CHAPTER III: ELECTROKINETICS

I- Introduction

"Electrokinetics" refers to the study of the motion of electric charges.

All matter is composed of atoms. Each atom comprises of a central nucleus with a number of electrons associated with it. Each electron carries a negative charge of electricity and this negative charge is balanced by an equal and opposite positive charge on the nucleus. The atom, as a whole is electrically neutral. Electrons are bound in their orbit by the attraction of the protons (bound electrons). But by the application of some external force such as a magnetic field, friction, or chemical action, the electrons in the outer band can become free of their orbit (free electrons) and they can move from one atom to the next atom. So as result, an electron flow is produced.

II- Electrical Conductor

It is the substance which allows the electric current to pass through it. Metals are conductors because they have "free" electrons, which are not bound to metal atoms.

The substance which does not allow electricity is called non-conductor (insulator).

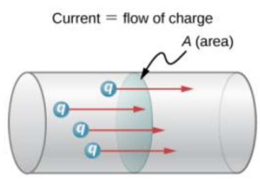
An insulator around the outside of the copper conductor is provided to keep electrons in the conductor.

III- Electric Current

It is defined to be the rate at which charges flow in the same general direction: charge per unit time that flows through an area. It is designated by the symbol "I". The current is measured in amperes "A" and it is defined to be:

- **average current :** $I_{avg} = \frac{\Delta q}{\Delta t}$
- **instantanuous current :** $I = \frac{dq}{dt}$

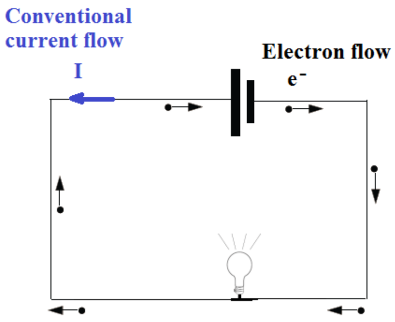
$$1\ A = \frac{1\ C}{1\ S}$$



dq is the amount of charge passing through a given area in time dt.

- Direction of Current

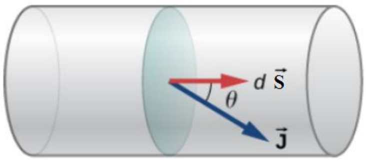
The direction of the current is defined as the direction in which a positive charge would move. So, the current flows from positive to negative. However, the electrons flow from negative to positive.



An electron flowing from – to + gives rise to the same "conventional current" as a proton flowing from + to -

IV- Current Density

The current density $\vec{J}$ is current per area: charge per area and time. It is a vector quantity. The direction of the current-density vector J is the direction of motion of positive charge and hence of the current, and is opposite to the direction of motion of negative charge.

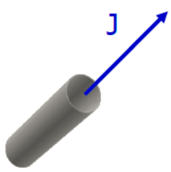


unit of J: A/m^2

$$dI = \vec{J}.d\vec{S} = J.dS.\cos\theta$$

$$\Rightarrow I = \iint_{Area} \vec{J}.d\vec{S}$$

A cross section A of wire:



$$dI = \vec{J}.d\vec{S} = J.dS \Rightarrow J = \frac{dI}{dS}$$

$$I = \iint_{Area} \vec{J}.d\vec{S} = J.S \Rightarrow J = \frac{I}{S}$$

During a time Δt free charges move a distance: $d = v_d\,\Delta t$ (v: drift velocity of the charges)

The volume of charge that moves past a point is: $V = S.d = S.\,v_d\,.\Delta t$

$$\text{Number of free charges in the volume} = \text{number of charges per volume (n)} \times \text{volume (V)}$$

$$N = n\,V = n.\,S.\,v_d\,.\Delta t$$

$$\text{Total charge of free charges} = \text{number of charges (N)} \times \text{charge per carrier(q)}$$

$$\Delta\, q = N.q = n.S.v_d\,.\Delta\, t.q$$

$$\Rightarrow I_{avg} = \frac{\Delta q}{\Delta t} = \frac{n.S.v_d\,.\Delta\, t.q}{\Delta t}$$

$$\Rightarrow \mathbf{I_{avg} = n.q.S.v_d}$$

$$\Rightarrow \mathbf{I = n.q.S.v_d}$$

$$\mathbf{J = \frac{I}{S} \Rightarrow J = n.q.v_d}$$

$$\mathbf{\Rightarrow \vec{J} = n.q.\overrightarrow{v_d}}$$

If the charge carriers are electrons, **q= - e** so that:

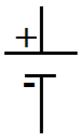
$$\mathbf{\vec{J} = -n.e.\overrightarrow{v_d}}$$

The (–) sign demonstrates that the velocity of the electrons is antiparallel to the conventional current direction.

V- Voltage

The force required to make electicity flow through a conductor is called a difference in potential, electromotive force (emf), or more simply referred to as voltage. Voltage is designated by the letter “E”, or the letter “V”.

The unit of measurement for voltage is volts which is also designated by the letter “V”

Voltage Circuit Symbol: 

VI- Charge conservation and continuity

Suppose the current flows through a closed surface. The continuity equation:

$$\vec{\nabla}.\vec{J} + \frac{\partial \rho}{\partial t} = 0 \Rightarrow \mathrm{div}\,\vec{J} + \frac{\partial \rho}{\partial t} = 0$$

It guarantees conservation of electric charge in the presence of currents.

When current is steady with time, ρ is constant: $\frac{\partial \rho}{\partial t} = 0 \Rightarrow \mathrm{div}\,\vec{J} = 0$

VII- Ohm's law

VII.1- Microscopic Ohm’s law

Electric fields cause charges to move. So, an electric field applied to some material will cause currents to flow in that material. The relation between current density and applied electric field is:

$$\mathbf{\vec{J} = \sigma_c \vec{E}}$$

σ_c : material's electrical conductivity.

The conductivity can be expressed as:

$$\boldsymbol{\sigma_c = \frac{n.e^2.\tau}{m_e}}$$

n: is the number of charges per unit volume

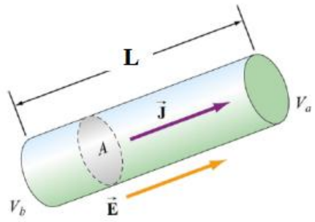
τ: is the average characteristic time between successive collisions.

Unit of σ_c: $\frac{A/m^2}{V/m} = \frac{A}{V.m} = \frac{1}{\Omega\, m}$

The electrical resistivity is : $\rho = \frac{1}{\sigma_c}$ (ρ is not volume density)

VII.2- Resistance

Suppose a potential difference $\Delta V = V_b - V_a$ between the ends of the wire, creating an electric field **E** and a current I. Assuming $\vec{E}$ to be uniform, we then have:



$\vec{E} = -\overrightarrow{grad}\ V \Rightarrow dv = -E\, dL \Rightarrow V_a - V_b = -E.L$

$$\Rightarrow \Delta V = V_b - V_a = E.L$$

$$J = \sigma_c E$$

$$\Rightarrow J = \sigma_c \frac{\Delta V}{L}$$

$$J = \frac{I}{S} \Rightarrow \frac{I}{S} = \sigma_c \frac{\Delta V}{L}$$

$\Rightarrow \Delta V = \frac{L}{\sigma_c . S} I \Rightarrow \mathbf{\Delta V = R.I}$ (global or "macroscopic" form of Ohm's law)

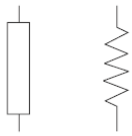
$$\mathbf{R = \frac{\Delta V}{I} = \frac{L}{\sigma_c . S}}$$

R is the resistance of the conductor. Resistance is the opposition to the flow of electricity within a circuit. The SI unit of R is Ohm (Ω)

$$\rho = \frac{1}{\sigma_c} = \frac{S.R}{L} \Rightarrow \mathbf{R = \frac{\rho.L}{S}}$$

We call the conductance G: $\mathbf{G = \frac{1}{R}}$

- **Resistance Circuit Symbols:**

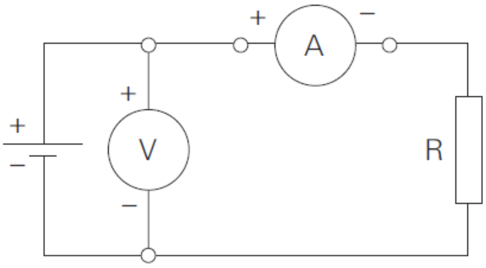


VII.3- Electric Circuit

It presents a fundamental relationship exists between current, voltage, and resistance. A simple electric circuit consists of a voltage source (battery), some type of load (light (resistance)), and a conductor (electrical wire)t o allow electrons to flow between the voltage source and the load.

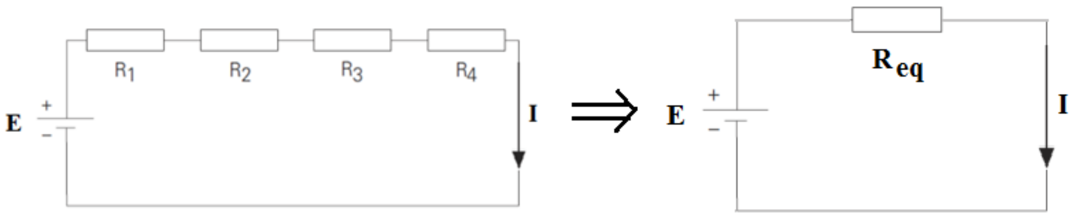
An additional component has been added to this circuit, a switch. If the switch is open, the path is incomplete and the lamp will not illuminate. Closing the switch completes the path, allowing electrons to leave the negative terminal and flow through the light to the positive terminal.

- To measure the current, we use the amperemeter, connected in series with the circuit.
- To measure the electric potential difference between two points, we use voltmeter, connected in parallel.



VII.4- Resistance in a series Circuit

A series circuit is formed when any number of resistors are connected end to end so that there is only one path for current to flow. The resistors can be actual resistors or other devices that have resistance.



The mathematical formula for equivalent resistance in series is:

$$R_{eq} = \sum_{i=1}^{n} R_i$$

$$R_{eq} = R_1 + R_2 + R_3 + R_4$$

Current in a Series Circuit: The Current is the same anywhere it is measured in a series circuit.

$$I = \frac{E}{R_{eq}}$$

Voltage in a Series Circuit: the total voltage applied to the circuit equals the sum of the voltage across the resistances of a closed circuit.

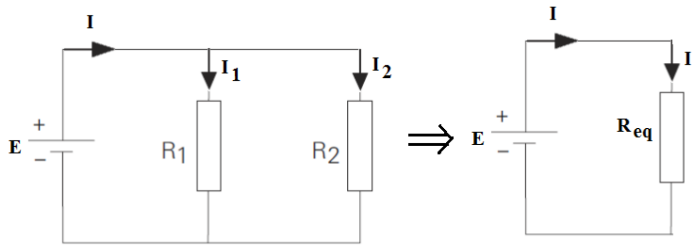
$$E = \sum_{i=1}^{n} V_i$$

$$E = V_1 + V_2 + V_3 + V_4$$

$$V_i = R_i I$$

VII.5- Resistance in a Parallel Circuit

A parallel circuit is formed when two or more resistances are placed in a circuit side-by-side. So, the current can flow through more than one path.



Formula for Unequal There are two formulas to determine total resistance for

Resistors in a Parallel Circuit: $\frac{1}{R_{eq}} = \sum_{i=1}^{n} \frac{1}{R_i}$

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2}$$

Voltage in a Parallel Circuit: the voltage is the same across each resistor.

$$E = V_1 = V_2 = V_3 = \cdots V_n$$

Current in a Parallel Circuit: Current flowing through a parallel circuit divides and flows through each branch of the circuit. Total current in a parallel circuit is equal to the sum of the current in each branch.

$$I = I_1 + I_2 + I_3 + \cdots + I_n$$

In the presented circuit: $I = I_1 + I_2$

VIII- Joule's law

Resistors of all types, like any electrical or electronic component possessing resistance, exhibit power dissipation when electric current travels through. Power is defined as the rate of energy transfer of energy from electric charge carriers to heat as those charge carriers pass through the device. Power is symbolized by the variable P .

Its unit of measurement (Watt) is symbolized by the letter W, a Watt being defined as one Joule of energy transferred per second of time.

$$P = E.I = R.I^2 = \frac{E^2}{R} \text{ (Joule's Law)}$$

IX- Applications of Ohm's Law

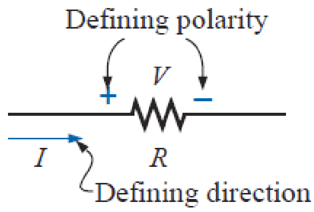
The main applications of Ohm's law are:

- To determine the voltage, resistance or current of an electric circuit.
- Ohm's law maintains the desired voltage drop across the electronic components.
- Ohm's law is also used in DC ammeter and other DC shunts to divert the current.

Remark

For an isolated resistive element, the polarity of the voltage drop is as shown in the opposite figure for the indicated current direction. In general, the flow of charge is from a high (+) to a low (-) potential.

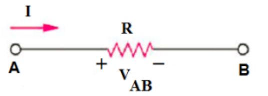
A reversal in current will reverse the polarity.



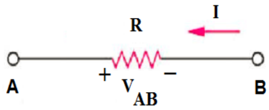
(a) (b)

Sign convention

$$V_{AB} = V_A - V_B = RI$$



$$V_{AB} = V_A - V_B = -RI$$



X- Kirchoff's law

X.1- Definition:

a- Node: is a place on the circuit where two or more circuit elements join.

b- Branch: is the portion of the circuit between two nodes.

c- Loop: is any closed path in a circuit.

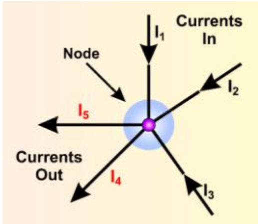
X.2- Kirchhoff's Current Law

The algebraic sum of all currents at any node in a circuit is equal to zero.

Mathematically: $\sum_{n=1}^{N} I_n = 0 \qquad (\sum_{n=1}^{N} I_n \text{ (In)} = \sum_{m=1}^{M} I_m \text{ (Out)})$

The word “algebraic” means that we must associate the proper sign with each of the currents. That A current entering a node will get a positive (+) sign when used in KCL; a current leaving a node will get a negative sign (-).

Example



$$I_1 + I_2 + I_3 + (-I_4 + (-I_5)) = 0$$

Or $\quad : I_1 + I_2 + I_3 = I_4 + I_5$

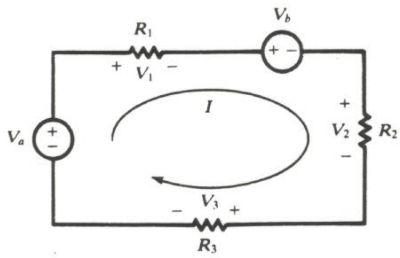
X.3- Kirchhoff’s Voltage Law

Kirchhoff’s voltage law (KVL) states that the algebraic sum of all voltages around a closed path (or loop) is zero.

Mathematically: $\quad \sum_{n=1}^{N} V_n = 0$

Example

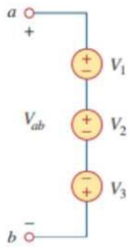
a- Applying the KVL equation for the circuit of the following figure:



$$-V_a + R_1 I + V_b + R_2 I + R_3 I = 0$$

Or: $V_a = R_1 I + V_b + R_2 I + R_3 I$

b- KVL is used to combine the voltage sources present in series.



$$-V_{ab} + V_1 + V_2 - V_3 = 0$$

$$\Rightarrow V_{ab} = V_1 + V_2 - V_3$$

XI- Limitations of Ohm’s Law

Following are the limitations of Ohm’s law:

- Ohm’s law is not applicable for unilateral electrical elements like diodes and transistors as they allow the current to flow through in one direction only.
- For non-linear electrical elements with parameters like capacitance, resistance etc the ratio of voltage and current won’t be constant with respect to time making it difficult to use Ohm’s law.