# CHAPTER 01: ELECTROSTATICS

## I-1 INTRDUCTION

*Electrostatics is a branch of physics that deals with the study of stationary electric charges and the behavior of objects that are electrically charged but are not in motion. It primarily focuses on understanding the forces and effects produced by electric charges at rest.*

*The foundation of electrostatics lies in the concept of electric charge, which is a fundamental property of matter. There are two kinds of electric charges – positive and negative. Like charges repel each other, while opposite charges attract each other according to Coulomb's law. This law states that the force between two charged objects is directly proportional to the product of their charges and inversely proportional to the square of the distance between them.*

*One important phenomenon in electrostatics is the concept of electric fields. An electric field is the region around a charged object where its influence can be felt. Electric fields are produced by electric charges and exert forces on other charged objects placed within the field. The strength and direction of an electric field at any point depend on the magnitude and sign of the charges present.*

*Electrostatics also includes the study of electric potential and potential difference. Electric potential is the amount of work done in bringing a unit positive charge from infinity to a point in an electric field. Potential difference, on the other hand, is the difference in electric potential between two points in an electric field, often measured in volts. These concepts are crucial in understanding the behavior of charged particles and electric circuits.*

*The practical applications of electrostatics are widespread. They range from everyday phenomena, such as the attraction or repulsion of objects after rubbing them together, to more complex applications like the operation of electrostatic precipitators used in air pollution control, inkjet printers, and the technology behind Van de Graaff generators.*

*In summary, electrostatics is a branch of physics that studies the behavior of electric charges at rest, exploring the forces and effects they produce. Understanding this field is vital for various practical applications and provides the foundation for further exploration of electromagnetism.*

## I-2 CONCEPT OF CHARGE

*All matter is made of certain elementary particles, the three most common are electrons, protons and neutrons. While the proton and neutrons are tightly confined within the nucleus only, the electrons are loosely bound in atom and are in a cloud around a* *nucleus. The protons have a positive charge, the electrons have a negative charge whereas the neutrons are neutral.*

*The charge is an intrinsic property that allowed the body to attract a smalls piece like a paper, by certain force called electric force. Like gravitational force where two masses are attracted each other. The charge is an 'electric mass'. There are two types of charge, negative and positive*

*If we a rod of glass is rubbed on a silk cloth, it acquires charge and attract a fragment of paper, this attraction is due to this acquired charge.*

## I-3 EXPERIENCES

### A- Experience 01

*When the rod of glass is rubbed to a silk cloth, it acquires a charge. Taking another rod of glass and rubbing it in the same manner. When we bring them to vicinity of each other, they will repel*

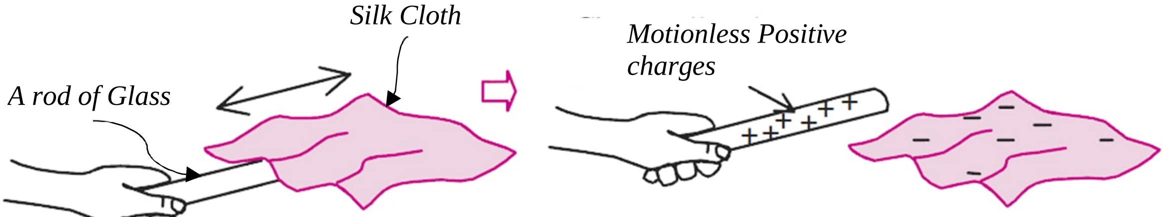



*Fig.1-a*

*The glass rods have the same type of charge, it repels each other*

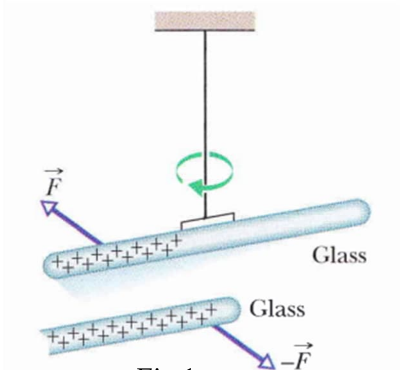



*Fig.1-*

### B- Experience 02

*When the rod of plastic is rubbed to a fur, it acquires a charge. Taking another rod of plastic and rubbing it in the same manner. When we bring them to vicinity of each other, they will repel*

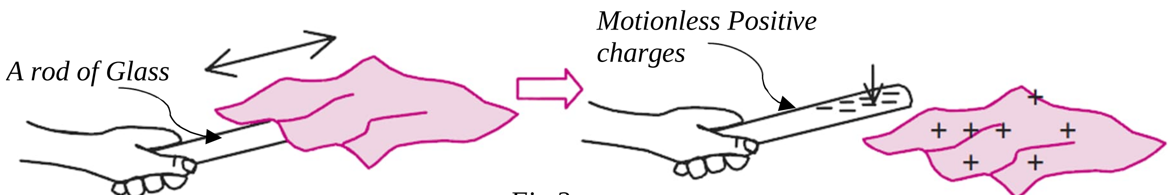



*Fig.2-a*

The plastic rods have the same type of charge, it repels each other

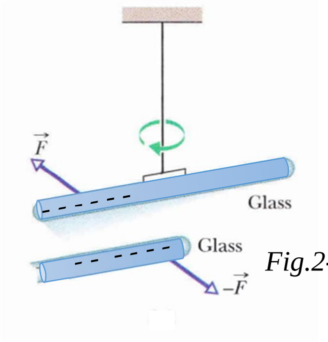


Fig.2-b

## C-Experience 03

*When the rod of plastic is rubbed to a fur, it acquires a charge. Taking another rod of glass and rubbing it to the silk cloth. When we bring them to vicinity of each other, they will attract*

Fig.3-a

*The plastic rods and the glass rod have different types of charge, it attracts each other*

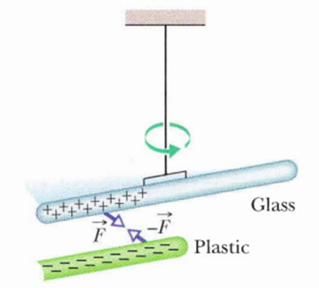


Fig.3-b

<u>Result</u>

*From these three experiences, we deduce that the charges are present in two kinds. like charges repel each other and unlike charges attract each other.*

## D- Experience 04

*If we rub a plastic rod to fur, it acquires a negative charge, same, if we take a glass rod and rub it to the silk, it acquires a positive charge. If we touch a small neutral conductor ball with one of the rods, that ball will be attracted for few second then it will repelled. This can be explained by the fact, that during a contact there is a transfer of charges from the rod to the ball. After that the excess charge will be distributed on the surface, and the two body will be identically charged. So, they repel each other.*

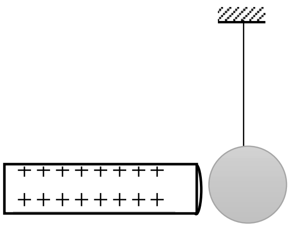


Fig.4-a

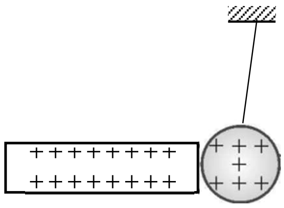


Fig.4-b

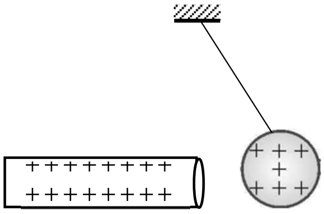
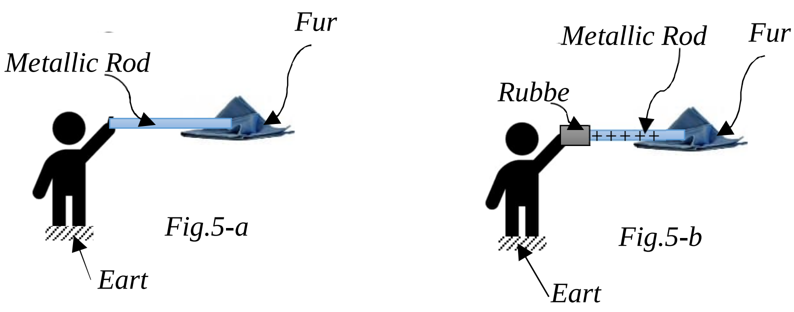


Fig.4-c

## E-Experience 05

*We try to electrify a metallic rod when we our feet touch a ground. The rod cannot be electrified, because the excess charge will flow from the rod to the earth and conversely,* *it depends on the kind of charge we put on it, and it hold neutral. But if we take it with rubber or nylon, then the charge cannot flow to or from the earth and the rod can be electrified*




*The insulator can be easily charged. Because the charge putting on the body remains in the location of contact and don't spread along the body like metal.*

## I-4 METHOD OF CHARGING (ELECTRIFYING)

*It exists several ways to electrify the body by any kind of charges*

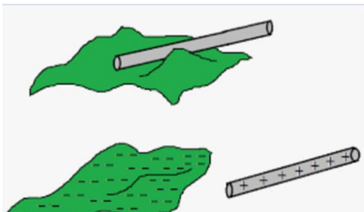


*Fig.6*

### I-4-1 CHARGING BY FRICTION

*In this method, when two bodies are rubbed together some electrons are transferred from one to other. As result, one body becomes positively charged while the other becomes negatively charged.*

### I-4-2 CHARGING BY CONDUCTION (BY CONTACT)

*When a charged body is in direct contact with another neutral body, some of the excess of charges will be transferred from the uncharged body to the charged body until they reach certain equilibrium (same potential for metal)*

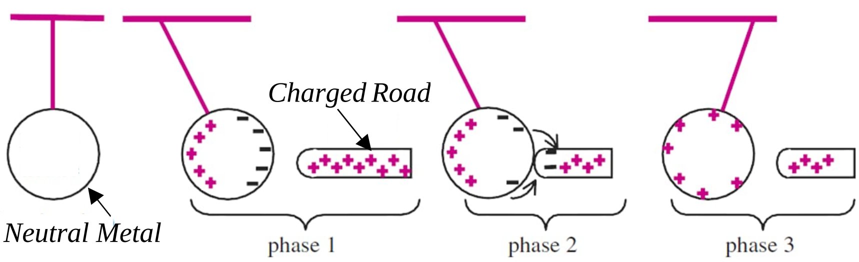



*Fig.*

## I-4-3 CHARGING BY IDUCTION (NO CONTACT)

*Whenever a charged body (conductor or insulator) is brought near a neutral conductor, the charged body will attract the opposite charge and repel similar charge present in the neutral body. This process is called induction. When the neutral conductor is connected to the earth, the charge repelled will be flown within. After removal the connected wire to earth first, we remove, then, the charged body. Initially neutral, the body will be charged. This way of charging is called electrifying by induction.*

*Notice that during electrification, only electrons are involved.*

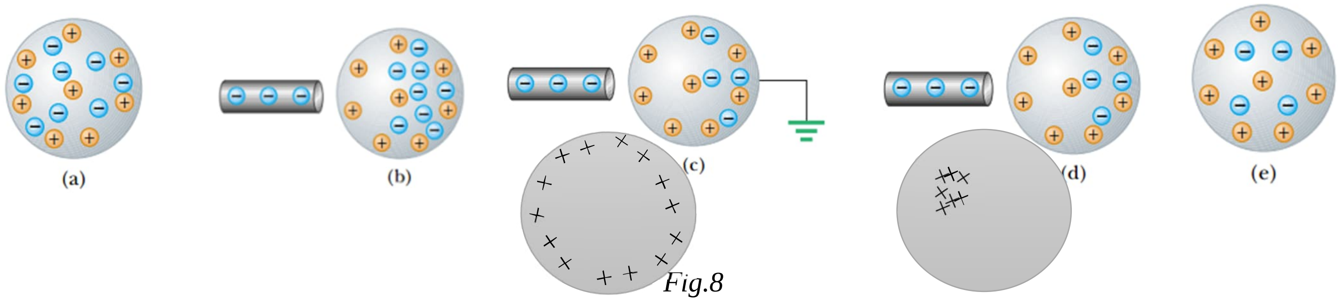



*Fig.8*

*Conducto* *Fig.9* *Insulato*

*When we put the charge on the body. If it is localized in the point of contact the body is an insulator. But if the charge is spread on the surface, the body is a conductor*

## I-5 PROPERTY OF CHARGE

1- *The charge is quantized. The charge of anybody or that transferred is equal to basic unit of charge, denoted '* $\boldsymbol{e = 1.9\ 10^{-19}\ C}$*, or its integral multiples '*$\boldsymbol{Q = ne}$*'. The unit of charge are the COULOMB.*

   $\boldsymbol{Q}$*: is the charge*

   $\boldsymbol{n}$*: is an integer* $\boldsymbol{n = \mp 1, \mp 2, \mp 3 \mp \cdots}$

2- *In an isolated system, the charge for two bodies in interaction, is conserved. No charges are created or destroyed. The amount of charge which is transferred is same. The charge given by one body equals the charge received by the other body. Within an isolated system consisting of many charged bodies, due to interactions among the bodies, charges may get redistributed but it is found that the total charge of the isolated system is always conserved. Conservation of charge has been established experimentally.*

3- *The charge is present in two kinds. Negative and positive charge (*$+\boldsymbol{Q}$; $-\boldsymbol{Q}$*)*

4- *The charge is a scalar quantity, so, we can add them algebraically.*

$$\boldsymbol{Q_1 = n\,e\ ;\ Q_2 = m.e \quad \Rightarrow \quad Q_T = Q_1 + Q_2 = n\,e + m\,e = (n + m)e}$$

5- *Charge is relativistically invariant. That is,* $q_{rest} = q_{motion}$.

*6- Moving charge produces magnetic field in addition to electric field.*

7- Accelerated charge radiates energy.

*Example*

*If $10^9$ electrons move out of a body to another body every second, how much time is required to get a total charge of Q =1 C on the other body?*

*Solution*

*In one second $10^9$ electrons move out of the body.*

*Therefore, the charge given out in one second is*

$$\boldsymbol{Q = 1{,}6\ 10^{-19} \times 10^{9}\,C = 1{,}6 \times 10^{-10}\,C}$$

*The time required to accumulate a charge of 1C can then be estimated to be:*

$$\boldsymbol{t = \frac{1C}{(1.6 \times 10^{-10}\,C/s)} = 6.25 \times 10^{9}\,s = 6.25 \times 10^{9}}$$

*In years*

$$\boldsymbol{t = \frac{6.25 \times 10^{9}\,s}{(365 \times 24 \times 3600)} = 198}\ years$$

*Thus, to collect a charge of one coulomb, from a body from which $10^9$ electrons move out every second, we will need approximately 200 years. One coulomb is, therefore, a very large unit for many practical purposes.*

*It is, however, also important to know what is roughly the number of electrons contained in a piece of one cubic centimeter of a material. A cubic piece of copper of side 1 cm contains about $2.5 \times 10^{24}$ electrons.*

## I-6 CHARGE DISTRIBUTION

*Sometimes we observe that the charge, instead of being point charge it is distributed over the entire. Such distribution can be uniform, random, or following a certain law. Depending on the extent, this distribution can be linear or surface or volume distribution.*

### I-6-1 DISCREET DISTRIBUTION

*When the charges are discreetly distributed, or there exist a set of point charge confined on whatever the extent of the body, the total charge is equal to the sum of all the locals point charges.*

$$\boldsymbol{Q = \sum_{i=1}^{n} q_i}$$

### I-6-2 CONTINUOUS DISTRIBUTION

*When the charge is continuously distributed, the total charge depends on the shape of body and its local distribution*

## A-LINEAR DISTRIBUTION

*Let the charge distributed along a line with a density '$\boldsymbol{\lambda}$', the total charge is given by the sum of all elements of charge of that body (example a very thin rod of length l)*

*To calculate the total charge, we determine the element of charge '$\boldsymbol{dq}$', and sum all this element along the length '$\boldsymbol{L}$'*

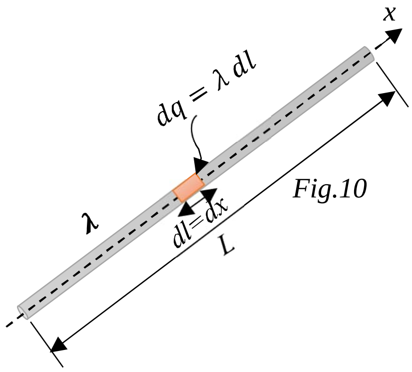


Fig.10

*The element of length '$\boldsymbol{dl}$' has an element of charge*

$$\boldsymbol{dq = \lambda\, dl}$$

*The charge of all body is given by:*

$$\boldsymbol{Q} = \int_0^{\boldsymbol{Q}} \boldsymbol{dq} = \int_0^{\boldsymbol{L}} \boldsymbol{\lambda\, dl} = \int_0^{\boldsymbol{L}} \boldsymbol{\lambda\, dx}$$

$\boldsymbol{Q}$*: Total charge of the body*

$\boldsymbol{\lambda}$*: Linear charge density (charge per unit surface)*

$\boldsymbol{L}$*: The length of the charged body*

*Notice that the body can be any curved form, so we integrate along that curve.*

## B-SURFACE DISTRIBUTION

*The charge distributed over a surface with a density '$\boldsymbol{\sigma}$', the total charge is given by the sum of all elements of charge of that body (example a very thin sheet)*

*To calculate the total charge, we determine the element of charge '$\boldsymbol{dq}$', and sum all this element along the surface '$\boldsymbol{S}$'*

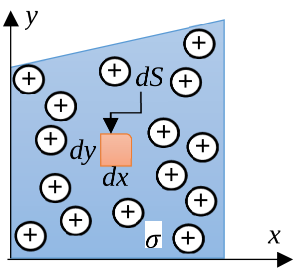


Fig.11

*The surface element '$\boldsymbol{dS}$' has an element charge '$\boldsymbol{dq = \sigma\, dS}$'*

$$\boldsymbol{dS = dx\, dy}$$

*The charge of all body is given by:*

$$\boldsymbol{Q} = \int_0^{\boldsymbol{Q}} \boldsymbol{dq} = \iint_{\boldsymbol{S}} \boldsymbol{\sigma\, dS}$$

$\boldsymbol{Q}$*: Total charge of the body*

$\boldsymbol{\sigma}$*: Surface charge density (charge per unit surface)*

$\boldsymbol{S}$*: The length of the charged body*

## C- VOLUME DISTRIBUTION

*The charge distributed in a volume with a density '$\boldsymbol{\rho}$', the total charge is given by the sum of all elements of charge of that body (example a sphere of radius R)*

*To calculate the total charge, we determine the element of charge '$\boldsymbol{dq}$', and sum all this element along the volume '$\boldsymbol{V}$'*

*The volume element '$\boldsymbol{dv}$' has an element charge '$\boldsymbol{dq = \rho\, dv}$'*

$$dv = dx\, dydz$$

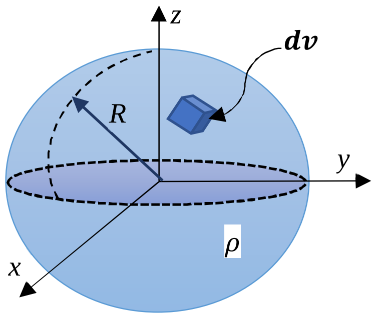


Fig.12

*The charge of all body is given by:*

$$Q = \int_0^Q dq = \iiint \rho\, dv$$

$\boldsymbol{Q}$: *Total charge of the body*

$\boldsymbol{\rho}$: *Volume charge density (charge per unit volume)*

$\boldsymbol{V}$: *The length of the charged body*

## 2 - INTERACTION OF CHARGES: ELECTRIC FORCE

*When describing, in the previously paragraphs, that charged bodies undergoes interaction either by repulsion or by attraction. This phenomenon is well quantified and measured. It was Coulomb who proposed an experiment to measure this effect. He concluded that the interaction forces between two charges is proportional to the amount of charge involved (depends on the charges), and also proportional to the inverse squared distance between them*

### 2-1 INTERACTION BETWEEN 2 POINT CHARGES IN VACUUM: COULOMB'S LAW

*The charges are assumed to be point, if their sizes are negligible or smaller compared to the distance of interaction (distance separating them). the size may be ignored and the charged bodies are treated as point charges. Coulomb's law is a quantitative statement about the force between two-point charges.*

*Coulomb measured the force between two-point charges and found that it varied inversely as the square of the distance between the charges and was directly proportional to the product of the magnitude of the two charges and acted along the line joining the two charges.*

*Two-point charges '$\boldsymbol{q_1}$', '$\boldsymbol{q_2}$' are separated by a distance '$\boldsymbol{r}$' in vacuum, the magnitude of the force '$\boldsymbol{F}$' between them is given by:*

$$F_{1/2} = k\frac{|q_1|\,|q_2|}{r^2} = \frac{1}{4\pi\epsilon_0}\frac{|q_1|\,|q_2|}{r^2}$$

$\boldsymbol{F_{1/2}}$: *The force due to the action of $\boldsymbol{q_1}$ on $\boldsymbol{q_2}$ $\boldsymbol{r}$:Distance separating the two charges*

$\boldsymbol{q_1}$: *The charge of a body 1*

$\boldsymbol{q_2}$: *The charge of a body 2*

$\boldsymbol{\epsilon_0}$: *permittivity of free space* $\epsilon_0 = 8{,}85\; 10^{-12}\, \frac{C^2}{N\, m^2} \quad \Rightarrow \quad c^2\, 10^{-7} = k \approx 9\; 10^9$

*$c$: The speed of light in vacuum* $\quad c = 2.99792458\, m/s$

*We use this formula and taking care that the like charges repel each other, and unlike charges attract each other.*

*The vectorial form of COULOMB's law, for two-point charges in interaction, is given by:*

$$\vec{F}_{1/2} = k\frac{q_1 q_2}{r^2}\vec{u}_r$$

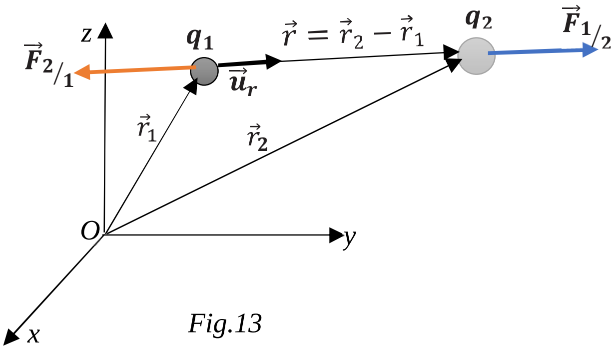



*Fig.13*

$\vec{F}_{1/2}$ *results from the action of* $q_1$*, the source, on* $q_2$*, the target. So the orientation of the unit vector* $\vec{u}_r$ *or the vector* $\vec{r}$ *is directed from the source to the target. The force lie along line joining the two point charges*

- *Because* $\vec{r} = r\,\vec{u}_r$*, the COULOMB's can be written in the form:*

$$\vec{F}_{1/2} = k\frac{q_1 q_2}{r^3}\vec{r} = k\frac{q_1 q_2}{|\vec{r}_2 - \vec{r}_1|^3}(\vec{r}_2 - \vec{r}_1)$$

- *The COULOMB's law obeys to reciprocity law (NEWTON's 3rd law)*

$$\vec{F}_{2/1} = -\vec{F}_{1/2}$$

- *Since the charges are of two types, and taking in consideration the reciprocity law, seen above, we can have the three representations, indicating below, for the forces of interaction,*

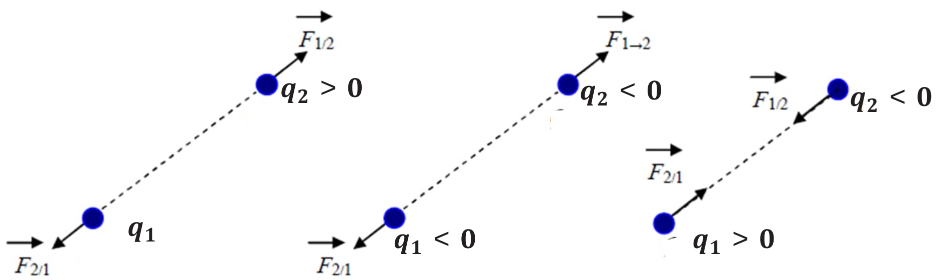



*Fig.14*

## 2-2 INTERACTION OF MULTIPLE PONCTUAL CHARGES: SUPERPOSITION PRINCIPLE

*Coulomb's law as we have stated it describes only the interaction of two-point charges. Experiments show that when two charges exert forces simultaneously on a third charge, the total force acting on that charge is the vector sum of the forces that the two charges*

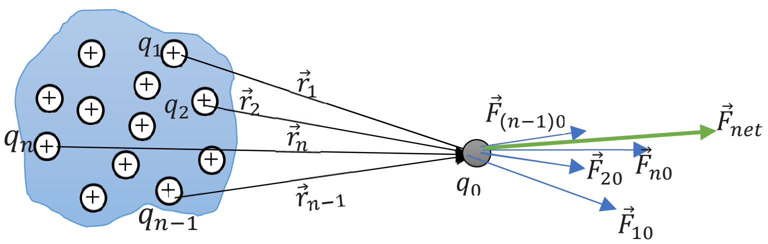



*Fig.15*

*would exert individually. This important property, called the* <u>*principle of superposition of forces*</u>*, holds for any number of charges. By using this principle, we can apply Coulomb's law to any collection of charges. Coulomb's law, as we have stated, should be used only for point charges in vacuum*

<u>*principle of superposition*</u>

*When more than two charges are interacting in a system of particles then net force on any given charge is the vector sum of all the individual forces acting on the given charge by all other charges considered independently.*

*Let found the net forces, due to* $'\boldsymbol{n}'$*discreet charges, on the charge* $\boldsymbol{q_0}$*. To do this we apply Coulomb's law for all pairs of charge.*

*The Coulomb's law for interaction between* $\boldsymbol{q_1}$ *and* $\boldsymbol{q_2}$*, gives*

$$\vec{F}_{1/0} = k\frac{q_1 q_0}{(r_1)^2}\vec{u}_1$$

$\boldsymbol{r_1}$ *: separation between the charges* $\boldsymbol{q_1}$ *and* $\boldsymbol{q_0}$

$\vec{\boldsymbol{u}}_1$*: unit vector of the line joining the two charges* $\boldsymbol{q_1}$ *and* $\boldsymbol{q_0}$

*Doing the same thing with the pairs* $(q_2; q_0)$*,* $(q_3; q_0)$*, …,* $(q_n; q_0)$*. We obtain*

$$\vec{F}_{2/0} = k\frac{q_2 q_0}{(r_2)^2}\vec{u}_2,\ \vec{F}_{3/0} = k\frac{q_3 q_0}{(r_3)^2}\vec{u}_3,\ \dots,\ \vec{F}_{n/0} = k\frac{q_n q_0}{(r_n)^2}\vec{u}_n$$

*The resultant force is given by the vectorial sum of all these forces*

$$\vec{F}_{net} = \vec{F}_{1/0} = k\frac{q_1 q_0}{(r_1)^2}\vec{u}_1 + k\frac{q_2 q_0}{(r_2)^2}\vec{u}_2 + k\frac{q_3 q_0}{(r_3)^2}\vec{u}_3 + \dots + k\frac{q_n q_0}{(r_n)^2}\vec{u}_n$$

$$\Longrightarrow \qquad \vec{F}_{net} = \sum_{i=1}^{n} k\frac{q_i q_0}{(r_i)^2}\vec{u}_i = kq_0 \sum\nolimits_{i=1}^{n} \frac{q_i}{(r_i)^2}$$

<u>*This represents the principle of superposition*</u>

## 2-3 ACTION OF CONTINUOUS DISTRIBUTION ON POINT CHARGE

*In above, we have considered the forces due to point charges, which are occupying very small physical space. In the following we will see the effect of continuous charge distribution, whether along a line, or over a surface or in a volume. As noted above* $'\boldsymbol{\lambda}'$*is the linear density* $'\boldsymbol{\sigma}'$ *the surface density and* $'\boldsymbol{\rho}'$*the volume density, we calculate the force due to the one cited distribution.*

### *- A line distribution of charge*

*Consider a line charge with a charge density* $\boldsymbol{\lambda}$ *extending from* $\boldsymbol{A}$ *to* $\boldsymbol{B}$ *along* $\boldsymbol{z}$*- axis. The charge element* $\boldsymbol{dq}$ *associate with the element* $\boldsymbol{dl = dz}$ *of the line is:* $\boldsymbol{dq = \lambda\, dl = \lambda\, dz}$

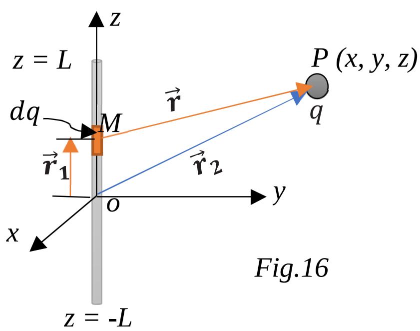


Fig.16

*Let take an element of charge* $\boldsymbol{dq}$ *on the rod, located at point* $\mathbf{M(0,0,z)}$*, so* $\vec{r}_1 = \overrightarrow{OM}$*.*

*Let the point charge* $\boldsymbol{q}$ *located at point* $\boldsymbol{P(x,y,z)}$*, so* $\vec{r}_2 = \overrightarrow{OP}$*.If we take the two-point charges* $\boldsymbol{q} = \boldsymbol{q_2}$ *and* $\boldsymbol{dq} = \boldsymbol{q_1}$ *they will interact between each other. That interaction is given by Coulomb's law such that:*

$$d\vec{F} = k\frac{q_1 q_2}{r^3}\vec{r} = k\frac{qdq}{|\vec{r}_2 - \vec{r}_1|^3}(\vec{r}_2 - \vec{r}_1)$$

*So, the interaction between* $\boldsymbol{q}$ *and* $\boldsymbol{dq}$ *produce an elementary force* $d\vec{F}$

$$d\vec{F} = k\frac{qdq}{\left|\overrightarrow{OP} - \overrightarrow{OM}\right|^3}\left(\overrightarrow{OP} - \overrightarrow{OM}\right) = \frac{1}{4\pi\epsilon_0}\frac{qdq}{\left|\overrightarrow{OP} - \overrightarrow{OM}\right|^3}\left(\overrightarrow{MP}\right)$$

*Now, what is the effect of the whole rod on the charge* $\boldsymbol{q}$*. Because each element of charge* $\boldsymbol{dq}$ *produce the elementary force* $d\vec{F}$*, then the total force, according to superposition principle, is the sum of the all-elementary forces produced by whole the rod.*

$$\vec{F} = \int_o^{\vec{F}} d\vec{F} = \int \frac{1}{4\pi\epsilon_0}\frac{q\left(\overrightarrow{MP}\right)}{\left|\overrightarrow{OP} - \overrightarrow{OM}\right|^3}dq$$

*From the figure 16, we have:*

$$\overrightarrow{MP} = (x_2 - x_1)\,\vec{\imath} + (y_2 - y_1)\,\vec{\jmath} + (z_2 - z_1)\,\vec{k}$$

$\overrightarrow{OM} = z\,\vec{k}$ *because the element* $\boldsymbol{dq}$ *is at point M (0,0, z)*

$$\overrightarrow{OP} = x_2\,\vec{\imath} + y_2\,\vec{\jmath} + z_2\,\vec{k}$$

$$\Rightarrow \overrightarrow{MP} = x_2\,\vec{\imath} + y_2\,\vec{\jmath} + (z_2 - z)\,\vec{k} \text{ and } \left|\overrightarrow{MP}\right| = r = \sqrt{x_2{}^2 + y_2{}^2 + (z_2 - z)^2}$$

*Since, the point charge* $\boldsymbol{q}$ *is fixed, then* $x_2$*,* $y_2$ *and* $z_2$ *are constants.*

*To determine the whole charge, of a rod, we vary* $\boldsymbol{dq}$ *along it.* $dq = \lambda\, dz$

$$\vec{F} = \int_{-L}^{+L}\frac{1}{4\pi\epsilon_0}\frac{q\left(\overrightarrow{MP}\right)}{\left|\overrightarrow{OP} - \overrightarrow{OM}\right|^3}\,dq = \int_{-L}^{+L}\frac{q}{4\pi\epsilon_0}\frac{\lambda\left((x_2 - x_1)\,\vec{\imath} + (y_2 - y_2)\,\vec{\jmath} + (z_2 - z)\vec{k}\right)}{\left((x_2 - x_1)^2 + (y_2 - y_2)^2 + (z_2 - z)^2\right)^{3/2}}\,dz$$

### - A surface distribution of charge

*Consider a surface charge with a charge density* $\boldsymbol{\sigma}$ *extending over a sheet. The charge element* $\boldsymbol{dq}$ *associate with the element* $\boldsymbol{ds} = \boldsymbol{dxdy}$ *of the surface is:* $\boldsymbol{dq} = \boldsymbol{\sigma\, ds} = \boldsymbol{\sigma\, dxdy}$

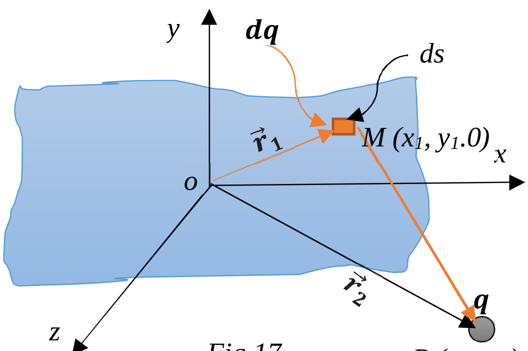


*Fig.17*

*So, the interaction between* $\boldsymbol{q}$ *and* $\boldsymbol{dq}$ *produce an elementary force* $d\vec{F}$

$$d\vec{F} = \frac{1}{4\pi\epsilon_0}\frac{qdq}{\left|\overrightarrow{OP}-\overrightarrow{OM}\right|^3}\left(\overrightarrow{MP}\right)$$

*The total force of interaction between the whole surface charge and the point charge is:*

$$\vec{F} = \int_o^{\vec{F}} d\vec{F} = \iint \frac{1}{4\pi\epsilon_0}\frac{qdq}{\left|\overrightarrow{OP}-\overrightarrow{OM}\right|^3}\left(\overrightarrow{MP}\right)$$

*In this case* $dq = \sigma\, ds$

$$\vec{F} = \iint \frac{\sigma q}{4\pi\epsilon_0}\frac{\left(\overrightarrow{MP}\right)}{\left|\overrightarrow{OP}-\overrightarrow{OM}\right|^3}ds$$

*In the cartesian coordinate* $ds = dxdy$*. Since the point charge is at rest, its coordinates are constants.*

$$\vec{F} = \iint_{(M_1)}^{(M_2)} \frac{\sigma q}{4\pi\epsilon_0}\frac{(x_2-x)\,\vec{\imath}+(y_2-y)\,\vec{\jmath}+z_2\vec{k}}{\left|\overrightarrow{OP}-\overrightarrow{OM}\right|^3}dxdy$$

*- A volume distribution of charge*

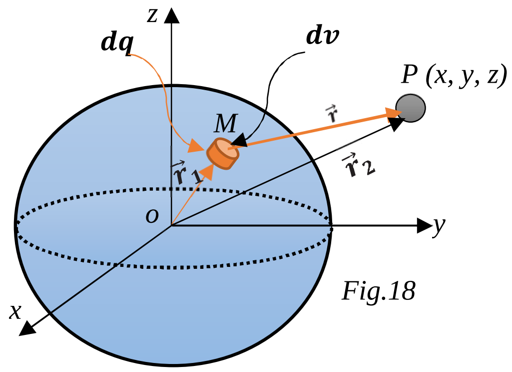


*Fig.18*

*Consider a volume charge with a charge density* $\rho$ *extending on a given volume. The charge element* $dq$ *associate with the element* $dv = dxdydz$ *of the volume is:*

$$dq = \rho\, dv = \rho\, dxdy$$

*So, the interaction between* $q$ *and* $dq$ *produce an elementary force* $d\vec{F}$

$$d\vec{F} = \frac{1}{4\pi\epsilon_0}\frac{qdq}{\left|\overrightarrow{OP}-\overrightarrow{OM}\right|^3}\left(\overrightarrow{MP}\right)$$

*The total force of interaction between the whole volume charge and the point charge is:*

$$\vec{F} = \int_o^{\vec{F}} d\vec{F} = \iiint \frac{\rho q}{4\pi\epsilon_0}\frac{\left(\overrightarrow{MP}\right)}{\left|\overrightarrow{OP}-\overrightarrow{OM}\right|^3}dv$$

*Limitation of Coulomb's law*

*1 – <u>Limitation for point charges</u>*

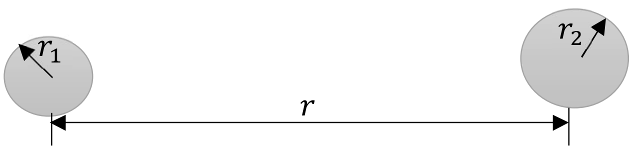


*Fig.19*

*When two conducting, charged spheres with charges* $q_1$ *and* $q_2$ *are placed in such a way that the separation distance between the is larger than the dimensions of this spheres. So, the charge seems to be points. The effect of charge induction is negligible. The Coulomb's law can be applied and gives the force* $(r >> q_1 \text{ and } q_2)$

$$\vec{F} = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^2}\vec{u}$$

*But if the two charges are brought closer, due to induction (each charge affect the other), the effectives centers of these charged spheres will shift from the geometrical center to point P₁ and P₂, and the effective separation distance is more than the distance between their geometrical centers $(r_{eff} > r)$, and the electric force is less than the force when the two charges are very distant.*

$$\vec{F}_{eff} = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^2{}_{eff}}\vec{u} \quad \Rightarrow \quad \vec{F} > \vec{F}_{eff}$$

*If the charges are closer, but they have opposite signs, the centers $P_1$ and $P_2$ are closer than the previous case. The effective distance of separation is less and the effective electric force will be greater*

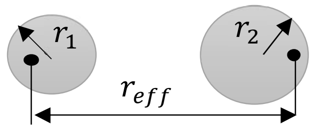



*Fig.20*

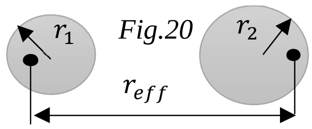



*Fig.21*

$$\vec{F}_{eff} = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^2{}_{eff}}\vec{u} \quad \Rightarrow \quad \vec{F} < \vec{F}_{eff}$$

*2 – <u>Limitation for static charges</u>*

*Coulomb's law is considered to be valid only for static charges because moving charges may involve magnetic interaction which is not accounted in this law.*

*If the two charges are moving, both the charges will also have an associated magnetic field in their surroundings. The net force will then be the vector sum of electrostatic force and the magnetic force exerted by the two charges on each other. Coulomb's law only accounts for the electrostatic force between the two charges so in condition of moving charges Coulomb's law does not give the net interaction force between the two charges.*

*If one of the charges it at rest, then the magnetic force is null, and the magnetic effect disappears, it remains only the electric effect. This produces the force deduced by the Coulombs law.*

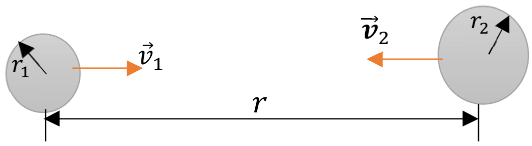



*Fig.22*

$v_1 > 0 \; and \; v_2 > 0 \quad \Rightarrow \quad \vec{F} = \vec{F}_{el} + F_{mag}$

$\vec{F}_{el}$: *The electrical forces produced by Coulomb's law*

$F_{mag}$: *The magnetic force due to the motion of the two charges* $(v_1 \neq 0 \; v_2 \neq 0)$

*The magnetism effect disappears if at least one of the two charges is at rest*

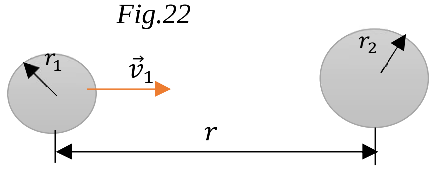



*Fig.23 - a*

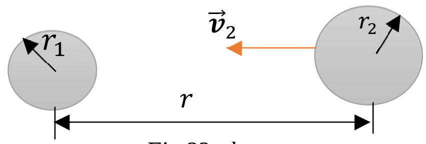



*Fig.23 - b*

## 3 – ELECTROSTATIC FIELD

### 3 -1 Concept of field (The Interaction: between Newton and Faraday)

*During the interaction between two systems, the resulting force is applied either by contact or remotely.*

*In the Newtonian notion, the interaction between two systems (A and B) is direct and instantaneous. The question is how does the first system know or perceive the presence of the second system. Following Faraday's approach, the interaction occurs indirectly with a delay between cause and effect. Thus, he introduced the notion of the field. The first system alters the space around it and creates a disturbance that spreads step by step until it affects the second system, so it receives the action of the first system.*

*The field is a quantity (vector or scalar) that spreads over an area of space. To detect this field, it must be tested by a second system (The presence of the earth's gravitational field is detected if a mass is released from a certain height. That mass executes an accelerated free fall motion)*

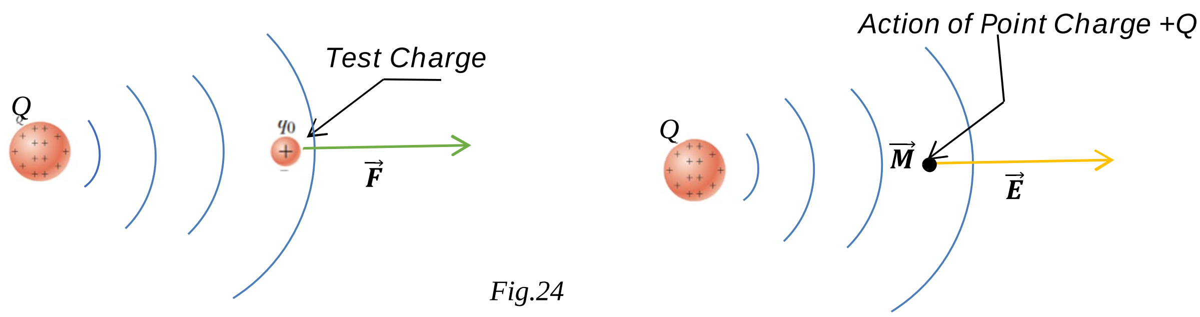



*Fig.24*

*In the case of an electrical charge, it creates a disturbance all around, that spreads and influences a second charge at a certain distance.*

*At each point in space the test load reacts by a motion in a well-defined direction, the field is a vector quantity.*

*In the electromagnetic field theory, it is based on the presence of an agent that intervenes in the interaction between the two objects. For the action at distance, the force of interaction does not require the presence of mechanisms other than the objects and the space between them, only the existence of objects is sufficient to trigger the action at distance between them. Just their presence exerts forces at distance that have an instantaneous effect through the space that separates them, i.e., the action takes place without the need for another agent other than the objects themselves, and also without considering any finite speed of the propagation of agent. This is the process of action at distance. we do not need another transmission agent.*

*Another conception sees that object, not in direct contact, must exert forces on each other through the presence of an intervening medium or mechanism that exists in the space between the objects, i.e., the first object creates a disturbance in the space immediately* *surrounding it. This disturbance propagates gradually through this space until it reaches the area directly surrounding the second object and acts upon it, causing a reaction to this disturbance. The transmission of the action occurs at a finite speed (in accordance with the theory of relativity). Thus, the field theory avoids the concept of action at distance and replaces it with the concept of action through continuous contact. By using highly sophisticated instruments, something is detected between the interacting and separated objects. What is detected is a field. This is the modern approach of field theory, which is the foundation for understanding the approaches to the world around us.*

*The electric force is an action at distance between charges, but is the instantaneous response (cause to effect)? Since there is no signal that exceeds the speed of light (according to the theory of relativity), the response to the action must therefore experience a delay.*

*This leads us to introduce the notion of the field. The charge will create a disturbance that propagates gradually until it is felt by the second charge (test charge) and it will react.*

*The field is therefore an entity that acts as a mediating agent of the force by bringing its action over a distance from one body to another.*

*Finally, we can say that the action at distance of one charge on other leads us to difficulties. If this charged, in action, is in motion, it is suddenly moved towards another charge. Since Coulomb's law varies inversely with the square of the distance separating them, there is an increase in the interaction force, but this variation is not felt immediately because there is no signal that goes beyond the finite speed of light. The charge will alter the space around it by disturbing it. This alteration will propagate for a moment and reach the target charge and affect it. This disturbance is called an electric field.*

### *3 -2* Electric field due to a charge Q

*Let us consider a point charge $\boldsymbol{Q}$ placed in vacuum, at the origin $\boldsymbol{O}$. If we place another point charge $\boldsymbol{q_0}$ at a point $\boldsymbol{P}$ which we call test charge, where $\overrightarrow{\boldsymbol{OP}} = \vec{\boldsymbol{r}}$, then the charge $\boldsymbol{Q}$ will exert a force on $\boldsymbol{q_0}$ as per Coulomb's law. We may ask the question: If charge $\boldsymbol{q_0}$ is removed, then what is left in the surrounding? Is there nothing? If there is nothing at the point $\boldsymbol{P}$, then how does a force act when we place the charge $\boldsymbol{q_0}$ at $\boldsymbol{P}$. The answer is given by introducing the concept of field. According to this, same we say above, that the charge $\boldsymbol{Q}$ produces an electric field everywhere in the surrounding. When another charge $\boldsymbol{q_0}$ is brought at some point $\boldsymbol{P}$, the field there acts on it and produces a force. The electric field produced by the charge $\boldsymbol{Q}$ at a point $\boldsymbol{r}$ is given as*

$$\vec{E} = \lim_{q_0 \to 0}\left(\frac{\vec{F}}{q_0}\right) = \lim_{q_0 \to 0}\left(\frac{\frac{1}{4\pi\varepsilon_0}\frac{Q}{q_0 r^3}\vec{r}}{q_0}\right)$$

$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}\vec{u}_r = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^3}\vec{r}$$

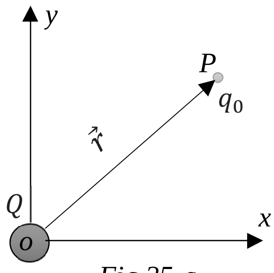


*Fig.25-a*

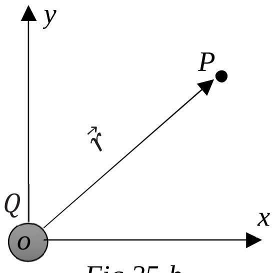


*Fig.25-b*

- *Note that the electric field $\vec{E}$ due to $Q$, is independent of $q_0$.This means that the field created at any point of the space surrounding, depends only on the charge itself and the distance to location or point considered. Thus, the electric field $\vec{E}$ is dependent on the space coordinate. For different positions over the space, we get different values of electric field $\vec{E}$.*
- *For a positive charge, the electric field will be directed radially outwards from the charge. On the other hand, if the source charge is negative, the electric field vector, at each point, points radially inwards.*
- *The magnitude of the electric field $\vec{E}$, due to charge $Q$, depends only on the distance $r$ from charge $Q$. Thus, at equal distances from the charge $Q$, the magnitude of its electric field $\vec{E}$ is same. The magnitude of electric field $\vec{E}$ due to a point charge $Q$ is thus same on a sphere with the point charge at its center, in other words, it has a spherical symmetry.*

### 3 -3 Electric field due to a several charges (principle of superposition)

*Consider a system of point charges $q_1$, $q_2$, ..., $q_n$ with respective position vectors $\vec{r}_1{}'$, $\vec{r}_2{}'$, ..., $\vec{r}_n{}'$ relative to some origin $O$, and $\vec{r}_1$, $\vec{r}_2$, ..., $\vec{r}_n$ with respect to a point P. Like the electric field at a point in space due to a single charge, electric field at a point in space due to the system of charges is defined to be the force experienced by a unit positive test charge placed at that point, without disturbing the original positions of charges $q_1$, $q_2$, ..., $q_n$. We can use Coulomb's law and the superposition principle to determine this field at a point P.*

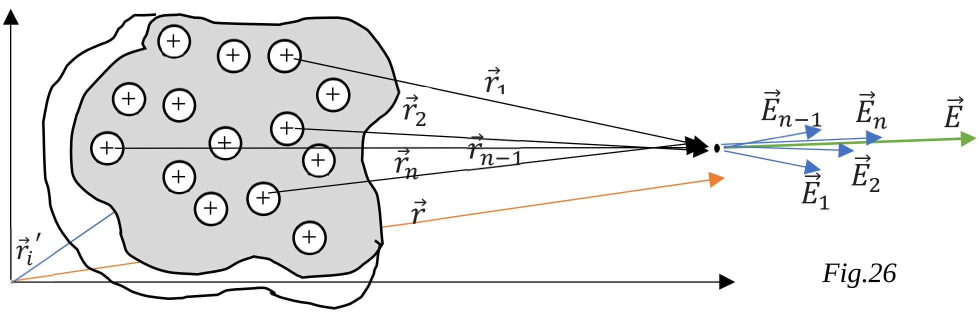


*Fig.26*

*Each charge produces its own electric field so the net field or the electric field created by all the charges is equal to the sum of these individual electric fields*

$$\vec{E} = \sum_{i=1}^{n} \frac{1}{4\pi\varepsilon_0} \frac{Q_i}{(\vec{r}-\vec{r}'_i)^2} \vec{u}_{r_i} = \frac{1}{4\pi\varepsilon_0} \sum_{i=1}^{n} \frac{Q_i}{{r_i}^3} \vec{r}_i$$

*This formula constitutes the superposition principle.*

*<u>When more than two charges are interacting in a system of particles then net force on any given charge is the vector sum of all the individual forces acting on the given charge by all other charges considered independently</u>*

### *3 -4* Electric field due to a continuous distribution of charge

*What is a continuous charge distribution? How can we calculate the electric field at any point* $\boldsymbol{P}$ *due to continuous charge distribution?*

*So far, we have dealt with only charged particles, a single particle or a simple collection of them. The situation in which an object is charged with a huge number of particles, more than we could ever even count.*

*The body charged can be a line charge (straight or curved line) in which the charge is spread with a distribution density of charge* $\boldsymbol{\lambda}$*, or surface charge when this charge is spread over a surface with a surface density* $\boldsymbol{\sigma}$*. When we deal with the charge that is spread through a volume, we have a volume distribution with a density* $\boldsymbol{\rho}$

- *A charge with linear distribution with density* $\boldsymbol{\lambda} = \lim_{\Delta l \to 0} \left(\frac{\Delta q}{\Delta l}\right)$
- *A charge with surface distribution with density* $\boldsymbol{\sigma} = \lim_{\Delta s \to 0} \left(\frac{\Delta q}{\Delta s}\right)$
- *A charge with volume distribution with density* $\boldsymbol{\rho} = \lim_{\Delta v \to 0} \left(\frac{\Delta q}{\Delta v}\right)$

*To calculate the electric field, of any continuous distributed charge, at any point in the space, we proceed in the manner as follows*

- *Take an infinitesimal element (line, surface, volume) of the charged body*
- *Assign to this element considered, an infinitesimal charge* $'\boldsymbol{dQ}'$.
- *Apply the Coulomb's Law which gives the electric field at the desired point.*
- *The law gives you the element of field* $'\boldsymbol{d\vec{E}}'$ *that is created by the element of charge* $\boldsymbol{dQ}$ *considered as point charge.*
- *To calculate the whole effect, due to the total charge of the body, we sum those elements of field* $'\boldsymbol{d\vec{E}}'$*created by each infinitesimal charge* $'\boldsymbol{dQ}$.

- *Because the huge number of these elements. The charge distributed is considered as continuous, and we use the integral form instead of the discrete sum*

### 3 -4 -1 Line distribution

*Let calculate the electric field created by a long thin rod with a linear distribution of charges with a density* $\boldsymbol{\lambda}$

*To calculate the electric field du the charged rod. We take an element - dl - that assigned a charge - dQ - which produces a field in any point P of the space, by using Coulomb's Law and the integrate on the whole element to have the total field in that point.*

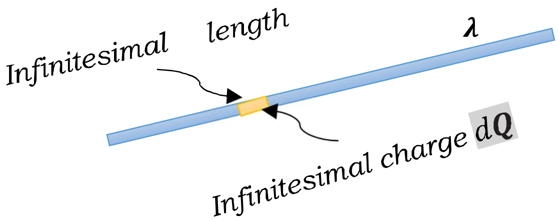


Fig.27

### Worked example

*What is the electric field produced, by a very long thin rod, in a point P?*

*The charge is distributed linearly along the rod oriented in 'z' direction.*

*Let an element of charge* $\boldsymbol{dq = \lambda\, dz}$ *located on the 'z' axis at the height* $'\boldsymbol{z}'$ *it is considered point charge, an infinitesimal electric field* $'\boldsymbol{d\vec{E}}'$*created at point P which is given by Coulomb's law as:*

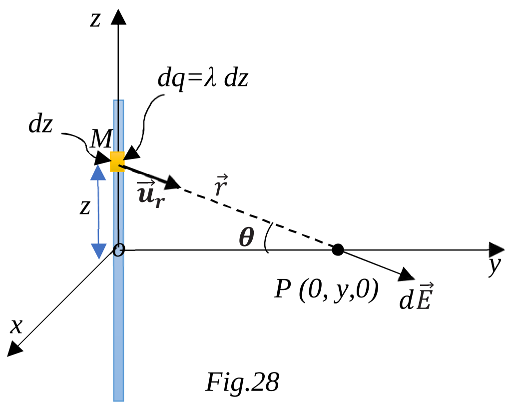


Fig.28

$$\boldsymbol{d\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{dq}{r^2}\vec{u}_r}$$

*From the geometry of figure, we have:* $\Rightarrow$

$$\boldsymbol{\vec{r} = \overrightarrow{MP} = \overrightarrow{MO} + \overrightarrow{OP} = y\,\vec{j} - z\,\vec{k}} \qquad \Rightarrow \qquad \boldsymbol{r = \sqrt{y^2 + z^2}}$$

*And*

$$\boldsymbol{\vec{u}_r = cos\theta\,\vec{j} + sin\theta\,\vec{k}}$$

*We can write the element of field* $\boldsymbol{d\vec{E}}$ *in the cartesian system as:*

$$\boldsymbol{d\vec{E} = d\vec{E}_y + d\vec{E}_z = \frac{1}{4\pi\varepsilon_0}\frac{\lambda\,dz}{r^2}\left(cos\theta\,\vec{j} + sin\theta\,\vec{k}\right)}$$

*To determine the total field, in the point P, due to the whole charge of the rod. We sum over all the length with very huge number of point charge. This means we will integrate over that rod.*

$$\boldsymbol{\vec{E} = \int_{-\infty}^{+\infty}\frac{1}{4\pi\varepsilon_0}\frac{\lambda}{r^2}\left(cos\theta\,\vec{j} + sin\theta\,\vec{k}\right)dz}$$

$$\boldsymbol{\vec{E} = \left(\int_{-\infty}^{+\infty}\frac{\lambda}{4\pi\varepsilon_0}\frac{cos(\theta)}{\left(\sqrt{y^2+z^2}\right)^{3/2}}dz\right)\vec{j} + \left(\int_{-\infty}^{+\infty}\frac{\lambda}{4\pi\varepsilon_0}\frac{sin(\theta)}{\left(\sqrt{y^2+z^2}\right)^{3/2}}dz\right)\vec{k}}$$

*To compute this integral we must use one variable z or* $\boldsymbol{\theta}$

*From the figure we see that:*

$$tg(\theta) = \frac{z}{y} \qquad \Rightarrow \ (1 + tg^2\theta)d\theta = \frac{1}{y}dz$$

$$cos(\theta) = \frac{y}{\sqrt{y^2+z^2}} \qquad \text{and} \qquad sin\,(\theta) = \frac{z}{\sqrt{y^2+z^2}}$$

*The integration limits:* $z \longrightarrow \pm\infty \quad \Rightarrow \quad \theta \longrightarrow \pm\frac{\pi}{2}$

$$\vec{E} = \left(\int_{-\infty}^{+\infty} \frac{\lambda}{4\pi\varepsilon_0} \frac{y}{\left(\sqrt{y^2+z^2}\right)^{3/2}} dz\right)\vec{j} + \left(\int_{-\infty}^{+\infty} \frac{\lambda}{4\pi\varepsilon_0} \frac{z}{\left(\sqrt{y^2+z^2}\right)^{3/2}} dz\right)\vec{k}$$

$$\vec{E} = I_1\,\vec{j} + I_2\,\vec{k}$$

*The second integral gives straight forward a null result*

$I_1$*This integral is somehow difficult, we use the angular variable* $\boldsymbol{\theta}$

$$\vec{E} = \left(\int_{-\infty}^{+\infty} \frac{\lambda}{4\pi\varepsilon_0} \frac{cos(\theta)}{\left(y\sqrt{1+\left(\frac{z}{y}\right)^2}\right)^{3/2}} dz\right)\vec{j} = \int_{-\frac{\pi}{2}}^{+\frac{\pi}{2}} \frac{\lambda}{4\pi\varepsilon_0} \frac{cos(\theta)}{y} dz$$

*The field* $\vec{E}$ *is oriented in y direction and have the following expression*

$$\vec{E} = \frac{\lambda}{4\pi\varepsilon_0 y}\vec{j}$$

### 3 -4 -2 Surface distribution

*To calculate the field created at point P by certain surface distributed charge with a density* $\boldsymbol{\sigma}$*. We proceed in the same manner like above. i.e., we take an infinitesimal element of surface which produces an elementary field in point P. The total effect is the integration on all surface namely the whole charge.*

### Worked example

*We can compute the field created, by the disc, in the point P*

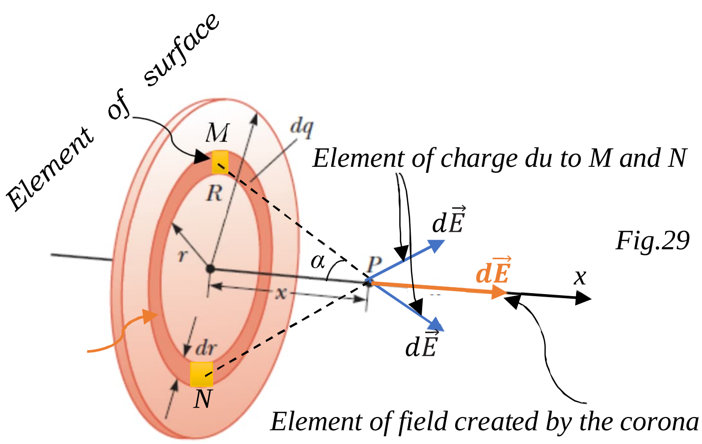



Fig.29

We can use the element of charge due to the corona. Or the charge of an element of surface ds containing the charge (M) dq = $\boldsymbol{\sigma}$ ds and that diametrically opposite (N). So, the element of the surface ds has the charge dq = $\boldsymbol{\sigma}$ ds produces an electric field:

$$d\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{dq}{r^2}\vec{u}_r = \frac{1}{4\pi\varepsilon_0}\frac{\sigma ds}{|\overrightarrow{MP}|^2}\vec{u}_r$$

We see for the two elementary charges in M an N can create a field which is oriented along the axis of the disc

$$d\vec{E} = \left(\frac{2}{4\pi\varepsilon_0}\frac{\sigma}{|\overrightarrow{MP}|^2}\cos(\alpha)\, r\, dr\, d\theta\right)\vec{\imath} \qquad |\overrightarrow{MP}| = |\overrightarrow{NP}|$$

$$0 \le r \le R \;\; ; \quad 0 \le \theta \le 2\pi$$

$$\vec{E} = \int_0^{2\pi}\int_0^{R}\left(\frac{2}{4\pi\varepsilon_0}\frac{\sigma}{|\overrightarrow{MP}|^2}\cos(\alpha)\, r\, dr\, d\theta\right)\vec{\imath}$$

The expression of electric field due to the total charge is given by:

$$\vec{E} = \frac{\sigma x}{2\pi\varepsilon_0}\left(\frac{1}{|x|} - \frac{1}{\sqrt{x^2+R^2}}\right)\vec{\imath} \quad \Rightarrow \quad \vec{E} = \begin{cases} -\dfrac{\sigma}{2\pi\varepsilon_0}\left(1 + \dfrac{x}{\sqrt{x^2+R^2}}\right)\vec{\imath} & if \;\; x > 0 \\ \dfrac{\sigma}{2\pi\varepsilon_0}\left(1 - \dfrac{x}{\sqrt{x^2+R^2}}\right)\vec{\imath} & if \;\; x < 0 \end{cases}$$

### 3 -4 -3 Volume distribution

To calculate the electric field created at point P by certain volume distributed charge with a density $\boldsymbol{\rho}$. We proceed in the same manner like above. i.e., we take an infinitesimal element of volume which produces an elementary field $d\vec{E}$ in point P. The total effect is the integration on all volume, which means the whole charge.

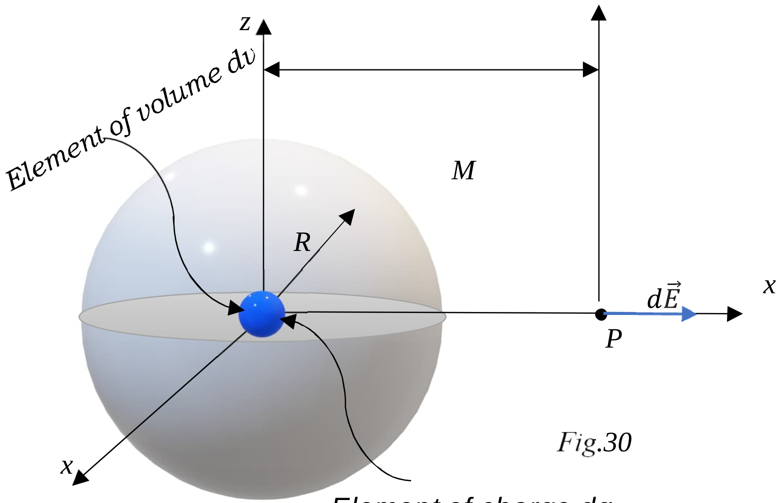


Fig.30

Let take an element of volume (an infinitesimal sphere or radius $\boldsymbol{dr}$) which has the charge dq = $\boldsymbol{\rho}$ dv. This element of charge produces an infinitesimal electric field $d\vec{E}$ at point P. Where this point is at distance x from the center.

$$dv \;\longrightarrow\; dq \;\longrightarrow\; d\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{dq}{x^2}\vec{u}_r = \frac{1}{4\pi\varepsilon_0}\frac{\rho\, dv}{x^2}\vec{u}_r$$

But the element of the volume is: $dv = 4\pi r^2 dr$, *Then the total field created at point P from whole charged sphere is:*

$$\vec{E} = \int d\vec{E} = \int_o^R \frac{1}{4\pi\varepsilon_0} \frac{\rho\, 4\pi r^2}{x^2} dr\, \vec{u}_r$$

## 3 - 5 Relationship between electric field and electric force

*To determine the interaction between charges, the force is calculated between the two charges. One cannot calculate the effect due to a single charge. Which the electric field can do. So, the relation that exists between the effect of the charge which create an electric field (the effect) and the response of another charge in field caused the first one is then:*

$$\vec{F} = Q\vec{E}$$

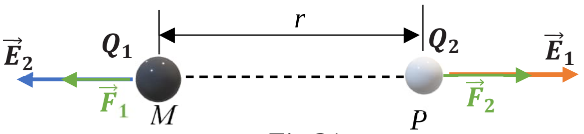


*Fig.31*

$\vec{F}$: *The force experienced by the charge* $Q$

$\vec{E}$: *The field created by other charge than* $Q$

$Q$: *The charge which feel the electric field*

*From the figure we see that:*

$\vec{E}_1$: *The field created by the point charge* $Q_1$ *at M at the point P*

$\vec{F}_2$: *The force experienced by the charge* $Q_2$ *at P due to the point charge* $Q_1$ *at M*

$\vec{E}_2$: *The field created by the point charge* $Q_2$ *at P at the point M*

$\vec{F}_1$: *The force experienced by the charge* $Q_1$ *at M due to the point charge* $Q_2$ *at P*

*One of the charges create the electric field, the other responds when it is immersed in this field.*

## 3 -6 Graphic Representation of the Electric Field: Field lines

### 3 -6 – 1 What is a field line

*We have studied electric field in the last section. It is a vector quantity and can be represented as we represent vectors. Let us try to represent E due to a point charge pictorially. Let the point charge be placed at the origin. Draw vectors pointing along the direction of the electric field with their lengths proportional to the strength of the field at each point. Since the magnitude of electric field at a point decreases inversely as the square of the distance of that point from the charge, the vector gets shorter as one goes away from the origin, always pointing radially outward. Figure 31 shows such a picture. In this figure, each arrow indicates the electric field, i.e., the force acting on a unit positive charge, placed at the tail of that arrow. Connect the arrows pointing in one direction and the resulting figure represents a field line. We thus get many field lines, all pointing outwards from the point charge.*

*Have we lost the information about the strength or magnitude of the field now, because it was contained in the length of the arrow? No. Now the magnitude of the field is* *represented by the density of field lines. $\vec{E}$ is strong near the charge, so the density of field lines is more near the charge and the lines are closer. Away from the charge, the field gets weaker and the density of field lines is less, resulting in well-separated lines. Another person may draw more lines. But the number of lines is not important. In fact, an infinite number of lines can be drawn in any region. It is the relative density of lines in different regions which is important. We draw the figure where we wish to estimate the density of field lines, one has to consider the number of lines per unit cross-sectional area, perpendicular to the lines. Since the electric field decreases as the square of the distance from a point charge and the area enclosing the charge increases as the square of the distance, the number of field lines crossing the enclosing area remains constant, whatever may be the distance of the area from the charge. We started by saying that the field lines carry information about the direction of electric field at different points in space. Having drawn a certain set of field lines, the relative density (i.e., closeness) of the field lines at different points indicates the relative strength of electric field at those points. The field lines crowd or tighten where the field is strong and are spaced apart where it is weak. Figure 32 shows a set of field lines. We can imagine two equal and small elements of area placed at points R and S normal to the field lines there. The number of field lines in our picture cutting the area elements is proportional to the magnitude of field at these points. The picture shows that the field at R is stronger than at S.*

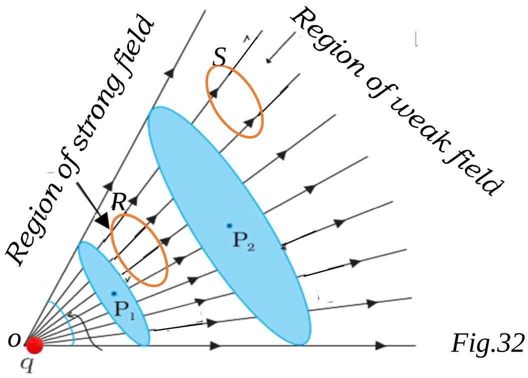



*Fig.32*

*<u>Electric lines of force in an electric field can be defined as paths, straight or curved, along which a unit positive charge tends to move, if it is free to do so. These are imaginary lines that we draw to visualize a real electric field, which they represent. This concept was introduced by Michael Faraday</u>*

### 3 -6 – 2 Equation of field line

The lines field are the geometric representation of the electric filed in different point of the space. The vector electric field is always tangent to that curve (line). To determine the equation of that line we us this property, we can then find it.

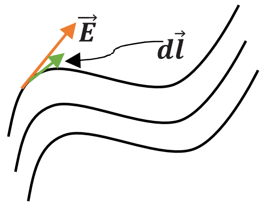



Fig.33

Let $\vec{E}$ be the electric field

$$\vec{E} = E_x\vec{i} + E_y\vec{j} + E_z\vec{k}$$

And $d\vec{l}$ be the element of displacement along the curve

$$d\vec{l} = dx\,\vec{i} + dy\,\vec{j} + dz\,\vec{k}$$

The vector field $\vec{E}$ is parallel to $d\vec{l}$, so

$$\vec{E} \wedge d\vec{l} = 0 \quad \Rightarrow \quad (E_y dz - E_z dy)\,\vec{i} - (E_x dz - E_z dx)\,\vec{j} + (E_x dy - E_y dx)\,\vec{k} = 0$$

$$\begin{cases} E_y dz - E_z dy \\ E_x dz - E_z dx \\ E_x dy - E_y dx \end{cases} \quad \Rightarrow \quad \begin{cases} \frac{E_y}{dy} = \frac{E_z}{dz} \\ \frac{E_x}{dx} = \frac{E_z}{dz} \\ \frac{E_y}{dy} = \frac{E_x}{dx} \end{cases}$$

$$\frac{E_x}{dx} = \frac{E_y}{dy} = \frac{E_z}{dz}$$

Find the equation of the vector $\vec{A}$ if the field is given by: $\vec{A} = 4z\,\vec{j} - 3y\,\vec{k}$

We have $A_x = 0$; $A_y = 4z$ ; $A_z = -3y$

$A_x = 0 \;\Rightarrow\; x = Const = C_1$

$\frac{A_y}{dy} = \frac{A_z}{dz} \;\Rightarrow\; \frac{z}{dy} = -\frac{3y}{dz} \;\Rightarrow\; 4z\,dz = -3y\,dy$

The equation of the field line of a vector $\vec{A}$ is:

$\frac{y^2}{4} + \frac{z^2}{3} = C_2$ it is the equation of the ellipse

$$\begin{cases} x = Const = C_1 \\ \frac{y^2}{4} + \frac{z^2}{3} = C_2 \end{cases}$$

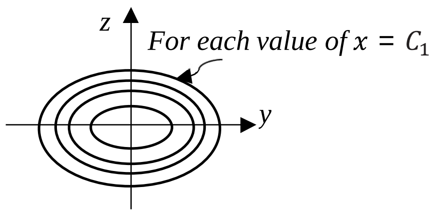



Fig.1-a

### 3 -6 – 3 Properties of electric lines of force

The lines of electric have certain properties

1- Every field line of field is a continuous and smooth curve, starting from a positively charged body and ending on a negatively charged body. If there is a single charge, they may start or end at infinity.
2- Electric field lines are generally not closed curves or loops. This follows from the conservative nature of electric field.

3- *The tangent to the electric lines at any point gives the direction of the electric field intensity.*

4- *The electric field lines never intersect each other*

5- *The field lines are closely spaced in the regions of high electric field intensity and widely separated in the regions of low electric field intensity.*

6- *The magnitude or the intensity of the electric field in a region, is proportional to the number of lines per unit cross-sectional area, held perpendicular to the field lines (which is the number density of field lines).*

7- *Where there is a uniform electric field (i.e., direction is same for all lines and magnitude of field intensity is same), the field lines will be represented by parallel lines.*

8- *The lines of forces are always normal to the surface of a conductor while leaving the conductor or ending on it.*

9- *Electric lines of force do not pass through a conductor, as there is no electric field inside a conductor due to stationary charges*

10- *Electric lines of force can pass through an insulator*

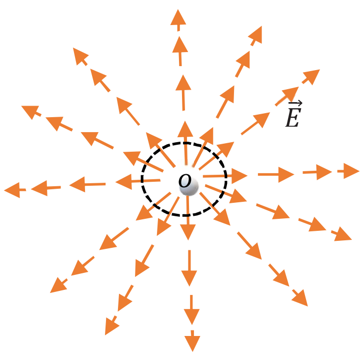
## Field lines for a negative and positive charge

*In the expression of the electric field due to a point charge, it depends on the distance from the charge to the point where it is sensed (considered point). For the positive charge the field is pointing outwards*

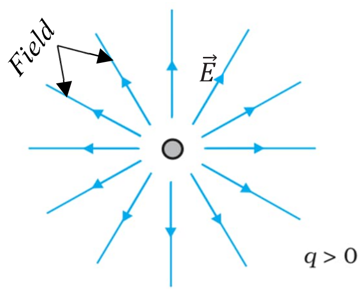
$$\vec{E}(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}\,\vec{u}_r$$

*From the same direction the field decrease in going out of the charge centered on point O taken as origin. But for the same distance (dashed circle) from the charge, the field is same in all directions.*

*There is an elegant way of representing this field in different position. We draw this field in different point with a smooth curve which we call field lines*

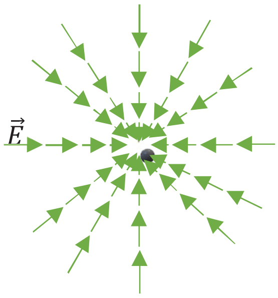



*Representation of field vector*
*Fig.34-a*



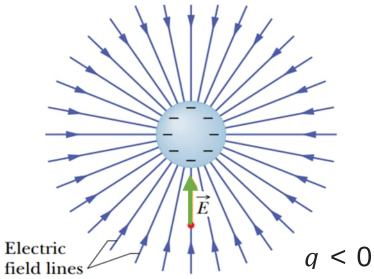

*Representation of field lines*
*Fig.34-b*

*Notice that for the negative point charge, the field is pointing inwards (from the point P to charge*




*Representation of field vector*

*Fig.35-a*




*Representation of field lines*

*Fig.35-b*

## Field lines for two equals opposite point charges (dipole)

*We know from equation of electric field and properties of field line, that the field is outwards for positive charge and inward for positive charges. So, the line is emanating*

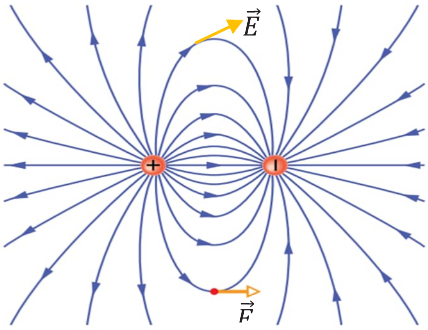



*Fig.36-a*

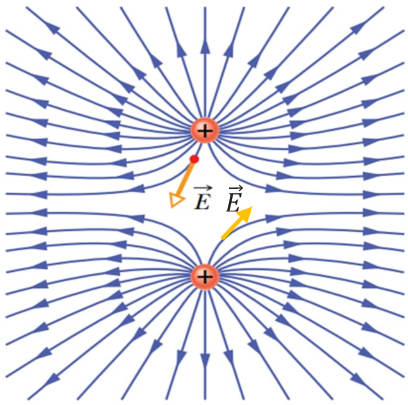



*Fig.36-b*

*from positive charge and arrive on the negative charge.*

*Notice that the field lines for two same negative point charges is similar to representation of line field for two equals positive point charges, only the difference is that the direction of that line or vector field is inwards towards the charge.*