CHAPTER 02: ELECTRIC CURRENT

7 – 1 - INTRODUCTION

In the previous chapter we have study the charge that are in static state (at rest) what we called electrostatics. What we can say when the charges are moving through the material (in general the conductor)? This motion creates what we call a current. Electric charges (positive and negative) in motion, with respect to an observer, constitute electric current. The charges in motion can be ions and their motion produce ion current. This current happens in the electrolyte, the plasma, the charged fluid… If the charges in motion are the free electrons like a metal, we obtain a conduction current.

The conductors like silver, aluminum, copper etc.… have a large number of free electrons. In an isolated piece of conductor, these free charges are in random motion with high speed in the order of $\boldsymbol{10^6}$ ***m/s (****Fig.72). In a hypothetical surface in that isolated piece of metal, these free electrons pass through it in both directions. So, there no there is no net transport of charge. Then there is no electric current*

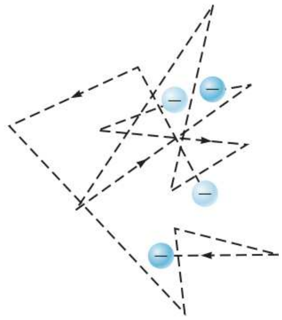


Fig.72

When a potential difference is applied across the conductor, an electric field is set up in this conductor, which causes the free charges to move in the opposite direction if they are electrons. This motion is not straight-ahead but in zigzag due to a collision between these charges and the collision with the atoms of the metal. The velocity at which the charge is displacing in the direction (or opposite) of the electric field is called drift velocity $\boldsymbol{v_d}$.

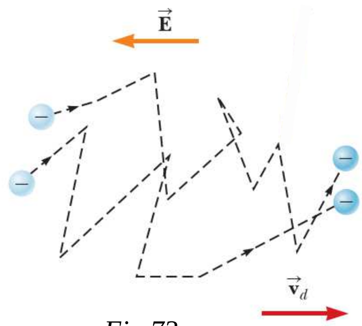


Fig.73

The amount of charge that pass through a perpendicular surface (cross-sectional area) at unit time define the average current created by these charges

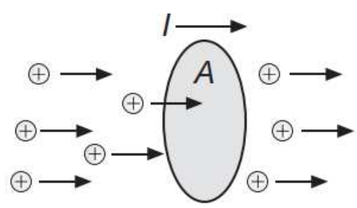


Fig.74

$$I_{avg} = < I > = \Delta Q / \Delta t$$

The current is a scalar quantity, because it obeys to the rule of adding scalar. The unit is the Ampere (A). The conventional direction is that of the positive charges or in the direction of the field created in the media. In the metal the direction of the electric current is in the opposite direction of the motion of the free electrons.

*If the magnitude and direction of current does not vary with time, it is said to be direct current (**DC**). This type of current is provided by a Cell, a battery or DC dynamo.*

*If a current is periodic (with constant amplitude) and has half cycle positive and half negative, it is said to be alternating current (**AC**). This type of current is generally sinusoidal in nature. An **AC** dynamo provides this type of current.*

7 – 2 - MOBILITY, CURRENT DENSITY, CONDUCTIVITY, RESISTIVITY AND OHM's LAW: A MICROSCOPIC VIEW

*Let a piece of conductor in cylindric form. When the switch **S** is open there no field in the conductor. The motion of free electrons is random. When the switch **S** is closed, the potential difference **V** is applied to the conductor and an electric field is established. This field causes the free electrons to move in the opposite direction with a drift velocity $\boldsymbol{v_d}$ but in zigzag way.*

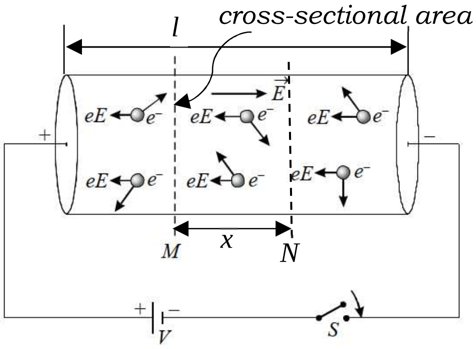


Fig.75

The free electron experiences a force $\boldsymbol{F} = \boldsymbol{e}\,\boldsymbol{E}$ in magnitude. It begins to accelerate, but not indefinitely. Due to its trajectory in zigzag, it collides other free electrons present in the conductor. Then its speed will decrease. Again, it accelerates and collides with the other free electrons and so on during its journey along the conductor. The time between two collisions is the relaxation time $\boldsymbol{\tau}$. This collision that causes the decrease in speed is assimilated to the force which opposes the motion and is proportional to the speed. $\vec{\boldsymbol{F}}_{\boldsymbol{f}} = -\boldsymbol{k}\,\vec{\boldsymbol{v}}$

The distance traveled by the charge carrier between two successive collision is called mean free path $\boldsymbol{l}$.

So, the velocity at which the free charge is: (The order of drift velocity is 10^{-4} m/s. It is very small in comparison to thermal speed)

$$\boldsymbol{v_d} = \frac{\boldsymbol{l}}{\boldsymbol{\tau}}$$

If we consider the distance l between the two collisions, the free charge experiences the electric force $\boldsymbol{F} = \boldsymbol{qE}$ that accelerate it

$$F_{net} = qE - k\,v = ma \quad \Rightarrow \quad a = \frac{dv}{dt} = \frac{qE - k\,v}{m}$$

This is the equation which is a differential equation that allows us to found the drift speed by resolving this equation (by the method of variables separation), we can find the solution

$$\vec{v} = \frac{q\vec{E}}{k} + \vec{v}_0 e^{-\frac{k}{m}t} = \frac{q\vec{E}}{k} + \vec{v}_0 e^{-\frac{t}{\tau}}$$

$\tau = m/k$ *is the relaxation time: time between two successive collisions*

When the time progress the term $e^{-\frac{t}{\tau}}$ *tends to zero, then the speed of the charge is the drift velocity:*

$$\vec{v}_d = \vec{a}\,t = \vec{a}\,\tau = \frac{q}{k}\vec{E}$$

We see that the speed is proportional to the field:

$$\vec{v}_d = \mu\vec{E} \quad \Rightarrow \quad v_d = \mu E$$

$\mu = \frac{q}{k}$ *is the* ***mobility****, which express how ease the charge carriers can flow in the medium and contribute in the electric current*

Now calculate the charges contained in the volume ΔV*. For this let* S *be cross-sectional area of the volume between the planes* M *and* N *which is* ΔV*, and* n *the density of charge per unit volume*

$$\Delta V = S.x$$

The charge ΔQ *that pass through the cross section in the interval time* Δt*, is the charge that is contained in the volume* ΔV

But

$$x = v_d\tau = \frac{qE}{m}\tau^2$$

Then, the charge ΔQ *is equal to the number of charges per unit volume multiplied by the volume* ΔV*. If the charge of each carrier is* q

$$\boldsymbol{\Delta Q = n\Delta V.q = nS.x.q = nq.S.v_d.\tau. = nqS\frac{qE}{m}\tau^2}$$

If the displacement expressed by $\boldsymbol{v_d}$ *and the surface are not orthogonal, the charge through the cross section is*

$$\boldsymbol{\Delta Q = nq\,\tau\,\vec{S}\circ\vec{v}_d}$$

Let $\boldsymbol{\rho} = \boldsymbol{nq}$*, be a density of charge*

$$\boldsymbol{\Delta Q = \rho\,\tau\,\vec{S}\circ\vec{v}_d = \left(\rho\,\vec{v}_d\circ\vec{S}\right)\tau}$$

The average current is the charge passing through the cross section in the time $\boldsymbol{\Delta t} = \boldsymbol{\tau}$

$$\boldsymbol{I = \frac{\Delta Q}{\Delta t} = \rho\,\vec{v}_d\circ\vec{S}}$$

If we are interested to small current which delimit the small region in the conductor. We will use the infinitesimal variations of these quantities. This gives us the instantaneous current

$$\boldsymbol{i = \lim_{\Delta t\to 0}\left(\frac{\Delta Q}{\Delta t}\right) = \frac{dQ}{dt}}$$

The conventional direction is given by the direction of motion of the positive charges or in the direction of the electric field present in the material

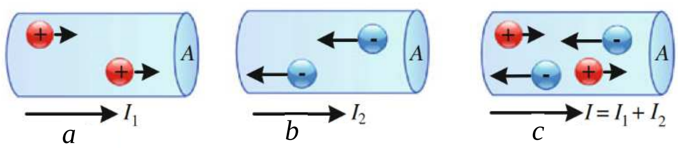
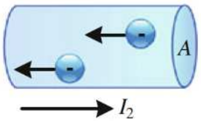


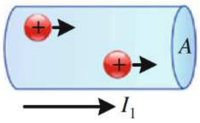
Fig.76

In a given time of **10 s**, **40** *electrons pass from right to left. In the same interval of time* **40** *protons also pass from left to right. Is the average current zero? If not, then find the value of average current.*

No, the average current is not zero. Direction of current is the direction of motion of positive charge or in the opposite direction of motion of negative charge. So, both currents are from left to right and both currents will be added.



Electrons pass from right to left $\Longrightarrow$ the current is directed to right



Protons pass from left to right $\Longrightarrow$ the current is directed to right

The direction of the two currents $\boldsymbol{I_1}$ and $\boldsymbol{I_2}$ are in the same direction. So, the currents will be added $\boldsymbol{I = I_1 + I_2}$

$$I = \frac{q_1}{t_1} + \frac{q_2}{t_2} = \frac{40\,e}{10} + \frac{40\,e}{10} = 8\,e = 1.28\ 10^{-18} A$$

1- CURRENT DENSITY

In an isolated conductor, the charge carriers have a random motion and the no drift speed and the current is zero. But when applying a potential difference an electric field is established inside the conductor and that causes free electrons to flow. There is the response to the action of the electric field on these charge carriers. Since the current is a scalar quantity, it can not be directly related to this field. So, we need to define a vector quantity for current measurement establishing a relation with electric field strength. That quantity, noted $\vec{J}$ is a current density at any point inside a conductor, which has the same direction as the velocity of the moving charges if they are positive and the opposite direction if they are negative. The magnitude is equal to current per unit infinitesimal area at that point, the area held normal to the direction of flow of current. The current density can be represented with a similar set of lines as we done with electric field, which we can call streamlines

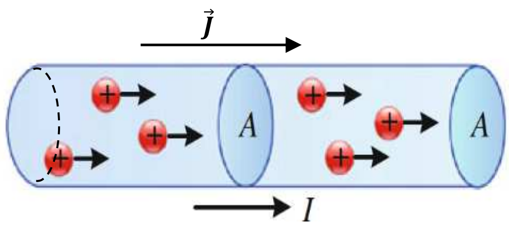


Fig.77

We have

$$I = \frac{\Delta Q}{\Delta t} = \rho\,\vec{v}_d \circ \vec{S}$$

$\vec{J}$ is equal to current per unit infinitesimal area at that point.

Let the surface element be $\boldsymbol{d\vec{S}}$, the current density is $\vec{J}$ and the electric current in that point of the conduct is

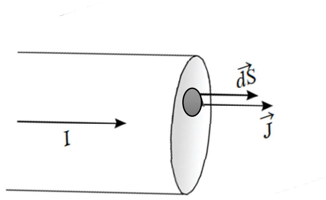


Fig.78

$$dI = \vec{J} \circ d\vec{S}$$

The total current in the conductor is

$$I = \iint \vec{J} \circ d\vec{S}$$

If the current density and the inside of the conductor are uniform, then

$$I = \vec{J} \circ \vec{S}$$

2- *MICROSCOPIC OHM's LAW*

We can deduct from the expression of the electric field, that the density of current is related to the microscopic parameters as

$$\vec{J} = \rho\,\vec{v}_d = \frac{\rho q \tau}{m}\vec{E} = \frac{nq^2\tau}{m}\vec{E}$$

$$\vec{J} = \sigma\vec{E} = \frac{nq^2\tau}{m}\vec{E} \quad or \quad \vec{E} = \frac{1}{\sigma}\vec{J} = \rho_e.\vec{J}$$

This constitutes the Ohm's Law

$\sigma = \frac{nq^2\tau}{m}$ *is called the* ***conductivity*** *of the medium.*

Its reciprocal is the ***resistivity*** $\rho_e = \frac{1}{\sigma}$

Remark: don't confuse between the density of charge ρ *and the resistivity* ρ_e

The relaxation time is $\tau = m/k$, *then* $\sigma = \frac{nq^2}{m}\tau$

3- RESISTOR AND RESISTANCE

With the increase of temperature, the thermal agitation increases and the collisions become more frequent. The conductivity decreases which means that the resistance of the conductor increases. Conversely if the temperature decreases, the resistance also, decreases. There is some material, when the temperature is small, the conductance become very large and no resistance exist. These materials are called supra-conductor material.

The resistivity, by consequence, the resistivity varies with temperature, in many cases this relation is given by:

$$\rho_e = \rho_{e0}(1 + \alpha\,T + \beta\,T^2)$$

$\boldsymbol{\rho_e}$: *the resistivity at any temperature*

$\boldsymbol{\rho_{e0}}$: *the resistivity at any temperature 0°*

$\boldsymbol{T}$: *the temperature at which the resistivity will be calculated.*

$\boldsymbol{\alpha}$ *and* $\boldsymbol{\beta}$ *are temperature coefficients of the resistance (they are positive for metal but negative for non-metal)*

The reciprocal of the conductance is called resistance.

4- RESISTANCE AND OHM's LAW IN MACROSCOPIC FORM

Let take a conductor of cylindrical form. When there is no penitential difference between its ends, there is no field and the current is zero. When there is a potential difference, an electric field is setup in the conductor, the charges (electrons) start drifting and the current is established, then

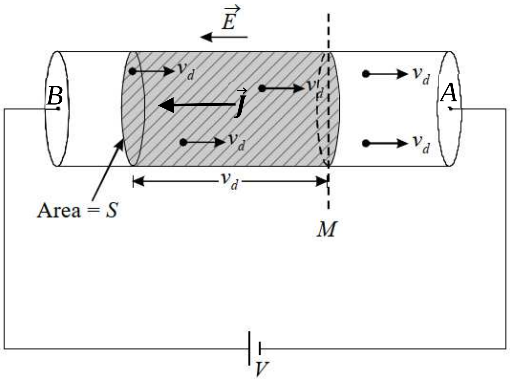


Fig.79

$$V_B - V_A = -\int \vec{E} \circ d\vec{l} = \int \rho_e.\vec{J} \circ d\vec{l} = \int \rho_e.J\,.dl$$

But $\boldsymbol{J = I/S}$

$$V_B - V_A = \frac{\rho_e.\, l}{S} I \quad \Rightarrow \quad \frac{V_B - V_A}{I} = \rho_e \frac{l}{S}$$

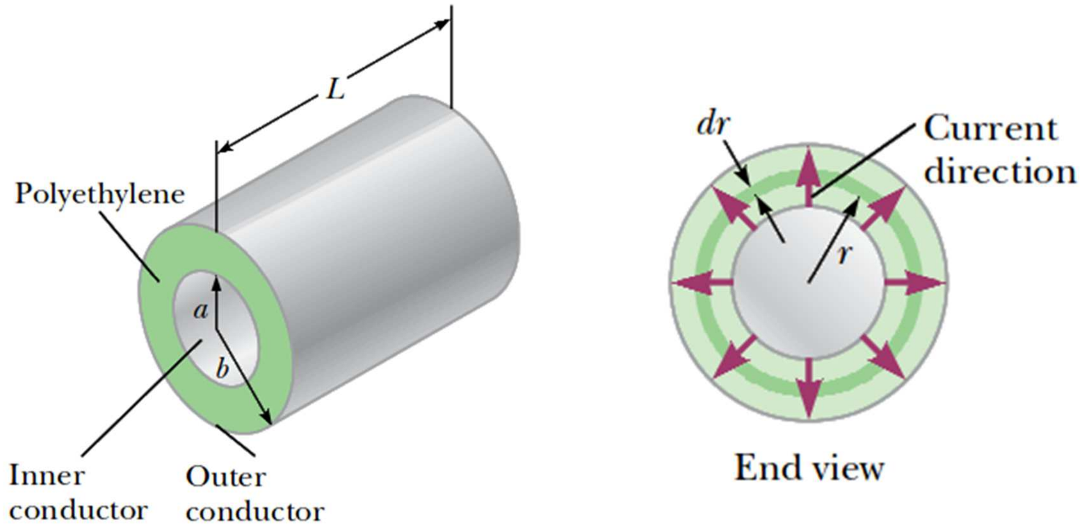
The Ohm's Law states that the potential difference is proportional to the current and the proportional coefficient is noted $\boldsymbol{R}$ *called the resistance*

$$R = \rho_e \frac{l}{S}$$

<u>*Example*</u>

A coaxial cable consists of two concentric cylindrical conductors. The region between the conductors is completely filled with polyethylene plastic as shown in Figure. Current leakage through the plastic, in the radial direction, is unwanted. (The cable is designed to conduct current along its length, but that is not the current being considered here.) The radius of the inner conductor is $\boldsymbol{a} = \boldsymbol{0.500\ cm}$, *the radius of the outer conductor is* $\boldsymbol{b} = \boldsymbol{1.75\ cm}$, *and the length is* $\boldsymbol{L} = \boldsymbol{15.0\ cm}$. *The* resistivity of the plastic is $\boldsymbol{\rho_e} = \boldsymbol{1.10^{13}}\ \Omega.\boldsymbol{m}$.

Calculate the resistance of the plastic between the two conductors.



Imagine two currents as suggested in the text of the problem. The desired current is along the cable, carried within the conductors. The undesired current corresponds to leakage through the plastic, and its direction is radial.

Because the resistivity and the geometry of the plastic are known, we categorize this problem as one in which we find the resistance of the plastic from these parameters. We have a more complicated geometry in this situation. Because the area through which the charges pass depends on the radial position, we must use integral calculus to determine the answer.

We divide the plastic into concentric cylindrical shells of infinitesimal thickness $\boldsymbol{dr}$. *Any charge passing from the inner to the outer conductor must move radially through this shell. Use a differential form of equation of the resistance, replacing* $\boldsymbol{l}$, *with* $\boldsymbol{dr}$ *for the length variable:* $\boldsymbol{dR = \rho\, dr / A}$, *where* $\boldsymbol{dR}$ *is the resistance of a shell of plastic of thickness* $\boldsymbol{dr}$ *and surface area* $\boldsymbol{S}$.

$$\boldsymbol{dR = \rho\, dr / S = (\rho / 2\pi r L)\, dr}$$

Integrate this expression from $\boldsymbol{r = a}$ *to* $\boldsymbol{r = b}$:

$$\boldsymbol{R = \int dR = \int_a^b \frac{\rho}{2\pi L} \frac{dr}{r}}$$

$$\boldsymbol{R = \frac{\rho}{2\pi L} ln\left(\frac{b}{a}\right) = 1.33\ \ 10^{13}}\ \Omega$$

Let's compare this resistance to that of the inner copper conductor of the cable along the $\mathbf{15.0}\ \boldsymbol{cm}$ *length.*

Use Equation 27.10 to find the resistance of the copper cylinder:

$$\boldsymbol{R_{Cu}} = \boldsymbol{\rho}\frac{\boldsymbol{l}}{\boldsymbol{S}} = \mathbf{1.7\,3\ 10^{-8}}.\frac{\mathbf{0.15}}{\boldsymbol{\pi}(\mathbf{5\ 10^{-3}})^{2}} = \mathbf{3.2\ \ 10^{-5}}\ \Omega$$

This resistance is **18** *orders of magnitude smaller than the radial resistance. Therefore, almost all the current corresponds to charge moving along the length of the cable, with a very small fraction leaking in the radial direction.*

What if?

Suppose the coaxial cable is enlarged to twice the overall diameter with two possible choices:

(1) the ratio b/a is held fixed, or (2) the difference $b\ 2\ a$ is held fixed.

For which choice does the leakage current between the inner and outer conductors increase when the voltage is applied between them?

Answer For the current to increase, the resistance must decrease. For choice (1), in which b/a is held fixed, Equation (1) shows that the resistance is unaffected. For choice (2), we do not have an equation involving the difference b 2 a to inspect. Looking at Figure 27.8b, however, we see that increasing b and a while holding the difference constant results in charge flowing through the same thickness of plastic but through a larger area perpendicular to the flow. This larger area results in lower resistance and a higher current.

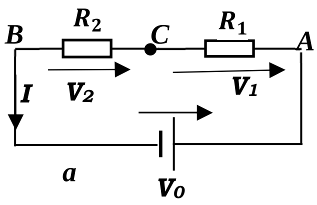
5- GROUPING OF RESISTANCE AND EQUIVALENTE RESISTANCE

If there is a several resistors and we want to connect them. The way in which they are assembled can be that all resistors are in series or all are in parallel or we mix them between the two configurations. Finally, we can replace these resistors which play the same role us the all resistors which we call an equivalent resistor.

A- SERIES COMBINATION OF RESISTORS

Le begin only with two resistors having the resistances $\boldsymbol{R_1}$ *and* $\boldsymbol{R_2}$*. We connect them one after the other (in series) to the source of voltage* $\boldsymbol{V_0}$ *(battery).*

In this configuration the potential difference delivered by the source is shared between the two resistors, and the current that is the same for the two.



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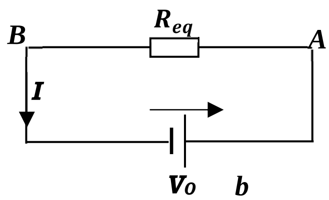


Fig.80

We have

$$V_{AB} = V_A - V_B = (V_A - V_C) + (V_C - V_B)$$

$$V_0 = V_1 + V_2$$

But $V_1 = R_1 I_1 = R_1 I$ *and* $V_2 = R_2 I_2 = R_2 I$ *Ohm's LAW*

Then

$$V_0 = R_1 I_1 + R_2 I_2 = (R_1 + R_2) I$$

In the equivalent circuit

$$V_0 = R_{eq} I$$

Since the same current and same voltage. $R_{eq} = R_1 + R_2$

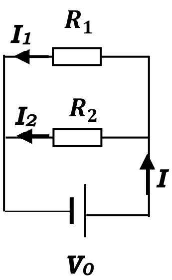
For several resistors $R_1, R_2, \ldots, R_n$*, the equivalent resistance is given by*

$$R_{eq} = \sum_{i=1}^{n} R_i$$

B- PARALLEL COMBINATION OF RESISTORS

Le begin only with two resistors having the resistances R_1 *and* R_2*. We connect them side by side (in parallel) to the source of voltage* V_0 *(battery).*

In this configuration the potential difference delivered by the source is same for the two resistors, and the current that is shared between the two.



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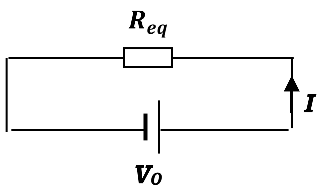


We have

$$I = I_1 + I_2$$

But $\boldsymbol{I_1 = V_1/R_1 = V_0/R_1}$ *and* $\boldsymbol{I_2 = V_2/R_2 = V_0/R_2}$ *Ohm's LAW*

Then

$$\boldsymbol{I = I_1 + I_2 = \frac{V_0}{R_1} + \frac{V_0}{R_2} = V_0\left(\frac{1}{R_1} + \frac{1}{R_2}\right)}$$

In the equivalent circuit

$$\boldsymbol{I = V_0/R_{eq}}$$

Since the same current and same voltage.

$$\boldsymbol{\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2}}$$

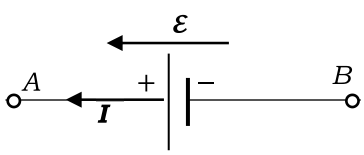
For several resistors $\boldsymbol{R_1}$, $\boldsymbol{R_2}$, …, $\boldsymbol{R_n}$, *the equivalent resistance is given by*

$$\boldsymbol{\frac{1}{R_{eq}} = \sum_{i=1}^{n} \frac{1}{R_i}}$$

7 – 3 – CELLS, BATTERY AND THE ELECTROMOTIVE FORCE

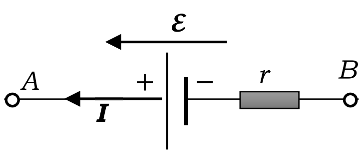
In the above description of the charge carriers in the conductor, we have said that, for an isolated conductor, there is no current be there is no drift of the charges. But when there is a certain potential difference, an electric field is setup and a current is established. Then, the charges are drifting with a velocity $\boldsymbol{\vec{v}_d}$. *To maintain this type of motion of charges, the conductor must be connected to a source of voltage which produce an electromotive force, which allow these charges to drift. As long as the source is connected to the conductor, the current will exist. But how to maintain the charges flowing from the higher potential to lower potential and bring these charges again to high potential and so on. This is done by the work produced by the battery. The influence that makes current flowing from lower potential to higher potential inside the battery is called electromotive force (e.m.f)*

For the ideal cell the potential difference between the ends ***A*** *and* ***B*** *is equal to the e.m.f of the source*



$$\boldsymbol{V_A - V_B = \mathcal{E}}$$

In the practical or real case, due to the motion of ions in the cell there are a collision, which can be assimilated to a resistance that we call internal resistance ***r***



the potential difference between the ends ***A*** *and* ***B*** *is equal to the e.m.f of the source minus the potential drop of the internal resistance*

$$V_A - V_B = \mathcal{E} - r\,I$$

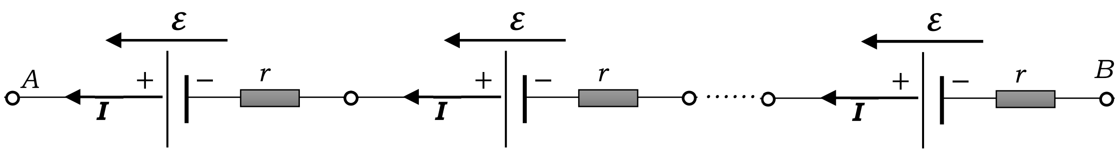
7 – 3 –1 GROUPPING OF CELLS

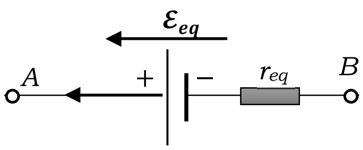
There are two principal configurations in grouping the cells, in series or in parallel

GROUPPING OF CELLS IN SERIES

For simplicity, we take a cells with same e.m.f $\mathcal{E}_1 = \mathcal{E}_2 = \ldots = \mathcal{E}_n = \mathcal{E}$, *and same internal resistances* $r_1 = r_2 = \ldots = r_n = r$. *This grouping is similar to a battery with e.m.f equal to the equivalent e.m.f of the grouping cells, and the internal resistance is equal to the equivalent internal resistance of the cells*

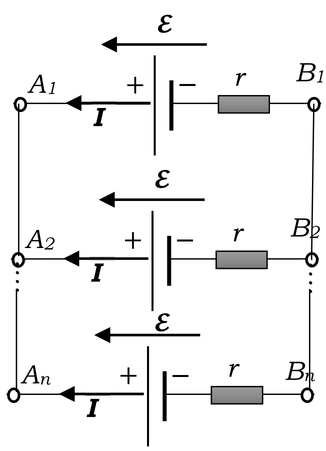
$$E = \mathcal{E}_{eq} = \sum_{i=1}^{n} \mathcal{E}_i = n\mathcal{E} \qquad\qquad r_{eq} = \sum_{i=1}^{n} r_i = nr$$





GROUPPING OF CELLS IN PARALLEL

If we take a cells with same e.m.f $\mathcal{E}_1 = \mathcal{E}_2 = \ldots = \mathcal{E}_n = \mathcal{E}$, *and same internal resistances* $r_1 = r_2 = \ldots = r_n = r$. *This grouping is similar to a battery with e.m.f equal to the equivalent e.m.f of the grouping cells, and the internal resistance is equal to the equivalent internal resistance of the cells*



$$E = \mathcal{E}_{eq} = \mathcal{E} \qquad\qquad \frac{1}{r_{eq}} = \sum_{i=1}^{n} \frac{1}{r_i} \quad \Rightarrow \quad r_{eq} = \frac{r}{n}$$

7 – 4 KIRCHOF's LAW

When we have a single source, the current is found by using Ohm's law and the combinations of the resistors. But when we are in presence of multiloop with several sources. To resolve the problem, there is several methods. Kirchhoff introduced the law of analyzing this situation. He introduced two rules, one based on the charge conservation, the other on the energy conservation

First let's express some definitions

Element: is device in the circuit like resistor, capacitor, inductor voltage source, …, etc.

(EFAB, BE, BCDE)

Branch: is a set of elements serialized (elements between two nodes)

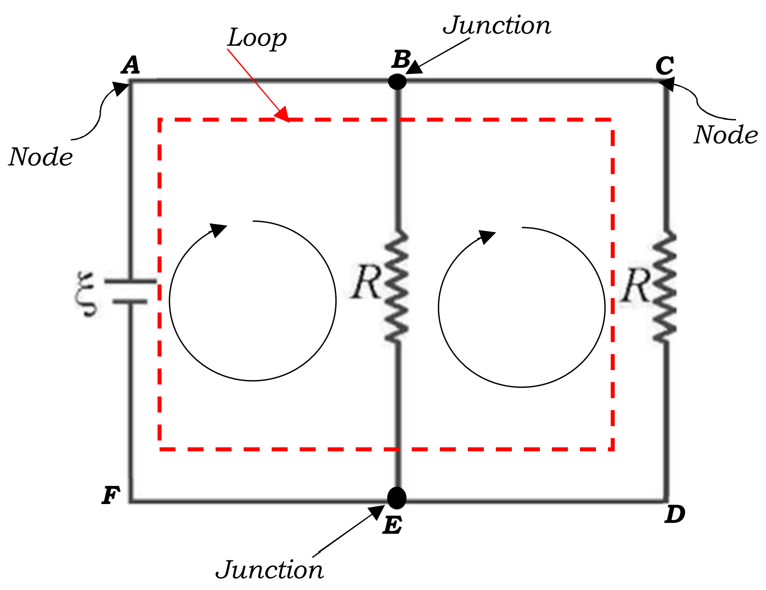
Node: is a point in circuit at which at least two elements (passive or active) are joined.

(A, C, D, F)

Junction: is a point in circuit at which at least two elements (passive or active) are joined (meeting point of at least of three branches)

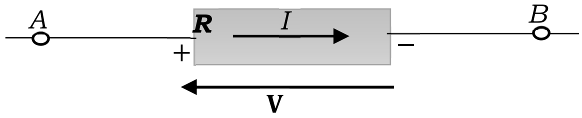
Loop: a set of branches that constitute a closed path. No element or node encountered more than once

Mesh: Is the loop that contain no other loop within it (subdivided on other loops)



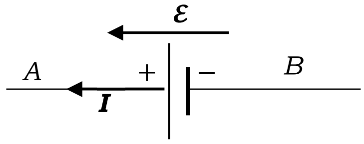
<u>Sign Convention</u>

The current I through the element (resistor) is oriented in the direction of the field from the high potential to low potential



I: The current

V: potential drop



$\boldsymbol{V = V_+ - V_- = V_A - V_B}$

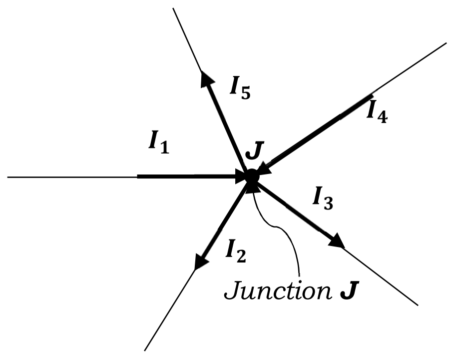
Inside the voltage source, the current is directed from low potential to high potential

Let pass now to state the Kirchhoff's Laws

1- *<u>Junction rule (Kirchhoff's Current Law KCL)</u>: This law is based on the charge conservation and states that:*

"At any junction, the sum of the currents entering the junction is equal to the sum of currents leaving the junction"

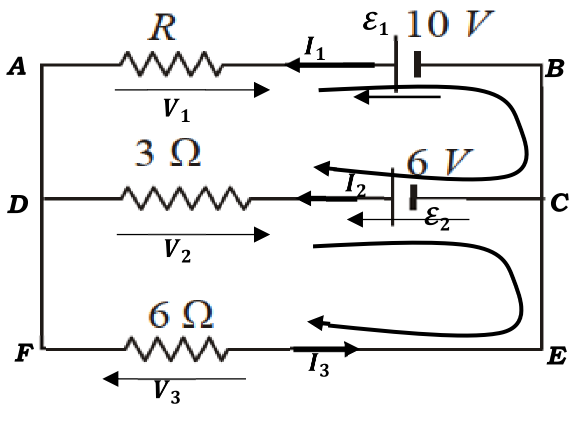
$$\left(\sum_i \boldsymbol{I_i}\right)_{In} = \left(\sum_j \boldsymbol{I_j}\right)_{Out}$$



- *In the figure side, the currents $\boldsymbol{I_1}$ and $\boldsymbol{I_4}$ are entering the junction **J***
- *The current $\boldsymbol{I_2}$, $\boldsymbol{I_3}$ and $\boldsymbol{I_5}$ are leaving the junction **J***

From junction rule

2- *Loop rule (Kirchhoff's Voltage Law KVL): This law is based on the energy conservation and states that:*



"The algebraic sum of changes in potential (potential drops) around any closed loop involving resistors and cells in the loop is zero"

$$\sum_i (\boldsymbol{V_i} + \boldsymbol{\mathcal{E}_i})$$

To calculate the current in each branch and the potential difference between each element (resistor), we proceed in the manner bellow

In the figure we have 3 branches DABC, CD, CEFD. So, we must have three equations to resolve the problem (the current of branches).

The minimal number of meshes (loops) is given by the relation:

$$\boldsymbol{M = B - J + 1}$$

$\boldsymbol{M}$**:** *Number of meshes (loops)*

$\boldsymbol{B}$**:** *Number of branches*

$\boldsymbol{J}$**:** *Number of junctions*

The number of mesh is 3, ABCDA; CEFDC; ABCEFDA

In the example above ***B = 3****, the junction is* ***J = 2****, then* $\boldsymbol{M} = \mathbf{2}$**.**

We need to have 2 equations for the loop rule

1- *Represent and choose an arbitrary direction of the current of branches*

2- *Represent an arbitrary direction in which the loop (mesh) is traversed or traveled.*

3- *Represent the direction of the f.e.ms in the circuit. The convention is from low potential to high potential. While the direction in the elements (resistors) the potential difference is directed inversely to the current through that element.*

4- *Apply the junction rule (KCL), after the choice of junctions. In the figure above, let choice The junction to be C and D which are symmetric. Then we take one junction say C*

The current entering the junction is $\boldsymbol{I_3}$**.**

The current leaving the junction are $\boldsymbol{I_1}$ *and* $\boldsymbol{I_2}$

The KCL $(\sum_i \boldsymbol{I}_i)_{\boldsymbol{In}} = \left(\sum_j \boldsymbol{I}_j\right)_{\boldsymbol{Out}}$ *gives :* $\boldsymbol{I_1 + I_2 = I_3}$

5- *Represent the directions of e.m.fs and potential drops like mentioned above.*

6- *Apply the second rule which is the loop rule or KVL in considering the above sign convention. When the direction of the potential drop or the e.m.f is same as the travel, we affect them by the sign plus (+). In contrast, when they are in the opposite direction, the sign is minus (-)*

Loop ABCDA: $\boldsymbol{\mathcal{E}_1}$ *and* $\boldsymbol{V_2}$ *are in the opposite direction of travel. But* $\boldsymbol{\mathcal{E}_2}$ *and* $\boldsymbol{V_1}$ *are in the same direction*

$$\mathcal{E}_2 - \mathcal{E}_1 + V_1 - V_2 = 0 \qquad \Longrightarrow \qquad \mathcal{E}_2 - \mathcal{E}_1 = V_2 - V_1$$

Loop *CEFDC*: $\boldsymbol{V_2}$ *and* $\mathbf{V_3}$ *are in the opposite direction of travel. But* $\boldsymbol{\mathcal{E}_2}$ *is in the same direction*

$$-\mathcal{E}_2 + V_2 + V_3 = 0 \qquad \Longrightarrow \qquad \mathcal{E}_2 = V_3 + V_2$$

7- *Using the Ohm's law* $\mathbf{V} = \boldsymbol{R.I}$

8- *The equations from the 2 loops becomes:*

$$\mathcal{E}_2 - \mathcal{E}_1 = R_3 I_2 - R\, I_1 \qquad and \qquad \mathcal{E}_2 = R_6 I_3 + R_3 I_2$$