Chapter II

LOGIC AND SET

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1. SET THEORY

Set is collection of objects which have same or equal characteristic. The objects in a set are called elements or members of the set

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- **Symbols in set theory**

x ∈ A	x is an element of A; x lies in A, x belongs to A x is in A
x ∉ A	x is not an element of A, x does not lie in A x does not belong to A, x is not in A
x, y ∈ A	(both) x and y are elements of A , …lie in A … belong to A, … are inA
x, y∉ A	(neither) x nor y is an element of A, … lies in A , … belongs to A, … is in A
∅	the empty set (= set with no elements)
A = ∅	A is an empty set
A ≠ ∅	A is non-empty
A=B	A in equal to B
A ⊂ B	A is sub set of B, A is contained in B
A ⊆ B	A is subset of or equal to B
A ⊃ B	A is superset of B, A contains B
A ∪ B	the union of (the sets) A and B, A union B A cup/joint B
A ∩ B	A intersection B, the intersection of (the sets) A and B A cap/meet B
A\B	The difference between A and B
A × B	A times B, the cartesian product of (the sets) A and B, A cross B
A ∩ B = ∅	A is disjoint from B, the intersection of A and B is empty
{x | . . .}	The set of all x such that
C	The set of all complex numbers

Q	The set of all rational numbers
R	The set of all real numbers

Practice

Fill the blank with the right words

1. .... is a set which has no member
2. The number of distinct objects in a set is called the …………… of the set
3. The cardinality of empty set is …………
4. If a set has finite member, we called the set ………… set. Otherwise, we called the set …………… set
5. …………………….. contains those elements that belong to A or to B (or to both)
6. ………………………contains those elements that belong to both A and B
7. ……………………….contains the ordered pairs (a, b), where a (resp., b) belongs to A (resp., to B)
8. …………………………..contains all ordered n-tuples of elements of A

2. LOGIC

- **Symbols in logic theory**

$S \vee T$	S or T, the disjunction of S and T
$S \wedge T$	S and T, the conjunction S and T
$S \Rightarrow T$	S implies T ; if S then T
$S \Leftrightarrow T$	S is equivalent to T ; S if and only if (iff) T
$\neg S$	not S
$\forall x \in A$ ...	for each [= for every=for all] x in A . . .
$\exists x \in A$ . . .	there exists [= there is=for some] an x in A (such that) . . .
$\exists! x \in A$ . . .	there exists [= there is] a unique x in A (such that) . . .
$\nexists x \in A$ . . .	there is no x in A (such that). . .

Examples

$x > 0 \wedge y > 0 \Rightarrow x + y > 0$	if both x and y are positive, so is x + y
$\nexists x \in Q\ x^2 = 2$	no rational number has a square equal to two
$\forall x \in R\ \exists y \in Q\ \|x - y\| < 2/3$	for every real number x there exists a rational number y such that the absolute value of x minus y is smaller than two thirds

Practice

Fill the blank with the right words from the list given below.

negation, true, consequent, disjunction statement, biconditional, statement, implication, disjunction statement.

1. .... is a sentence that is either..................................... or false.
2. If a statement is true, then its .... is false.
3. A compound statement that use the word —or is called ....

4. A compound statement that is true only when statements that make it are true is called ....
5. If the antecedent is true, and the .... is false, then an .... is false.
6. A compound statement which is true if the two statements that combined have the same truth value is called………………

Exercise 1

1. Let A = {a,b,c}. What can you tell about :

- a and c
- f
- {b,c}
- { }
- 8

2. How do we say these mathematical terms?

- $A \setminus (B \cap C) = (A \setminus B) \cup (A \setminus C)$
- $A \subseteq (B \setminus C) \cap E$
- $A = A \setminus (A \setminus B)$
- $A = (A \setminus B) \cup (A \cap B)$
- 5. $(D \setminus E) \cap (D \cap E) = \varnothing$

Exercise 2

Read out the following sentences:

1. $S \Rightarrow (H \wedge U)$
2. $(S \Rightarrow H) \wedge U$
3. $((N \vee G) \wedge (\neg N) \Rightarrow (G \Rightarrow N)$
4. $(P \Rightarrow Q) \wedge (Q \Rightarrow R) \Leftrightarrow (P \Rightarrow R)$
5. $\forall x, Ax \Rightarrow Mx$

$\exists x, \neg Cx \vee Dx$