Mohamed Boudiaf University of Msila.
Faculty of sciences
Field : Sciences of matter (SM)
1st year LMD   Semester 02.

جامعـة محمد بوضياف ـالمسيلـة
كليـة العلـوم
ميدان : علوم المادة
السنة الأولى ل م د - السداسي 02

Physics 02: Electricity and magnetism

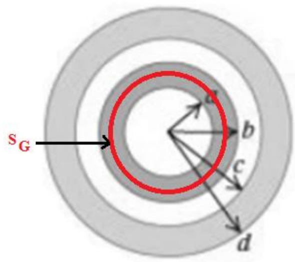
University Year 2023-2024

Series N° 02: CONDUCTORS

EXERCISE 01

1- Determination of the total charge on the inner surface of the small shell.

According to Gauss's law:



$$\emptyset = \oint\!\!\oint \vec{E}.d\vec{S} = \frac{Q_{enclosed}}{\varepsilon_0} = \frac{q_{inner}(SMALL\ SHELL)}{\varepsilon_0}$$

The electric field is zero everywhere inside the conductor

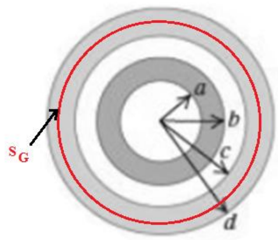
$$\vec{E} = \vec{0} \Rightarrow \emptyset = 0 \Rightarrow \boldsymbol{q_{inner}(SMALL\ SHELL) = 0}$$

2- Determination of the total charge on the outer surface of the small shell.

Since no charge resides on the inner surface of the small shell, the total charge of + $2q$ must reside on its outer surface.

3- Determination of the total charge on the inner surface of the large shell.

According to Gauss's law:



$$\emptyset = \oint\!\!\oint \vec{E}.d\vec{S} = \frac{Q_{enclosed}}{\varepsilon_0} = \frac{q(SMALL\ SHELL) + q_{inner}(LARGE\ SHELL)}{\varepsilon_0}$$

The electric field is zero everywhere inside the conductor

$$\vec{E} = \vec{0} \Rightarrow \emptyset = 0 \Rightarrow Q_{enclosed} = 0$$

$$Q_{enclosed} = q(SMALL\ SHELL) + q_{inner}(LARGE\ SHELL) = 0$$

$$\boldsymbol{\Rightarrow q_{inner}(LARGE\ SHELL) = -q(SMALL\ SHELL) = -2q}$$

4- Determination of the total charge on the outer surface of the large shell.

$$\boldsymbol{q(LARGE\ SHELL) = q_{inner}(LARGE\ SHELL) + q_{outer}(LARGE\ SHELL) = 4q}$$

$$\Rightarrow -2q + q_{outer}(LARGE\ SHELL) = +4q$$

$$\boldsymbol{\Rightarrow q_{outer}(LARGE\ SHELL) = +6q}$$

EXERCISE 03

A parallel–plate capacitor has circular plates of 8.2 cm radius and 1.3 mm separation.

1- Calculation of the capacitance.

For a parallel–plate capacitor: $\boldsymbol{C = \frac{\varepsilon_0 . S}{d}}$

For circular surface: $\boldsymbol{S = \pi r^2 \Rightarrow C = \frac{\varepsilon_0 . \pi r^2}{d} = \frac{8.85 \times 10^{-12} \times \pi \times (8.2 \times 10^{-2})^2}{1.3 \times 10^{-3}}}$

$$\boldsymbol{\Rightarrow C = 1.4 \times 10^{-10}\ F = 140\ pF}$$

$$\mathbf{(1\ pF = 10^{-12}\ F)}$$

2- The appeared charge on the plates if a potential difference of 120 V is applied.

$$\boldsymbol{Q = C\,V = 1.4 \times 10^{-10} \times 120}$$

$$\boldsymbol{\Rightarrow Q = 1.7 \times 10^{-8} C = 17\ nC}$$

$$\mathbf{(1\ nC = 10^{-9}\ C)}$$

EXERCISE 04

The total capacitance C_{eq} between A and B:

Association in series: $\boldsymbol{\frac{1}{C_{eq}} = \sum_{i=1}^{n} \frac{1}{C_i}}$

Association in parallel: $C_{eq} = \sum_{i=1}^{n} \boldsymbol{C_i}$

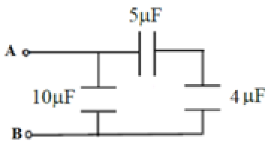


FIG 1

$\Rightarrow$

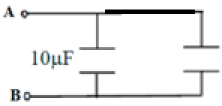


FIG 1

$\frac{4x5}{4+5} = 2.22$ µF $\Rightarrow$

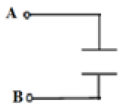


FIG 1

10+2.22= 12.22 µF

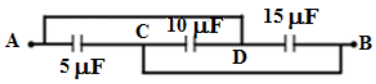


FIG 2

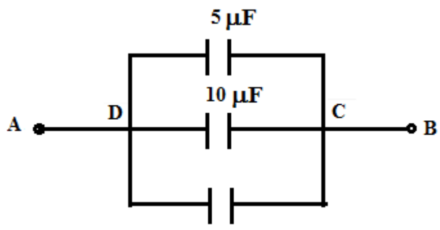
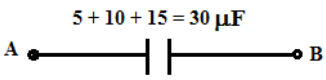


FIG 2

$\Rightarrow$



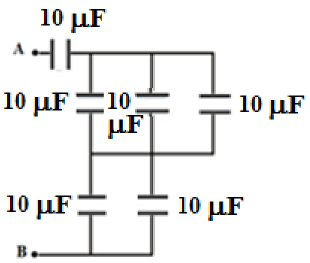


FIG 3

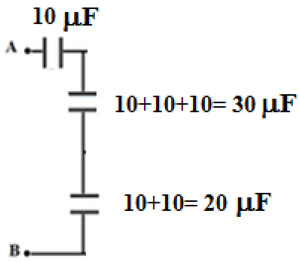
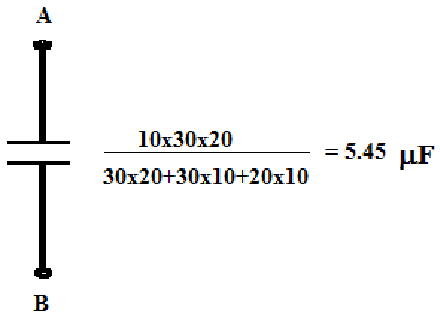


FIG 3

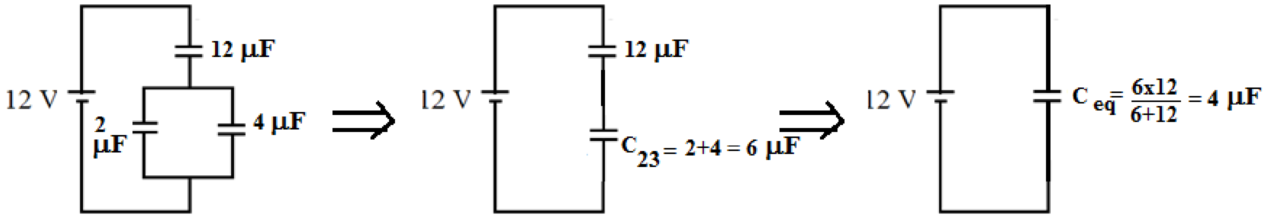


EXERCISE 05

$C_1 = 12\ \mu F$, $C_2 = 2\ \mu F$, $C_3 = 4\ \mu F$, V= 12 V

1- The charge and the voltage across each capacitor.

the equivalent capacitance between points A and B



$$C_{eq} = 4\ \mu\text{F}$$

$$Q = C_{eq}\,V = 4 \times 12 \Rightarrow \ Q = 48\ \mu\text{C}$$

$$\text{Q} = Q_1 = Q_{23} = 48\ \mu\text{C}\ (C_1 \textit{ and } C_{23} \textit{ are in series})$$

$$V_1 = \frac{Q_1}{C_1} = \frac{48}{12} \Rightarrow V_1 = 4\ V$$

$$\text{V} = V_1 + V_{23}\,, \qquad V_{23} = V_2 = V_3, \qquad Q_{23} = Q_2 + Q_3$$

$$\text{V} = V_1 + V_{23} \Rightarrow V_{23} = V - V_1 = 12 - 4 \Rightarrow V_{23} = 8\ V$$

$$\Rightarrow V_{23} = V_2 = V_3 = 8\ V$$

$$Q_2 = C_2 V_2 = 2 \times 8 \Rightarrow Q_2 = 16\ \mu\text{C}$$

$$Q_3 = C_3 V_3 = 4 \times 8 \Rightarrow Q_3 = 32\ \mu\text{C}$$

2-The energy stored in each capacitor.

$$E = \frac{1}{2C}\,Q^2 = \frac{1}{2}\,CV^2$$

So:

$$E_1 = \frac{1}{2}\,C_1 {V_1}^2 = \frac{1}{2}\,12 \times 4^2 = 96\ \mu\text{J}$$

$$E_2 = \frac{1}{2}\,2 \times 8^2 = 64\ \mu\text{J}$$

$$E_3 = \frac{1}{2}\, 4 \times 8^2 = 128\ \mu\text{J}$$

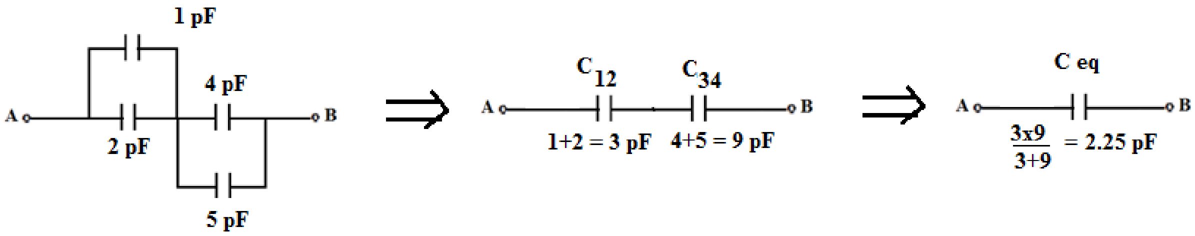
The total energy stored in this network is:

$$E = E_1 + E_2 + E_3 = 288\ \mu\text{J} = \frac{1}{2}\, C_{eq}V^2$$

EXERCISE 06

$C_1 = 1\, pF, C_2 = 2\, pF\ , C_3 = 4\, pF, C_4 = 5\, pF$

1- The equivalent capacitance between points A and B.



$$C_{eq} = 2.25\ \text{pF}$$

2- Calculate the charge on each capacitor if $V_{AB} = 12$ V.

$$Q = C_{eq}\, V = 2.25 \times 12 \Rightarrow\ Q = 27\ \text{pC}$$

$$\text{Q} = Q_{12} = Q_{34} = 27\ \text{pC}\ (C_{12}\ and\ C_{34}\ are\ in\ series)$$

$$\text{V} = V_{12} +\ V_{34}$$

$$V_{12} = V_1 = V_2, \qquad Q_{12} = Q_1 + Q_2$$

$$V_{12} = V_1 = V_2 \Rightarrow \frac{Q_1}{C_1} = \frac{Q_2}{C_2} \Rightarrow Q_1 = \frac{C_1}{C_2} Q_2$$

$$Q_{12} = Q_1 + Q_2 = \frac{C_1}{C_2} Q_2 + Q_2$$

$$\Rightarrow Q_2 = \frac{C_2}{C_1+C_2} Q_{12}$$

$$\Rightarrow Q_2 = \frac{2}{1+2} 27 \Rightarrow\ Q_2 = 18\ \text{pC}$$

$$Q_1 = \frac{C_1}{C_2} Q_2 \Rightarrow Q_1 = \frac{1}{2} 18 \Rightarrow Q_1 = 9\ pC$$

$$V_{34} = V_3 = V_4, \qquad Q_{34} = Q_3 + Q_4$$

$$V_{34} = V_3 = V_4 \Rightarrow \frac{Q_3}{C_3} = \frac{Q_4}{C_4} \Rightarrow Q_3 = \frac{C_3}{C_4} Q_4$$

$$Q_{34} = Q_3 + Q_4 = \frac{C_3}{C_4} Q_4 + Q_4$$

$$\Rightarrow Q_4 = \frac{C_4}{C_3+C_4} Q_{34}$$

$$\Rightarrow Q_4 = \frac{5}{4+5} 27 \Rightarrow \; Q_4 = 15 \text{ pC}$$

$$Q_3 = \frac{C_3}{C_4} Q_4 \Rightarrow Q_3 = \frac{4}{5} 15 \Rightarrow Q_3 = 12 \; pC$$

3- The voltage across each capacitor.

$$V_1 = \frac{Q_1}{C_1} = \frac{9}{1} \Rightarrow V_1 = 9 \; V$$

$$V_2 = \frac{Q_2}{C_2} = \frac{18}{2} \Rightarrow V_2 = 9 \; V$$

$$V_1 = V_2 (C_1 \; and \; C_2 \;\; are \; in \; parallel)$$

$$V_3 = \frac{Q_3}{C_3} = \frac{12}{4} \Rightarrow V_3 = 3 \; pV$$

$$V_4 = \frac{Q_4}{C_4} = \frac{15}{5} \Rightarrow V_1 = 3 \; pV$$

$$V_3 = V_4 (C_3 \; and \; C_4 \;\; are \; in \; parallel)$$

$$V = V_{12} + V_{34} \; (C_{12} \; and \; C_{34} \;\; are \; in \; parallel)$$