$

2- The electric field at point M on the z-axis a distance z from the center of the disk. (This part was resolved in the course)

$$dQ = \sigma \, . dS = \sigma \, \rho d\rho d\varphi$$



$$\vec{E} = \int d\vec{E} = k \iint \frac{\sigma \, \rho d\rho d\varphi}{r^2} \, \vec{u}$$

$$r^2 = \rho^2 + z^2$$

$$\vec{u_1} = \sin\theta \; \vec{u_\rho} + \cos\theta \, \vec{k}, \qquad \vec{u_2} = -\sin\theta \; \vec{u_\rho} + \cos\theta \, \vec{k}$$

$$\vec{E_1} = k \iint \frac{\sigma \, \rho d\rho d\varphi}{\rho^2 + z^2} \, (\sin\theta \; \vec{u_\rho} + \cos\theta \, \vec{k})$$

$$\vec{E_2} = k \iint \frac{\sigma \, \rho d\rho d\varphi}{\rho^2 + z^2} \, (-\sin\theta \; \vec{u_\rho} + \cos\theta \, \vec{k})$$

$$\vec{E} = \vec{E_1} + \vec{E_2} = k \iint \frac{\sigma \, \rho d\rho d\varphi}{\rho^2 + z^2} \, (2 \cos\theta \, \vec{k})$$

$$\cos\theta = \frac{z}{r} = \frac{z}{\sqrt{\rho^2 + z^2}}$$

$$\vec{E} = \vec{E_1} + \vec{E_2} = k \iint \frac{\sigma \, \rho d\rho d\varphi}{\rho^2 + z^2} \, (2 \frac{z}{\sqrt{\rho^2 + z^2}} \, \vec{k})$$

$$\vec{E} = \vec{E_1} + \vec{E_2} = 2kz \iint \frac{\sigma \, \rho d\rho d\varphi}{(\rho^2 + z^2)^{3/2}} \; \vec{k}$$

$$\vec{E} = \vec{E_1} + \vec{E_2} = 2k\sigma z \int_0^R \frac{\rho d\rho}{(\rho^2 + z^2)^{3/2}} \int_0^\pi d\varphi \, \vec{k}$$

$$\vec{E} = \vec{E_1} + \vec{E_2} = 2\pi k\sigma z \int_0^R \frac{\rho d\rho}{(\rho^2 + z^2)^{3/2}} \vec{k}$$

$$\vec{E} = \vec{E_1} + \vec{E_2} = 2\pi k\sigma z \left[\frac{-1}{\sqrt{\rho^2 + z^2}} \right]_0^R \vec{k}$$

$$\vec{E} = \vec{E_1} + \vec{E_2} = 2\pi k\sigma \, z(\frac{-1}{\sqrt{(R^2+z^2)}} + \frac{1}{z}) \, \vec{k} \Rightarrow \vec{E} = 2\pi k\sigma \, (1 - \frac{z}{\sqrt{(R^2+z^2)}}) \, \vec{k}$$

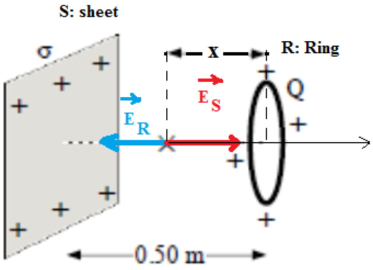
The electric field in the case of infinite plane.

Infinite plane $\Rightarrow R \rightarrow \infty \Rightarrow \vec{E} = 2\pi k\sigma \, (1 - \frac{z}{\sqrt{(\infty^2+z^2)}}) \, \vec{k}$

$$\Rightarrow \vec{E} = 2\pi k\sigma \, \vec{k} \Rightarrow \vec{E} = \frac{2\pi\sigma}{4\pi\varepsilon_0} \vec{k} \Rightarrow \vec{E} = \frac{\sigma}{2\varepsilon_0} \vec{k}$$

3- The electric field at the point midway between the ring and the sheet.

$$\vec{E} = \vec{E_S} + \vec{E_R}$$



According to the previous results:

- the electric field due to the positively charged sheet is:

$$\vec{E_S} = \frac{\sigma}{2\varepsilon_0}\vec{i}$$

- the electric field due to the positively charged ring is:

$$\vec{E_R} = -k\frac{Qx}{(R^2 + x^2)^{3/2}}\vec{i}$$

Where x=0.25 m is the distance from the center of the ring to and R is the radius of the ring.

$$\vec{E} = \vec{E_S} + \vec{E_R} = \frac{\sigma}{2\varepsilon_0}\vec{i} - k\frac{Qx}{(R^2+x^2)^{3/2}}\vec{i}$$

$$\Rightarrow \vec{E} = \vec{E_S} + \vec{E_R} = \left(\frac{\sigma}{2\varepsilon_0} - \frac{1}{4\pi\varepsilon_0}\frac{Qx}{(R^2+x^2)^{3/2}}\right)\vec{i}$$

$$\Rightarrow \vec{E} = \vec{E_S} + \vec{E_R} = \frac{1}{2\varepsilon_0}\left(\sigma - \frac{1}{2\pi}\frac{Qx}{(R^2+x^2)^{3/2}}\right)\vec{i}$$

$$\Rightarrow \vec{E} = \vec{E_S} + \vec{E_R} = \frac{1}{2\times 8.854\times 10^{-12}}\left(10^{-6} - \frac{1}{2\pi}\frac{2.5\times 10^{-6}\times 0.25}{(0.2^2+0.25^2)^{3/2}}\right)\vec{i}$$

$$\Rightarrow \vec{E} = -1.15\times 10^5\,\vec{i} \Rightarrow E = 1.15\times 10^5\,N/C$$

EXERCISE 02

$$\lambda = \lambda_0 cos\theta.$$

1- The relationship between λ_0, R and Q.

$$dQ = \lambda . dl = \lambda_0 cos\theta\, R d\theta$$

$$\Rightarrow dQ = \lambda_0 R cos\theta d\theta \Rightarrow Q = \lambda_0 R \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} cos\theta d\theta$$

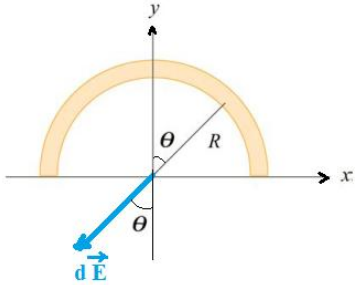
$$\Rightarrow Q = \lambda_0 R\,[sin\theta]_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \Rightarrow Q = 2\,\lambda_0 R$$

2- The force on the charged particle at the center of the semicircle is given by:

$$\vec{F} = q\,\vec{E}$$

The electric field at the origin $\vec{E}$.

$$\vec{E} = \int d\vec{E} = k\int \frac{dQ}{R^2}\vec{u}$$



$$\vec{E} = k\int \frac{\lambda_0 R cos\theta d\theta}{R^2}\vec{u}$$

$$\vec{u} = -sin\theta\,\vec{i} - cos\theta\,\vec{j}$$

$$\Rightarrow \vec{E} = k\int \frac{\lambda_0 R cos\theta d\theta}{R^2}(-sin\theta\,\vec{i} - cos\theta\,\vec{j})$$

$$\Rightarrow\begin{cases} \boldsymbol{E_x} = -k\int \frac{\lambda_0 R cos\theta d\theta}{R^2}\ sin\theta \\ \boldsymbol{E_y} = -k\int \frac{\lambda_0 R cos\theta d\theta}{R^2}\ cos\theta \end{cases} \Rightarrow\begin{cases} \boldsymbol{E_x} = -k\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{\lambda_0 R sin\theta cos\theta d\theta}{R^2} \\ \boldsymbol{E_y} = -k\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{\lambda_0 R cos^2\theta d\theta}{R^2} \end{cases}$$

$$\Rightarrow\begin{cases} \boldsymbol{E_x} = \frac{k}{2}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{\lambda_0(-2sin\theta cos\theta)d\theta}{R} \\ \boldsymbol{E_y} = -k\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{\lambda_0 R(1+cos2\theta)d\theta}{2R^2} \end{cases}$$

$$\int(-2sin\theta cos\theta)\ d\theta = cos^2\theta, \qquad \int(1+cos2\theta)\ d\theta = \theta + \frac{sin2\theta}{2}$$

$$\Rightarrow\begin{cases} \boldsymbol{E_x} = \frac{k\lambda_0}{2R}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}(-2sin\theta cos\theta)d\theta \\ \boldsymbol{E_y} = \frac{-k\lambda_0}{2R}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}1+cos2\theta)d\theta \end{cases}$$

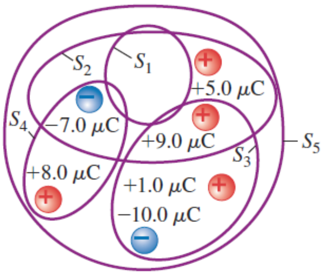
$$\Rightarrow\begin{cases} E_x = \frac{k\lambda_0}{2R}[cos^2\theta]_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \\ E_y = \frac{-k\lambda_0}{2R}\left[\theta + \frac{sin2\theta}{2}\right]_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \end{cases} \Rightarrow\begin{cases} E_x = 0 \\ E_y = \frac{-k\lambda_0}{2R}\left[\left(\frac{\pi}{2} + \frac{sin2(\frac{\pi}{2})}{2}\right) - (-\frac{\pi}{2} + \frac{sin2(-\frac{\pi}{2})}{2})\right] \end{cases}$$

$$\Rightarrow\begin{cases} E_x = 0 \\ E_y = \frac{-k\lambda_0}{2R}\pi = -\frac{\lambda_0}{2\times 4\pi\varepsilon_0 R}\pi \end{cases} \Rightarrow\begin{cases} E_x = 0 \\ E_y = -\frac{\lambda_0}{8R\varepsilon_0} \end{cases}$$

$$\Rightarrow \vec{\boldsymbol{E}} = -\frac{\boldsymbol{\lambda_0}}{\boldsymbol{8R\varepsilon_0}}\vec{\boldsymbol{j}}$$

Therefore the force on the charged particle at the point is given by

$$\vec{F} = q\,\vec{E} \Rightarrow \vec{\boldsymbol{F}} = -\frac{\boldsymbol{\lambda_0 q}}{\boldsymbol{8R\varepsilon_0}}\vec{\boldsymbol{j}}$$



EXERCISE 04

each enclose part of this plane.
Compute the electric flux through the five Gaussian surfaces S_1 , S_2 , S_3 , S_4 , and S_5.

$$\boldsymbol{\Phi} = \frac{\boldsymbol{Q_{enclosed}}}{\varepsilon_0}$$

$$\boldsymbol{\Phi_1} = \frac{\mathbf{0}}{\varepsilon_0} = \mathbf{0},\ \boldsymbol{\Phi_2} = \frac{(-7+9+5)10^{-6}}{8.85\times10^{-12}} = \mathbf{79}\times\mathbf{10^4}\ \boldsymbol{Nm^2/C},$$

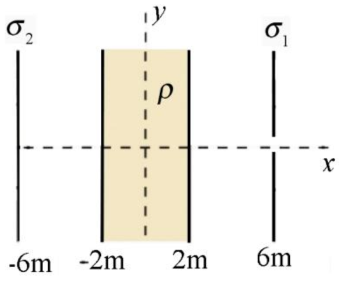
$$\boldsymbol{\Phi_3} = \frac{(-10+1+9)10^{-6}}{8.85\times10^{-12}} = \mathbf{0}\ \boldsymbol{Nm^2/C}\ ,\ \boldsymbol{\Phi_4} = \frac{(-7+8)10^{-6}}{8.85\times10^{-12}} = \mathbf{101.7}\times\mathbf{10^4}\ \boldsymbol{Nm^2/C}$$

$$\boldsymbol{\Phi_5} = \frac{(-7+9+5+8+10-10)10^{-6}}{8.85\times10^{-12}} = \mathbf{146.9}\times\mathbf{10^4}\ \boldsymbol{Nm^2/C}$$

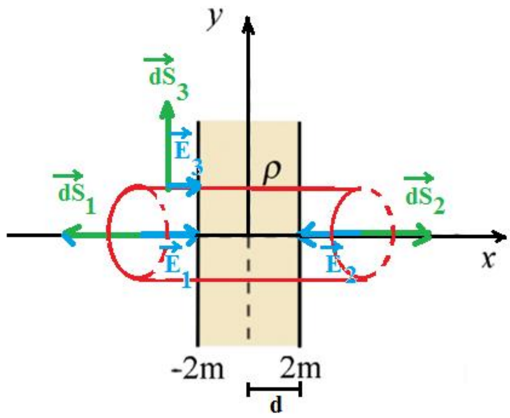
EXERCISE 05

In the accessible regions you've measured the electric field to be:

$$\vec{E}\,(x) = \begin{cases} \vec{0}\,, & x < -6\ m \\ \left(10\,\frac{N}{C}\right)\vec{\imath}, & -6\ m < x < -2\ m \\ \left(-10\,\frac{N}{C}\right)\vec{\imath}, & 2\ m < x < 6\ m \\ \vec{0}, & x > 6\ m \end{cases}$$



1- The charge density ρ of the slab.



$$\emptyset = \oint_S \vec{E}.d\vec{S} = \frac{Q_{enclosed}}{\varepsilon_0}$$

$$\Rightarrow \emptyset = \int \overrightarrow{E_1}.\overrightarrow{dS_1} + \int \overrightarrow{E_2}.\overrightarrow{dS_2} + \int \overrightarrow{E_3}.\overrightarrow{dS_3}$$

$$\emptyset =$$

$$\int E_1.dS_1 \cos 180 + \int E_2.dS_2 180 + \int E_3.dS_3 \cos 90$$

$$E_1 = E_2 = E$$

$$dS_1 = dS_2 = d\,S$$

$$\Rightarrow \emptyset = -2E\int dS = -2E\,.S = -2\,E\,\pi r^2$$

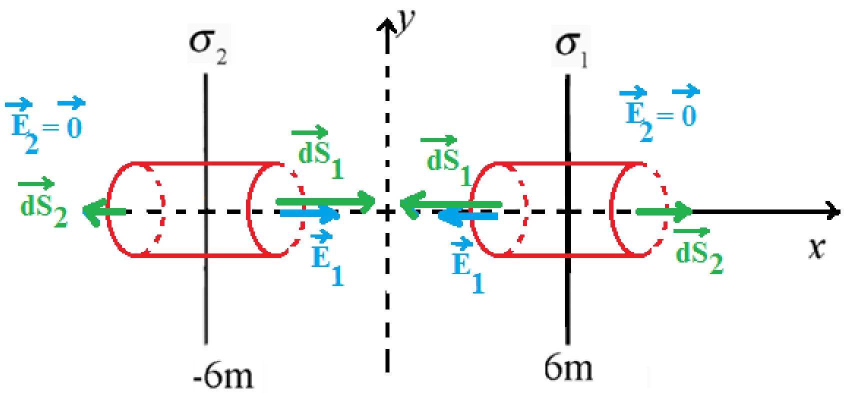
$$Q_{enclosed} = \rho \iiint dv = \rho \int_0^r rdr \int_0^{2\pi} d\theta \int_{-d}^{d} dl$$

$$\Rightarrow Q_{enclosed} = \rho\left(\frac{r^2}{2}\right)(2\pi)(2\,d) \Rightarrow Q_{enclosed} = 2\,\pi d r^2 \rho$$

$$\emptyset = \oint_S \vec{E}.d\vec{S} = \frac{Q_{enclosed}}{\varepsilon_0} \Rightarrow -2\,E\,\pi r^2 = \frac{2\,d\pi r^2\rho}{\varepsilon_0} \Rightarrow \rho = -\frac{1}{d}\varepsilon_0\,E \Rightarrow \rho = -\frac{1}{2}\times 8.85\times 10^{-12}\times 10$$

$$\Rightarrow \boldsymbol{\rho = -4.42\times 10^{-11}\ C/m^3}$$

2- The two surface charge densities σ_1 and σ_2 of the left and right charged sheets.



$$\emptyset = \oint_S \vec{E}.d\vec{S} = \frac{Q_{enclosed}}{\varepsilon_0}$$
$$\Rightarrow \emptyset = \int \overrightarrow{E_1}.\overrightarrow{dS_1} + \int \overrightarrow{E_2}.\overrightarrow{dS_2}$$

(the curved surface is perpendicular to the electric field)

For the sheet on the left:

We have $E_2 = 0 \Rightarrow \int \overrightarrow{E_2}.\overrightarrow{dS_2} = 0$

$$\emptyset = \int E_1 . dS_1 \cos 0$$
$$E_1 = E,$$
$$dS_1 = dS_2 = d\,S$$
$$\Rightarrow \emptyset = E \int dS = E\,.S = E\,\pi r^2$$
$$Q_{enclosed} = \sigma_2 \iint dS = \sigma_2 \int_0^r r dr \int_0^{2\pi} d\theta$$
$$\Rightarrow Q_{enclosed} = \sigma_2 (\frac{r^2}{2})(2\pi) \Rightarrow Q_{enclosed} = \sigma_2 \pi r^2$$

$\emptyset = \oint_S \vec{E}.d\vec{S} = \frac{Q_{enclosed}}{\varepsilon_0} \Rightarrow E\,\pi r^2 = \frac{\sigma_2 \pi r^2}{\varepsilon_0} \Rightarrow \sigma_2 = \varepsilon_0 E \Rightarrow \sigma_2 = 8.85 \times 10^{-12} \times 10$

$$\Rightarrow \boldsymbol{\sigma_2 = 8.85 \times 10^{-11}\ C/m^3}$$

In a similar manner :

$$\boldsymbol{\sigma_1 = 8.85 \times 10^{-11}\ C/m^3}$$

EXERCISE 06

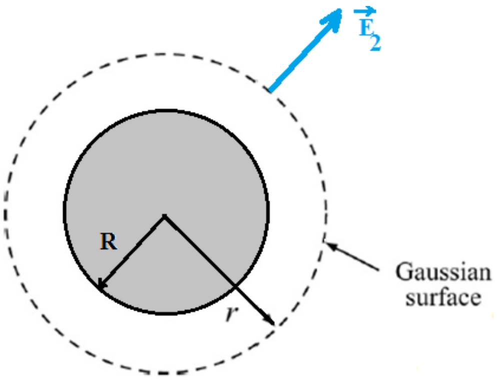
The electric field at a point outside the sphere: **r >R**

$\emptyset = \oiint \overrightarrow{E_2}.d\vec{S} = \frac{Q_{enclosed}}{\varepsilon_0}$

$\mathbf{dV} = r^2 dr \sin\theta\, d\theta\, d\Phi \Rightarrow$

$V = \int_0^R r^2\, dr \int_0^{\pi} \sin\theta\, d\theta \int_0^{2\pi} d\Phi$

$\Rightarrow V = \frac{4}{3}\pi R^3$



we can write: dV= $4\pi r^2 dr$

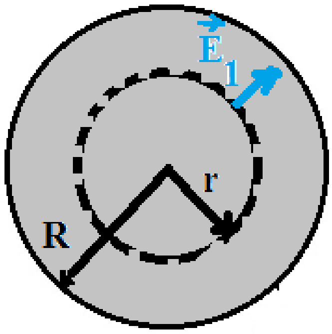
$\Rightarrow E_2 \times S = \frac{Q_{enclosed}}{\varepsilon_0}$

$\Rightarrow E_2 \times 4\pi r^2 = \frac{\rho \int_0^R 4\pi r^2 dr}{\varepsilon_0} = \frac{\rho(\frac{4}{3}\pi R^3)}{\varepsilon_0}$

$$\Rightarrow \boldsymbol{E_2 = \frac{Q}{4\pi\varepsilon_0}\frac{1}{r^2} = \frac{\rho(\frac{4}{3}\pi R^3)}{4\pi\varepsilon_0}\frac{1}{r^2} \Rightarrow E_2 = \frac{\rho R^3}{3\varepsilon_0}\frac{1}{r^2}}$$

The electric field at a point inside the sphere: **r <R**

$$\emptyset = \oiint \overrightarrow{E_1}.d\vec{S} = \frac{Q_{enclosed}}{\varepsilon_0}$$



$$\mathbf{dV}= \boldsymbol{r^2 dr} \ \mathbf{sin}\boldsymbol{\theta} \ \mathbf{d}\boldsymbol{\theta} \ \mathbf{d}\boldsymbol{\Phi} \Rightarrow$$

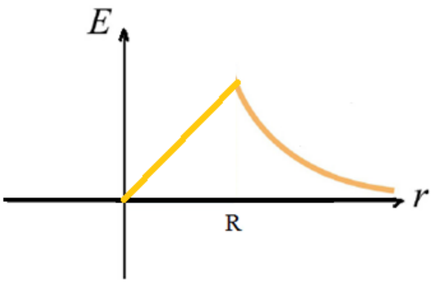
$$\text{V}=\int_0^R \boldsymbol{r^2 \, dr} \ \int_0^\pi \mathbf{sin}\boldsymbol{\theta} \ \mathbf{d}\boldsymbol{\theta} \int_0^{2\pi} \mathbf{d}\boldsymbol{\Phi}$$

$$\Rightarrow \text{V}= \frac{4}{3}\pi R^3$$

We can write: dV= $4\pi r^2 \text{dr}$

$$\Rightarrow E_1 \times S = \frac{Q_{enclosed}}{\varepsilon_0} \Rightarrow E_1 \times 4\pi r^2 = \frac{\rho \int_0^r 4\pi r^2 \text{dr}}{\varepsilon_0} = \frac{\rho(\frac{4}{3}\pi r^3)}{\varepsilon_0}$$

$$\Rightarrow \boldsymbol{E_1} = \frac{\rho(\frac{4}{3}\pi r^3)}{\boldsymbol{4\pi\varepsilon_0}} \frac{\mathbf{1}}{\boldsymbol{r^2}} \Rightarrow \boldsymbol{E_1} = \frac{\rho}{3\varepsilon_0} \boldsymbol{r}$$



The electric potential at a point outside the sphere: **r >R**

$$\vec{\mathbf{E}} = -\overrightarrow{\mathbf{grad}} \, \mathbf{V}$$

$$\boldsymbol{E_2} = \frac{\rho R^3}{3\varepsilon_0} \frac{1}{r^2} \Rightarrow \boldsymbol{E_2} = -\frac{\partial V_2}{\partial r} \Rightarrow \boldsymbol{dV_2} = -\boldsymbol{E_2 dr} \Rightarrow \boldsymbol{dV_2} = -\frac{\rho R^3}{3\varepsilon_0} \frac{1}{r^2} \boldsymbol{dr}$$

$$\Rightarrow \boldsymbol{V_2} = \frac{\rho R^3}{3\varepsilon_0} \frac{1}{r} + \mathbf{C_2}$$

$$\boldsymbol{V_2(\infty) = 0 \Rightarrow C_2 = 0} \quad \Rightarrow \boldsymbol{V_2} = \frac{\rho R^3}{3\varepsilon_0} \frac{1}{r}$$

The electric potential at a point outside the sphere: **r <R**

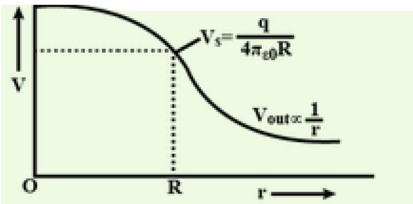
$$\vec{\mathbf{E}} = -\overrightarrow{\mathbf{grad}} \, \mathbf{V}$$

$$\boldsymbol{E_1} = \frac{\rho}{3\varepsilon_0} \boldsymbol{r} \Rightarrow \boldsymbol{E_1} = -\frac{\partial V_1}{\partial r} \Rightarrow \boldsymbol{dV_1} = -\boldsymbol{E_1 dr} \Rightarrow \boldsymbol{dV_1} = -\frac{\rho}{3\varepsilon_0} \boldsymbol{r \, dr}$$

$$\Rightarrow \boldsymbol{V_1} = -\frac{\rho}{6\varepsilon_0} \mathbf{r^2} + \mathbf{C_1}$$

$$\boldsymbol{V_2(R) = V_1(R)} \Rightarrow \frac{\rho R^2}{3\varepsilon_0} = -\frac{\rho}{6\varepsilon_0} \mathbf{R^2} + \boldsymbol{C_1} \Rightarrow \boldsymbol{C_1} = \frac{\rho R^2}{3\varepsilon_0}$$

$$\Rightarrow \boldsymbol{V_1} = -\frac{\rho}{6\varepsilon_0} \mathbf{r^2} + \frac{\rho R^2}{3\varepsilon_0}$$

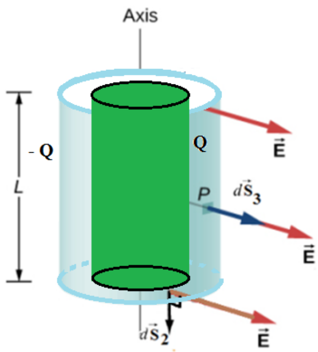


The variation of electric potential with distance in non-conducting sphere

EXERCISE 07

- The electric field in the all regions.

Case 1: r<a



$$\emptyset = \oint_S \vec{E}.d\vec{S} = \frac{Q_{enclosed}}{\varepsilon_0}$$

$$\Rightarrow E_1.2\pi rL = \frac{Q_{enclosed}}{\varepsilon_0} \Rightarrow E_1.2\pi rL = \frac{\iiint \rho\, .dV}{\varepsilon_0}$$

$$\Rightarrow E_1.2\pi rL = \frac{\rho\int_0^r \mathbf{rdr} \int_0^{2\pi} \theta\, \mathbf{d}\theta \int_0^L \mathbf{dz}}{\varepsilon_0} = \frac{\rho(\pi r^2 L)}{\varepsilon_0}$$

$$\Rightarrow E_1.2\pi rL = \frac{\rho(\pi r^2 L)}{\varepsilon_0}$$

$$\Rightarrow \boldsymbol{E_1} = \frac{\rho}{2\varepsilon_0}\mathbf{r}$$

r =a $\Rightarrow \boldsymbol{E_1(a)} = \frac{\rho}{2\varepsilon_0}\mathbf{a}$

Case 2: a <r <b

$$\Rightarrow E_2.2\pi rL = \frac{\rho\int_0^a \mathbf{rdr} \int_0^{2\pi} \theta\, \mathbf{d}\theta \int_0^L \mathbf{dz}}{\varepsilon_0}$$

$$\Rightarrow E_2.2\pi rL = \frac{\rho(\pi a^2 L)}{\varepsilon_0} \Rightarrow E = \frac{2kQ}{L}\frac{1}{r}$$

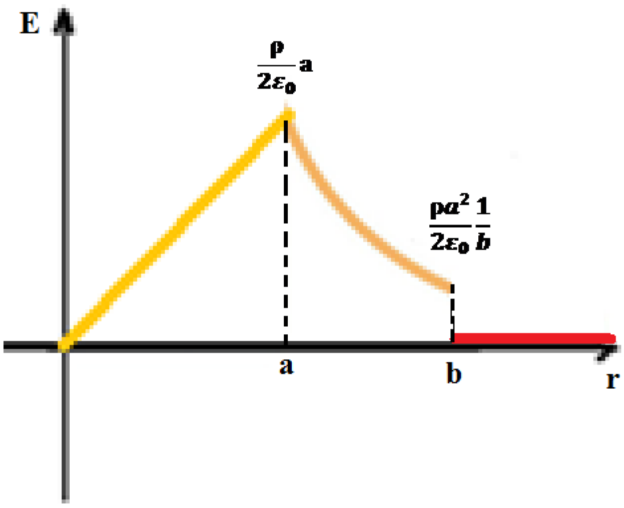
$$\Rightarrow E_2.2\pi rL = \frac{\rho(\pi a^2 L)}{\varepsilon_0} \Rightarrow \boldsymbol{E_2} = \frac{\rho a^2}{2\varepsilon_0}\frac{1}{r}$$

r= b$\Rightarrow \boldsymbol{E_2(b)} = \frac{\rho a^2}{2\varepsilon_0}\frac{1}{b}$

Case 3: b <r

$$\emptyset = \oint_S \vec{E_3}.d\vec{S} = \frac{Q_{enclosed}}{\varepsilon_0} = \frac{Q_1 + Q_2}{\varepsilon_0} = \frac{\boldsymbol{Q-Q}}{\boldsymbol{\varepsilon_0}} = 0$$

$$\Rightarrow \boldsymbol{E_3 = 0}$$



- The electric potential in the all regions.

$$\vec{E} = -\overrightarrow{\text{grad}}\, V$$

$$E = -\frac{\partial V}{\partial r} \Rightarrow dV = -E dr$$

Case 3: b <r

$$\Rightarrow E_3 = 0 \Rightarrow V_3 = C_3 = constant$$

$$V_3(\infty) = 0 \Rightarrow C_3 = 0 \quad \Rightarrow V_3 = 0$$

Case 2: a <r <b

$$E_2 = \frac{\rho a^2}{2\varepsilon_0}\frac{1}{r} \Rightarrow \int_{V_a}^{V_2} V_{\ 2} = -\frac{\rho a^2}{2\varepsilon_0}\int_a^r \frac{dr}{r} \Rightarrow V_2 - V_a = -\frac{\rho a^2}{2\varepsilon_0}\,[lnr]_a^r = \frac{\rho a^2}{2\varepsilon_0}(ln\frac{r}{a})$$

$$\Rightarrow V_2 = -\frac{\rho a^2}{2\varepsilon_0}\left(ln\frac{r}{a}\right) + V_a$$

$$r = b \Rightarrow V_2(b) = 0 \Rightarrow V_a = \frac{\rho a^2}{2\varepsilon_0}\left(ln\frac{b}{a}\right)$$

$$\Rightarrow V_2 = -\frac{\rho a^2}{2\varepsilon_0}\left(ln\frac{r}{a}\right) + \frac{\rho a^2}{2\varepsilon_0}\left(ln\frac{b}{a}\right) = \frac{\rho a^2}{2\varepsilon_0}\left[-ln\left(\frac{r}{a}\right) + ln(\frac{b}{a})\right]$$

$$= \frac{\rho a^2}{2\varepsilon_0}[-ln(r) + ln(b)]$$

$$\Rightarrow V_2 = \frac{\rho a^2}{2\varepsilon_0}\ ln\frac{b}{r}$$

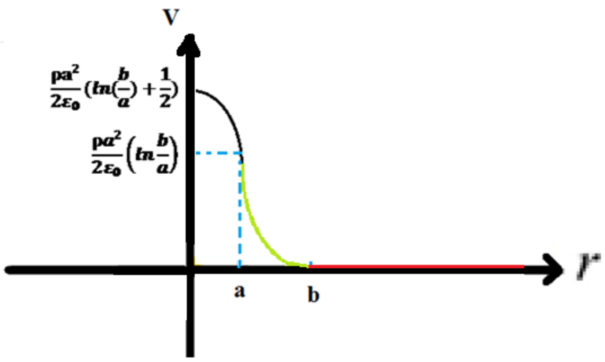
Case 1: r<a

$$E_1 = \frac{\rho}{2\varepsilon_0}\mathrm{r} \Rightarrow V_1 = -\frac{\rho}{4\varepsilon_0}\mathrm{r}^2 + \mathrm{C}_1$$

$$V_2(a) = V_1(a) \Rightarrow \frac{\rho a^2}{2\varepsilon_0}\left(ln\frac{b}{a}\right) = -\frac{\rho}{4\varepsilon_0}\mathrm{a}^2 + \mathrm{C}_1 \Rightarrow \mathrm{C}_1 = \frac{\rho a^2}{2\varepsilon_0}\left(ln\frac{b}{a}\right) + \frac{\rho}{4\varepsilon_0}\mathrm{a}^2$$

$$\Rightarrow C_1 = \frac{\rho a^2}{2\varepsilon_0}\left(ln\left(\frac{b}{a}\right) + \frac{1}{2}\right)$$

$$\Rightarrow V_1 = -\frac{\rho}{4\varepsilon_0}r^2 + \frac{\rho a^2}{2\varepsilon_0}\left(ln\left(\frac{b}{a}\right) + \frac{1}{2}\right) \Rightarrow V_1 = \frac{\rho}{2\varepsilon_0}\left[-\frac{1}{2}r^2 + a^2\left(ln\left(\frac{b}{a}\right) + \frac{1}{2}\right)\right]$$



EXERCISE 08

$\rho = \rho_0 \frac{r}{R}$, where ρ_0 is a constant and r is the distance from the center of the sphere.

1-a- The total charge of the sphere.

$$dQ = \rho\,.\,dV = \rho(4\pi r^2 dr) \Rightarrow \; Q = \int_0^R \rho(4\pi r^2 dr) = \int_0^R \left(\rho_0 \frac{r}{R}\right)(4\pi r^2 dr)$$

$$\Rightarrow \; Q = 4\pi \frac{\rho_0}{R}\int_0^R r^3 dr$$

$$\Rightarrow Q = \pi \rho_0 R^3$$

1-b- The magnitude of the electric field inside the sphere.

$$\emptyset = \oint_S \vec{E}.d\vec{S} = \frac{Q_{enclosed}}{\varepsilon_0}$$

$$\Rightarrow E_2.4\pi r^2 = \frac{4\pi\frac{\rho_0}{R}\int_0^r r^3 dr}{\varepsilon_0} \Rightarrow E_2.4\pi r^2 = \frac{4\pi\frac{\rho_0}{R}\left(\frac{r^4}{4}\right)}{\varepsilon_0} \Rightarrow E_2 = \frac{\rho_0}{4\varepsilon_0 R}r^2$$

$$\Rightarrow E_2 = \frac{\pi\rho_0 R^3}{4\pi\varepsilon_0 R.R^3}r^2 \Rightarrow E_2 = k\frac{Q}{R^4}r^2$$

2- The magnitude of the electric field outside the sphere

$$\emptyset = \oint_S \vec{E}.d\vec{S} = \frac{Q_{enclosed}}{\varepsilon_0}$$

$$\Rightarrow E_2.4\pi r^2 = \frac{Q}{\varepsilon_0} \Rightarrow E_2.4\pi r^2 = \frac{\pi\rho_0 R^3}{\varepsilon_0}$$

$$\Rightarrow E_2 = kQ\frac{1}{r^2}$$

$$\Rightarrow E_2 = \frac{\rho_0 R^3}{4\varepsilon_0}\frac{1}{r^2}$$