Physics 02: Electricity and magnetism

University Year 2023-2024

Series N° 03: ELECTROKINETICS

EXERCISE 01

- The radius of the cable (r).

We have: $V = R.I \Rightarrow R = \frac{V}{I} = \frac{1.6\times10^{-2}}{1200} \Rightarrow \boldsymbol{R = 1.33\times10^{-5}\Omega}$ **(R: resistance)**

$R = \rho\frac{L}{S} \Rightarrow S = \frac{\rho L}{R} \Rightarrow S = \frac{1.72\times10^{-8}\times0.24}{1.33\times10^{-5}} \Rightarrow \boldsymbol{S = 3.1\times10^{-4}\,m^2}$ **(S: surface)**

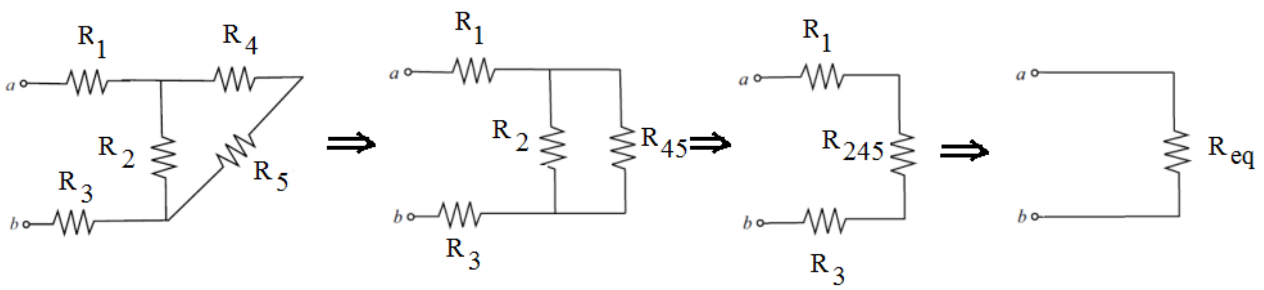
The cable has a circular cross-section, so that:

$$S = \pi r^2$$

$$\Rightarrow r = \sqrt{\frac{S}{\pi}} = \sqrt{\frac{3.1\times10^{-4}}{\pi}} \Rightarrow \boldsymbol{r = 9.93\times10^{-3}\,m = 9.93\,mm}$$

EXERCISE 02

The equivalent resistance between points a and b.



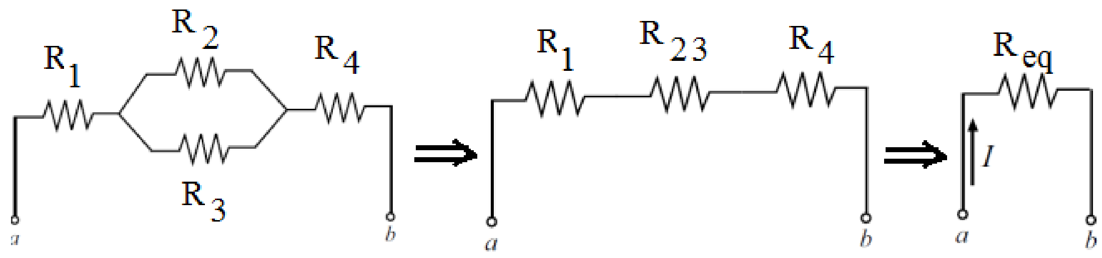
R_4 and R_5 are in series: $R_{45} = R_4 + R_5 = 5.1 + 3.5 \Rightarrow \boldsymbol{R_{45} = 8.6\,\Omega}$

R_{45} and R_2 are in parallel: $\frac{1}{R_{245}} = \frac{1}{R_{45}} + \frac{1}{R_2} = \frac{1}{8.6} + \frac{1}{1.8} \Rightarrow \boldsymbol{R_{245} = 1.5\,\Omega}$

R_1, R_{245} and R_3 are in series: $R_{eq} = R_1 + R_{245} + R_3 = 2.4 + 1.5 + 3.6 \Rightarrow \boldsymbol{R_{eq} = 7.5\,\Omega}$

EXERCISE 03

1- The equivalent resistance between points a and b



R_2 and R_3 are in parallel: $\frac{1}{R_{23}} = \frac{1}{R_2} + \frac{1}{R_3} = \frac{1}{7} + \frac{1}{10} \Rightarrow \boldsymbol{R_{23} = 4.12\ \Omega}$

$R_1,\ R_{23}$ and R_4 are in series: $R_{eq} = R_1 +\ R_{23} + R_4 = 4 + 4.12 + 9 \Rightarrow \boldsymbol{R_{eq} = 17.12\ \Omega}$

2- The current in each resistor if a potential difference of 34.0V is applied between points a and b

$$\boldsymbol{V_{ab} = R_{eq} I \Rightarrow I = \frac{V_{ab}}{R_{eq}} = \frac{34}{17.12} \Rightarrow I = 1.99\ A}$$

$R_1,\ R_{23}$ and R_4 are in series: **I=**$\boldsymbol{I_1 = I_{23} = I_4 = 2\ A}$

$$\boldsymbol{V_{ab} = V_1 + V_{23} + V_4}$$

$$V_1 = R_1.I_1 = 4 \times 1.99 = \boldsymbol{7.96\ V}$$

$$V_4 = R_4.I_4 = 9 \times 1.99 = \boldsymbol{17.90\ V}$$

$$\Rightarrow$$

$$V_{23} = V_{ab} - (V_1 + V_4) = 34 - (7.96 + 17.9) = \boldsymbol{8.1\ V}$$

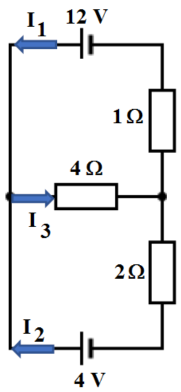
$$\boldsymbol{V_{23} = V_2 = V_3 = 8.1\ \ V}$$

$$V_2 = R_2.I_2 \Rightarrow I_2 = \frac{V_2}{R_2} = \frac{8.1}{7} \Rightarrow \boldsymbol{I_2 = 1.2\ A}$$

$$V_3 = R_3.I_3 \Rightarrow I_3 = \frac{V_3}{R_3} = \frac{8.1}{10} \Rightarrow \boldsymbol{I_2 = 0.8\ A}$$

EXERCISE 04

The currents I_1, I_2 and I_3.



According to **Kirchhoff's Current Law:**

$\boldsymbol{I_1 + I_2 = I_3}$………………..eq.1

According to **Kirchhoff's Voltage Law:**

From loop 1:

$\boldsymbol{12\ V = \ 4\ I_3 +\ 1 I_1}$ ……………………..eq.2

From loop 2:

$\boldsymbol{4\ V = \ 4\ I_3 +\ 2 I_2}$ ……………………….eq.3

eq.2 $\Rightarrow I_1 = 12 -\ 4\ I_3$

eq.3 $\Rightarrow I_2 = \frac{4 - 4\ I_3}{2}\ \Rightarrow\ I_2 = 2 -\ 2\ I_3$

$$I_1 + I_2 = I_3$$

$$\Rightarrow 12 - 4\,I_3 + 2 - 2\,I_3 = I_3$$

$\Rightarrow 14 = 7\,I_3$

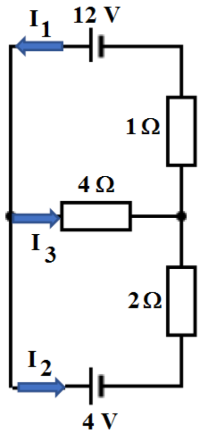
$$\Rightarrow \boldsymbol{I_3 = 2\ A}$$

$$I_1 = 12 - 4\,I_3 = 12 - 4 \times 2 \Rightarrow \boldsymbol{I_1 = 4\ \mathrm{A}}$$

$$I_2 = 2 - 2\,I_3 = 2 - 2 \times 2 \Rightarrow \boldsymbol{I_2 = -2\ \mathrm{A}}$$

I_2 is in the opposite chosen direction

$$\boldsymbol{I_1 = I_2 + I_3}$$



EXERCISE 05

The current flowing in the network.

We assume that the directions of currents are as follow:

According to Kirchhoff's Current Law:

$\boldsymbol{I_1 = I_2 + I_3}$………………..eq.1

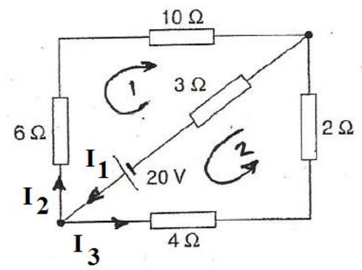
According to Kirchhoff's Voltage Law:

From loop 1:

$\mathbf{20\ V} = \boldsymbol{6\,I_2 + 10\,I_2 + 3\,I_1}$ ……………………..eq.2

From loop 2:

$\mathbf{20\ V} = \boldsymbol{4\,I_3 + 2I_3 + 3\,I_1}$……………………….eq.3



eq.2 $\Rightarrow 20 = 16I_2 + 3\,I_1 \Rightarrow I_2 = \frac{20-3I_1}{16}$

eq.3 $\Rightarrow 20 = 6\,I_3 + 3\,I_1 \Rightarrow I_3 = \frac{20-3I_1}{6}$

$$\boldsymbol{I_1 = I_2 + I_3}$$

$$\Rightarrow I_1 = \frac{20 - 3I_1}{16} + \frac{20 - 3I_1}{6}$$

$$\Rightarrow I_1 = \frac{6 \times (20 - 3I_1) + 16 \times (20 - 3I_1)}{16 \times 6}$$

$$\Rightarrow I_1 = \frac{6 \times 20 - 6 \times 3I_1 + 16 \times 20 - 16 \times 3I_1}{16 \times 6}$$

$\Rightarrow I_1 = \frac{440-66\,I_1}{96} \Rightarrow 96\,I_1 + 66\,I_1 = 440 \Rightarrow 162\,I_1 = 440 \Rightarrow I_1 = \frac{440}{162} \quad \Rightarrow \boldsymbol{I_1 = 2.72\,A}$

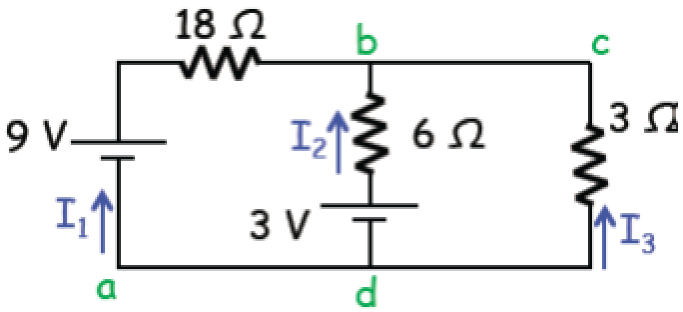
$$I_2 = \frac{20-3I_1}{16}$$

$$\Rightarrow I_2 = \frac{20-3\times 2.72}{16} \quad \Rightarrow \boldsymbol{I_2 = 0.74\,A}$$

$I_1 = I_2 + I_3 \Rightarrow I_3 = I_1 - I_2 = 2.72 - 0.74 \Rightarrow \boldsymbol{I_3 = 1.98\,A}$

EXERCISE 06

The current through each branch.



According to Kirchhoff's Current Law:

$\boldsymbol{I_1 + I_2 + I_3 = 0}$……………….eq.1

According to Kirchhoff's Voltage Law:

From loop 1:

$\mathbf{9\,V = 18\,I_1 - 6\,I_2 + 3\,V}$ ……………………..eq.2

From loop 2:

$\mathbf{3\,V = 6\,I_2 - 3I_3}$ ……..……………………….eq.3

eq.2 $\Rightarrow 6 = 18I_1 - 6\,I_2 \Rightarrow I_1 = \frac{6+6I_2}{18} \Rightarrow I_1 = \frac{1+I_2}{3}$

eq.3 $\Rightarrow 3 = 6\,I_2 - 3\,I_3 \Rightarrow I_3 = \frac{-3+6I_2}{3}$

$$\boldsymbol{I_1 + I_2 + I_3 = 0}$$

$$\Rightarrow \frac{1+I_2}{3} + \boldsymbol{I_2} + \frac{-3+6I_2}{3} = \mathbf{0}$$

$$\Rightarrow \frac{1+\boldsymbol{I_2}+3\boldsymbol{I_2}-3+6I_2}{18} = 0 \Rightarrow \boldsymbol{I_2} = \frac{\mathbf{2}}{\mathbf{10}} \Rightarrow \boldsymbol{I_2 = 0.2\,A = 200\,mA}$$

$I_1 = \frac{1+I_2}{3} \Rightarrow I_1 = \frac{1+0.2}{3} \quad \Rightarrow \boldsymbol{I_1 = 0.4\,A = 400\,mA}$

$I_3 = \frac{-3+6I_2}{3} \Rightarrow I_3 = \frac{-3+6\times 0.2}{3} \quad \Rightarrow \boldsymbol{I_3 = -0.6\,A = -600\,mA}$

$\boldsymbol{I_3}$: is in the opposite direction: $\boldsymbol{I_1 + I_2 = I_3}$