People's Democratic Republic of Algeria
الجمهورية الجزائرية الديمقراطية الشعبية
Ministry of Higher Education and Scientific Research
وزارة التعليم العالي و البحث العلمي

University Mohamed Boudiaf- M'sila
Faculty of Technology
Departement of Electrical Engineering

جامعة محمد بوضياف-المسيلة
كلية التكنولوجيا
قسم :الهندسة الكهربائية

Pedagogical Support

License 2nd year L2-S3

Heat Transfer

Prepared by:

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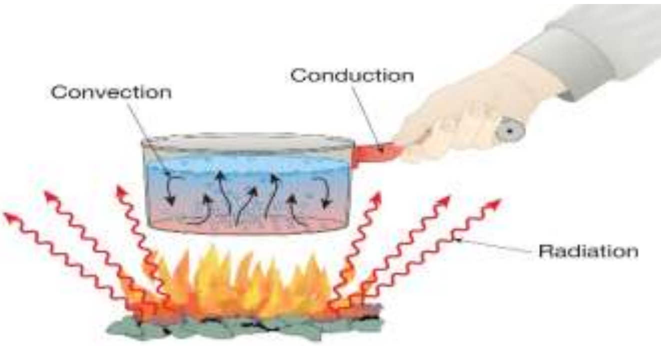
Associate professor



Academic year: 2023/2024

Preface

This document is a teaching aid on the subject of heat transfer intended for Renewable Energy second-year, students.

Course Objectives:

1. Students will be able to derive the governing equations of heat transfer.

2. Students will be able to analytically and numerically solve problems of heat conduction and convection.

3. Students will be able to analytically solve problems of radiative heat transfer.

Dr. Khaled Belhouchet

Semester: 3

Year: 2nd year Science and Technology (ST).
Course: Heat transfer.
Responsible of the course: Dr. Khaled BELHOUCHET

Material: Heat transfer

SHV: 45h00 (Weekly timetable: 1h30, Tutorial: 1h30)

Coefficient: 2

Teaching objectives:

Master the three modes of heat transfer (conduction, convection and radiation) and introduction to heat exchangers.

Recommended prior knowledge:

Have some knowledge of thermodynamics.

Material content:

Chapter 1: General information on heat transfer

Chapter 2: Steady-state conduction heat transfer

Chapter 3: Heat transfer by conduction in variable regime

Chapter 4: Heat transfer by convection

Chapter 5: Heat Transfer by Radiation

Assessment method: Continuous assessment: 40%; Exam: 60%

Contents

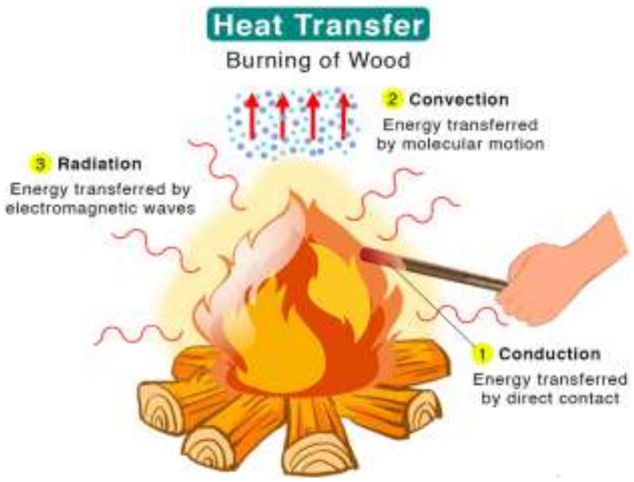
Figures List

General introduction

Heat transfer is a fundamental science that deals with the rate of energy transfer thermal. In engineering practice, understanding the transfer mechanisms of heat is becoming increasingly important, as heat transfer plays a crucial role in the design of vehicles, power plants, refrigerators, electronic devices, buildings and bridges, among others. Even a chef must have an understanding intuitive heat transfer mechanism to cook food “properly” adjusting the heat transfer rate. We may not know it, but we already use the principles of heat transfer to seek thermal comfort. We isolate our body by putting on heavy coats in winter, and we minimize heat gain by radiation by staying in shaded places in summer. We accelerate the cooling hot food by blowing on it and keeps us warm in cold weather by huddling and thus minimizing the exposed surface area.

Chapter 1

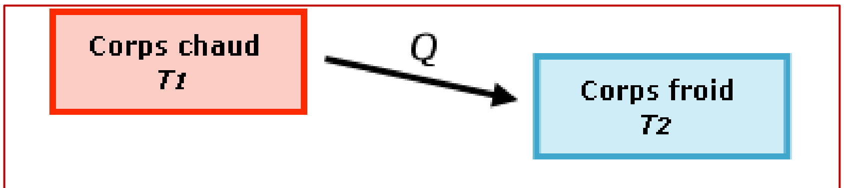
Heat transfer: General



Heat Transfer: General

1.1. Introduction

Heat transfer, which should be called thermal transfer, is defined as thermal energy in transit because of a temperature difference. Two bodies having the same temperature are said to be in thermal equilibrium. If their temperatures are different, the warmer body transfers thermal energy to the colder body, we say that there is heat transfer.



Le transfert thermique s'effectue du corps chaud vers le corps froid.

Heat transmission occurs through three basic mechanisms:

a) Conduction:

Thermal conduction, also called thermal diffusion, is a transfer of energy in a material medium without macroscopic movement, involving collisions of molecules (in fluids) or transfers of vibrations (in solids).

b) Convection:

Convection is a mode of heat transfer that involves the macroscopic movement of matter. This phenomenon occurs within flowing fluid media or between a solid wall and a moving fluid. There are two types of convection: natural and forced.

c) Radiation:

Every material body emits and absorbs energy in the form of electromagnetic radiation. The transfer of heat by radiation between two bodies separated by a vacuum or a semi-transparent medium occurs via electromagnetic waves, therefore without material support.

1.2. The importance of the heat transfer study

Heat transfer is energy in transit due to a difference in temperatures within a body or between different bodies. Whenever there is a difference in temperatures, energy is transferred from the higher temperature region to the lower temperature. According to the concepts of thermodynamics, the energy transferred as a result of a temperature difference is heat. The laws of thermodynamics concern the transfer of energy, but they only apply to systems in equilibrium (they can be used to predict the amount of energy needed to change a system from one equilibrium state to another), but they are not used to predict how quickly (time) these changes may take place. Heat transfer complements thermodynamic principles, providing analytical methods that predict this rate of heat transfer.

Example:

Heating a steel bar immersed in hot water: thermodynamic principles can be used to predict the final temperatures once the two systems have reached equilibrium and the amount of energy transferred between states of initial and final equilibrium, but this tells us nothing about the rate of heat transfer or the temperature of the bar after a certain time, or the time it takes to obtain a certain temperature at a certain position of the bar. Performing a heat transfer analysis allows you to predict the rate of heat transfer from the water to the bar and from this information you can calculate the bar temperature as well as the water temperature as a function of time.

To carry out a complete analysis of heat transfer, it is necessary to consider three different mechanisms: conduction, convection and radiation.

The design of heat exchange systems and energy conversion requires some familiarity with each of these mechanisms, as well as their interactions.

1.3. Fundamental concepts

1.3.1. Concept of heat

Heat is considered a special form of energy. According to the first law of thermodynamics, heat is equivalent to work; hence, they have the same unit: the Joule. It spreads naturally from the hottest environment to that with the lowest temperature, while keeping the volume constant. This constitutes the second law of thermodynamics.

a) Sensible heat:

When a body receives or releases heat, heats up or cools down without changing state, we then speak of sensible heat. The quantity of heat received by a body of mass m with heat capacity cp when it passes from an initial state at temperature T_1 to a final state at temperature T_2 is expressed by:

$$Q_{1\to 2} = m.c_p.(T_2 - T_1) \quad (1.1)$$

b) Latent heat:

We speak of latent heat when the body that receives or transfers heat uses it to change state, without its temperature changing. The quantity of heat that must be supplied to a material of mass m so that, at constant temperature, it changes state (solid -> liquid; liquid -> gas) is given by:

$$Q = m.\, L \quad (1.2)$$

Where L, is the Latent heat coefficient.

We then speak of "isothermal transformation" because the temperature of the system remains constant throughout the heat exchange process.

1.3.2. Temperature fields

We call temperature the physical quantity which measures the degree of heat of a medium, namely:

solids: it is the state of vibration of atoms inside a crystal lattice or movement of electrons for materials which have the ability to exchange electrons (metals for example);

fluids: this is the state of agitation of molecules. Temperature is expressed in degrees Kelvin (K), degrees Celsius (°C) or degrees Rankine (R).

$$\begin{aligned} T\left({}^{0}K\right) &= T\left({}^{0}C\right) + 273.15 \\ T\left({}^{0}F\right) &= 1.8 * T\left({}^{0}C\right) + 32 \\ T\left({}^{0}R\right) &= 1.8 * T\left({}^{0}C\right) + 491.67 \end{aligned} \quad (1.3)$$

The regime is permanent or stationary if the temperature field is independent of time $T = f\left(x, y, z, t \right)$. Otherwise, it is said to be variable or transient.

1.3.3. Isothermal surface

The locus of points having the same temperature at each instant is called isothermal surface. Two isothermal surfaces cannot intersect because we would then have two different temperatures at the same point, which is physically impossible.

In steady state: Isothermal surfaces are invariable.

In variable regime: They are mobile and deformable.

1.3.4. Temperature gradient

If we bring together all the points in space that have the same temperature, we obtain a surface called an isothermal surface. The temperature variation per unit length is maximum along the normal to the isothermal surface. This variation is characterized by the temperature gradient:

$$\overrightarrow{grad}(T) = \vec{n}.\frac{\partial T}{\partial n} \quad (1.4)$$

$\vec{n}$: Unit vector of the normal. : $\frac{\partial T}{\partial n}$ Derivative of the temperature along the normal.

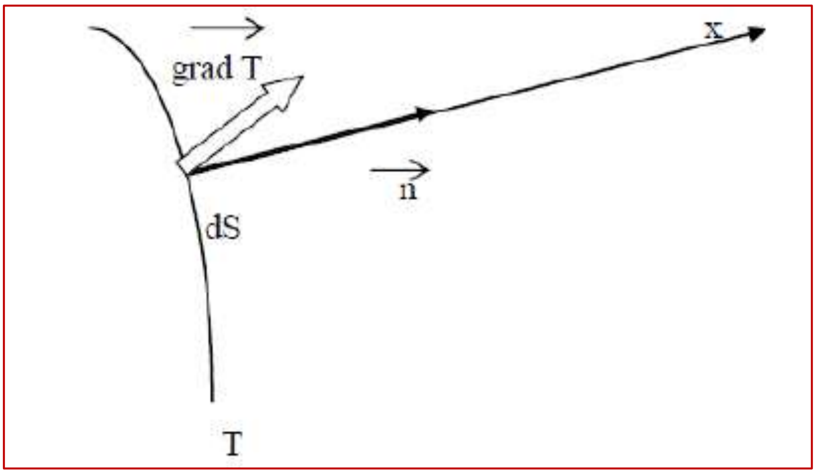


Figure 1. 1. Orthogonality of the gradient and the isotherm

1.3.5. Heat flux and density

The heat flow is the power exchanged per unit of time:

$$\Phi = \frac{dQ}{dt} \qquad [W] \quad (1.5)$$

The heat flux density is the quantity of heat transmitted per unit of time and per unit of area of the isothermal surface:

$$\varphi = \frac{dQ}{S.dt} = \frac{\Phi}{S} \qquad \left[W/m^2\right] \quad (1.6)$$

The heat source is defined by the thermal power it produces. In the case of a chemical reaction, it is expressed:

$$Q' = A_0 e^{\alpha t} \quad (1.7)$$

Where A0 and α se are constants.

1.4. The different modes of heat transfer

1.4.1. Heat transmission by conduction

This transfer of thermal energy, which does not require macroscopic movement of matter, is created by:

- molecular agitation (in gases and liquids)
- vibrations of crystal lattices (in non-conductive solids)
- the movement of free electrons (in conductive metals)

Jean-Baptiste Joseph Fourier (1768-1830) proposed in 1822 the law of conduction now known as Fourier's law. This relationship indicates that the heat flow is proportional to the temperature gradient and is in the direction of decreasing temperatures.

In the case of conduction, the flow is calculated by:

$$\Phi_S = -\lambda . S . \, gradT$$

(1.8)

For unidirectional conduction:

$$\overrightarrow{\Phi_S} = -\lambda . S . \frac{dT}{dx} \quad (1.9)$$

The (-) sign is a consequence of the second law of thermodynamics, according to which heat must flow towards the lowest temperature zone. The temperature gradient is negative if the temperature decreases for increasing values of x, therefore the heat transferred from the positive direction must be of positive magnitude, therefore a negative sign must be introduced in the right hand side of the equation former.

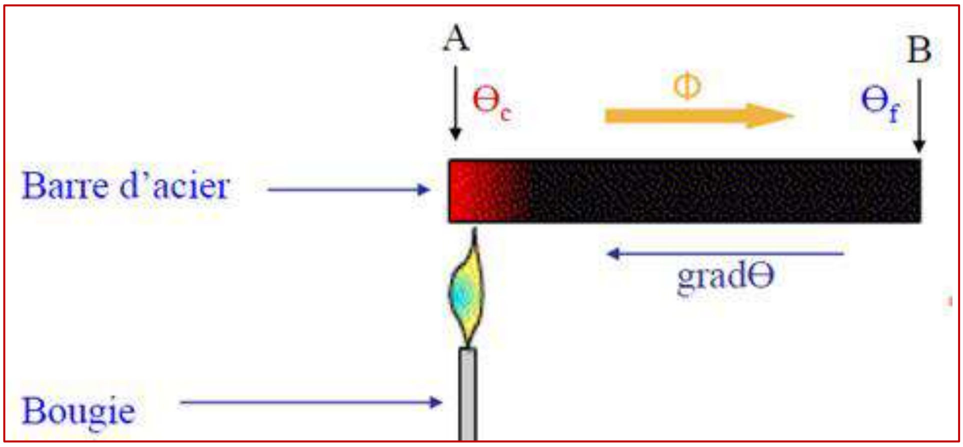


Figure 1. 2. Heating a metal bar at one of its ends

1.4.2. Heat transmission by convection

When a fluid at T_f comes into contact with a solid whose contact surface is at a temperature other than Ts, the process of thermal energy exchange is called convection. There are two types of convection.

a) Free or natural convection occurs when the driving force comes from the change in density in the fluid following contact with a surface at a different temperature, which gives rise to upward forces, with the fluid near the surface acquires a speed due only to this difference in density, without any external driving force.

b) Forced convection, occurs when an external driving force moves a fluid with a speed (v), over a surface which is found at a temperature Ts higher or lower than that of the fluid T f, because the speed of the fluid in Forced convection is higher than natural convection, therefore, a greater quantity of heat is transferred for a given temperature.

Newton's law is used to calculate heat transfer by convection:

$$\Phi_S = h.S.(T_p - T_\infty) \qquad (1.10)$$

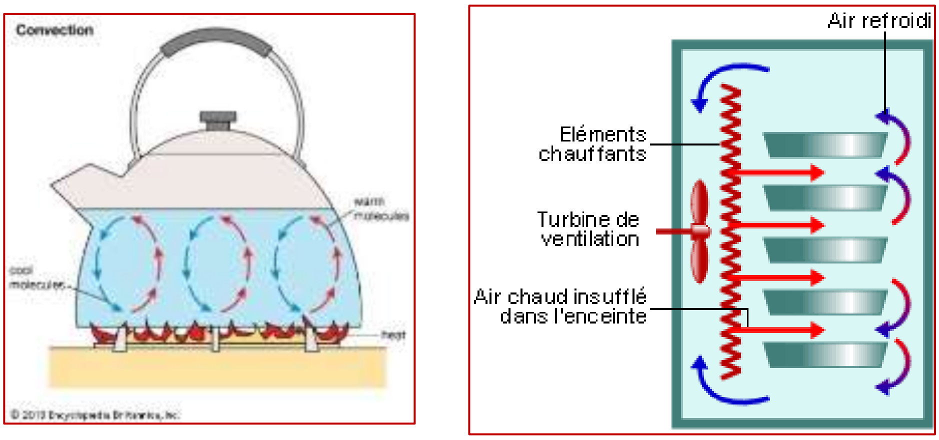


Figure 1. 3. Heat transfer by convection

1.4.3. Heat transmission by radiation

The maximum flux density emitted by a surface is given by the Stephan-Boltzmann law:

$$\Phi_{max\,i} = \sigma.T_p^4 \qquad (1.11)$$

σ : Stephan's constant = 5,669.10^{-8} W/m^2.K^4.

The maximum flux is obtained for an ideal surface (black body). However, real surfaces (gray bodies) have a certain emissivity (ε) which reduces the flux emitted by the surface:

$$\Phi_{réel} = \varepsilon.\sigma.T_p^4 \qquad (1.12)$$

In the case where this surface is surrounded by another surface at a temperature T_∞, the net heat exchange is then:

$$\Phi_{net} = \sigma.\varepsilon_p S.\left(T_p^4 - T_\infty^4\right) \quad (1.13)$$

Example: heating of a wall by solar radiation during the day, and heat emitted by the wall at night.

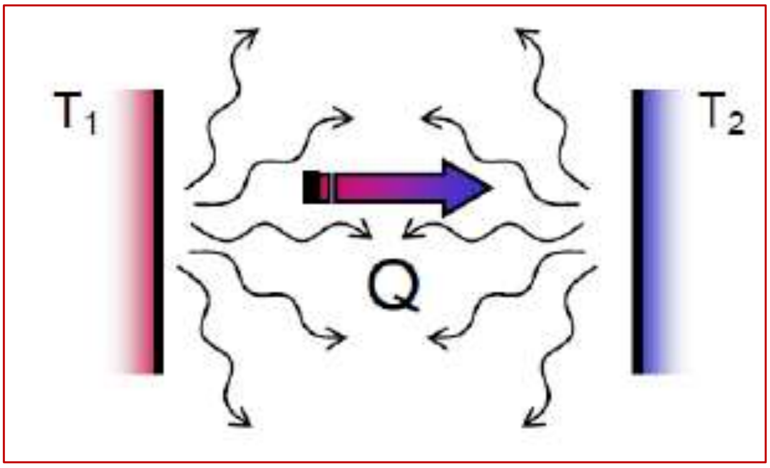


Figure 1. 4. Heat transfer by radiation

Noticed :

- ✓ In many thermal energy transformation problems, the three modes of heat transfer coexist but, generally, at least one of the three forms can be neglected, which will simplify the mathematical treatment of the transfer process;
- ✓ In the following figure, we can visualize the three modes of heat transfer at the same time.

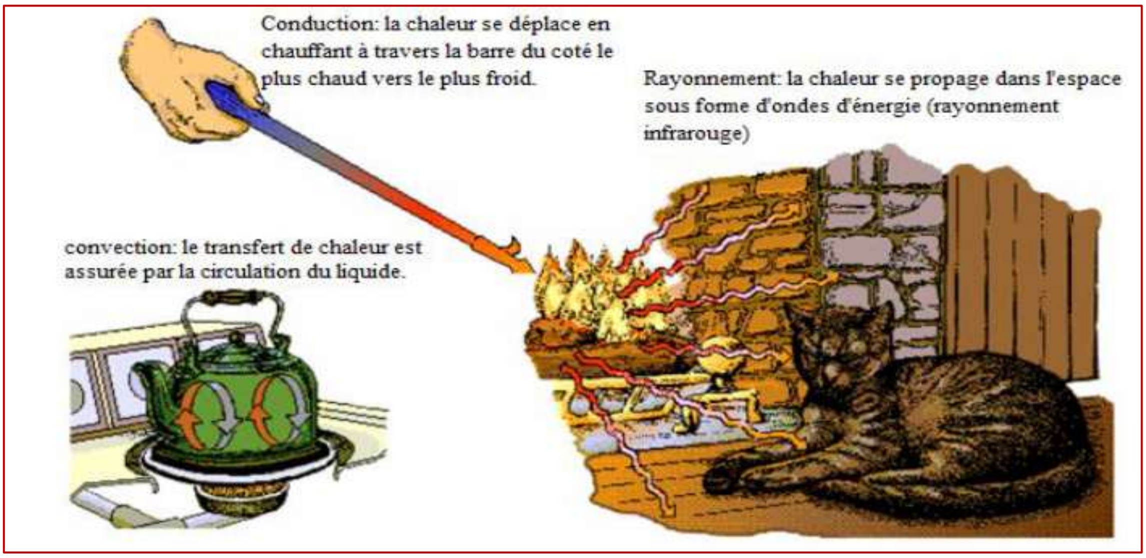


Figure 1. 5. Illustration of the three modes of heat transfer

1.5. Formulation of a heat transfer problem:

1.5.1. Energy balance:

Given a given control volume, at each instant, the energy conservation balance (mechanical energy + internal energy) on this volume is written:

(What goes in)- (What goes out)+ (What is generated)= (What stored)

$$E_e - E_s + E_g = E_{st} \quad (1.14)$$

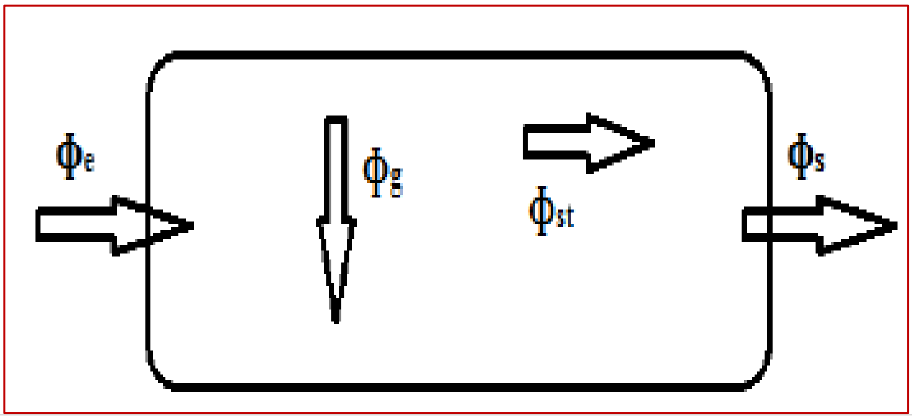


Figure 1. 6. Energy balance

The term "generated energy" should be considered in the broad sense: energy produced (in +) or consumed (in -). It is the same for the accumulation term (variation over time of the energy in the control volume). As part of the course, the energy balance will be limited to a thermal energy balance. The generation term, Eg, may appear in the following cases:

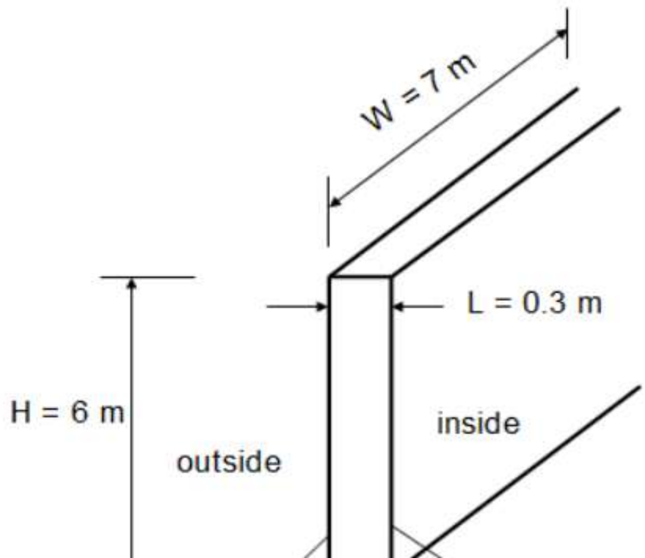
- exo or endothermic chemical reactions,
- nuclear reactions
- viscous dissipation
- Joule effect in an electrical resistance

Across a surface, the balance is simply reduced to:

$$E_e - E_s = 0 \quad (1.15)$$

Example 1.1

The wall of a house, 7 m wide and 6 m high is made from 0.3 m thick brick with $k = 0.6\ W/m\,K$. The surface temperature on the inside of the wall is 16°C and that on the outside is 6°C. Find the heat flux through the wall and the total heat loss through it.



1.1. Solution

For one-dimensional steady state conduction:

$$q = -k\frac{dT}{dx} = -\frac{k}{L}(T_i - T_o)$$

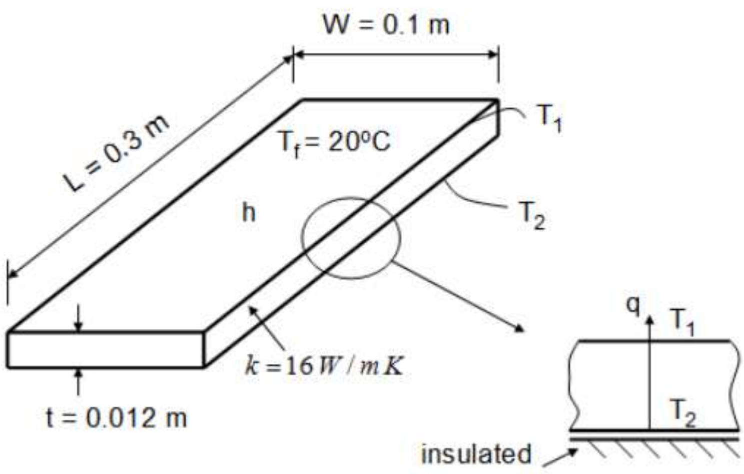
$$q = -\frac{0.6}{0.3}(16-6) = -20\ W/m^2$$

$$Q = qA = -20\times(6\times7) = -840\ W$$

The minus sign indicates heat flux from inside to outside.

Example 1.2

A plate 0.3 m long and 0.1 m wide, with a thickness of 12 mm is made from stainless steel ($k = 16\ W/m\,K$), the top surface is exposed to an airstream of temperature 20°C. In an experiment, the plate is heated by an electrical heater (also 0.3 m by 0.1 m) positioned on the underside of the plate and the temperature of the plate adjacent to the heater is maintained at 100°C. A voltmeter and ammeter are connected to the heater and these read 200 V and 0.25 A, respectively. Assuming that the plate is perfectly insulated on all sides except the top surface, what is the convective heat transfer coefficient?



Solution 1.2

Heat flux equals power supplied to electric heater divided by the exposed surface area:

$$q = \frac{V \times I}{A} = \frac{V \times I}{W \times L} = \frac{200 \times 0.25}{0.1 \times 0.3} = 1666.7\ W/m^2$$

This will equal the conducted heat through the plate:

$$q = \frac{k}{t}(T_2 - T_1)$$

$$T_1 = T_2 - \frac{qt}{k} = 100 - \frac{(1666.7 \times 0.012)}{16} = 98.75°C \qquad (371.75\ \text{K})$$

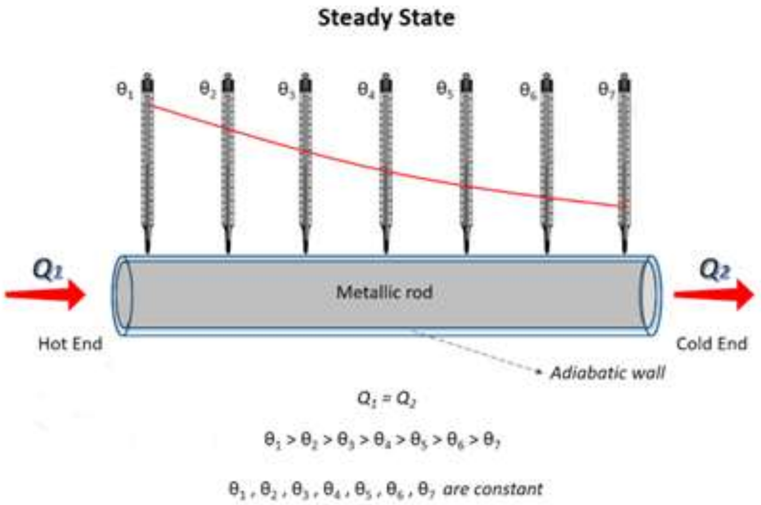
The conducted heat will be transferred by convection and radiation at the surface:

$$q = h(T_1 - T_f) + \sigma(T_1^4 - T_f^4)$$

$$h = \frac{q - \sigma(T_1^4 - T_f^4)}{(T_1 - T_f)} = \frac{1666.7 - 5.67 \times 10^{-8}(371.75^4 - 293^4)}{371.75 - 293} = 12.7\ W/m^2K$$

Chapter 2

Steady-state conduction heat transfer



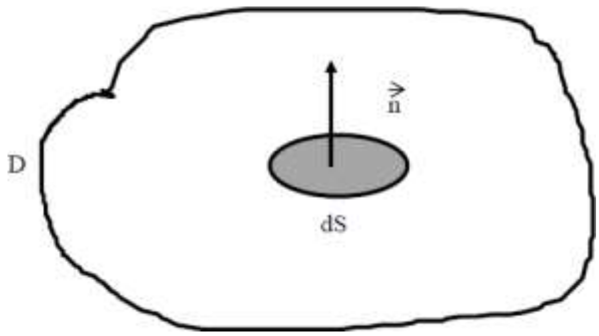
Steady-state conduction heat transfer

2.1. Introduction

It is the transfer of heat within an opaque medium, without movement of material, under the influence of a temperature difference. The propagation of heat by conduction inside a body occurs according to two distinct mechanisms: transmission by the vibrations of atoms or molecules and transmission by free electrons.

2.2. Fourier's Law

A solid medium D and an elementary surface dS oriented by its unit normal n



are considered in this case.

Figure 2. 1. solid medium.

FOURIER's law determines the quantity of heat d2Q which passes through a surface dS in the direction of the normal n and over a time interval dt:

$$d^2Q = -\lambda \cdot \overrightarrow{gradT} \cdot \vec{dS} \cdot n \cdot dt \qquad (2.1)$$

With ;

$\overrightarrow{gradT}$: Temperature gradient calculated by:

$$\overrightarrow{gradT}\begin{cases}\dfrac{\partial T}{\partial x}\\[4pt]\dfrac{\partial T}{\partial y}\\[4pt]\dfrac{\partial T}{\partial z}\end{cases}$$

λ: Thermal conductivity of the medium (W.m^{-1}.K^{-1}) We also have:

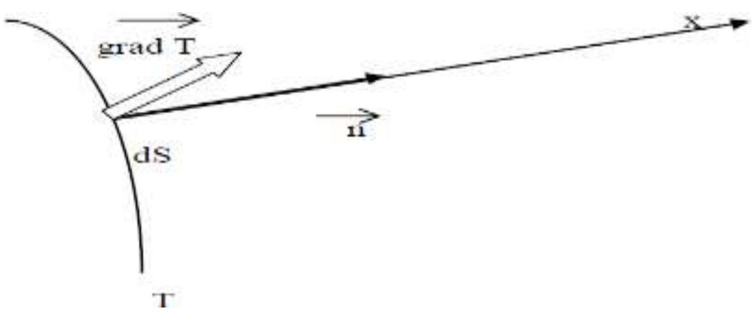
-The heat flow is:

$$d\Phi = \frac{d^2Q}{dt} = -\lambda \cdot \overrightarrow{gradT} \cdot dS \cdot \vec{n} \qquad (2.2)$$

-The heat flux density is:

$$d\varphi = \frac{d^2Q}{dt \cdot dS} = -\lambda \cdot \overrightarrow{gradT} \cdot \vec{n} \qquad (2.3)$$

The (-) sign indicates that in terms of heat flow, the gradient from largest to smallest is negative. If the surface dS is on an isothermal surface, the vectors $\overrightarrow{gradT}$ and n are collinear, as follows:



$$d^2Q = -\lambda \cdot \frac{dT}{dx} \cdot dS \cdot dt \qquad (2.5)$$

Ou $d\Phi = -\lambda \cdot \dfrac{dT}{dx} \cdot dS$ et $d\varphi = -\lambda \cdot \dfrac{dT}{dx}$

2.3. Thermal conductivity

The following table represents the usual thermal conductivity values.

Table 2.1: thermal conductivity.

Substances	λ en W/ m°C
Gaz à la pression atmosphérique	0,006 - 0,15
Matériaux solides isolants (Laine de verre, polystyrène, liège, amiante...)	0,025 - 0,18
Liquides non métalliques	0,075 - 0,60
Matériaux non métalliques (brique, pierre à bâtir, bois, béton..)	0,10 - 2,2
Métaux liquides	7,5 - 67
Alliages métalliques	12 - 100
Métaux purs	45 - 365

2.4. Heat equation

2.4.1 In Cartesian coordinates

In the one-dimensional case, this equation expresses the unidirectional heat transfer through a plane wall:

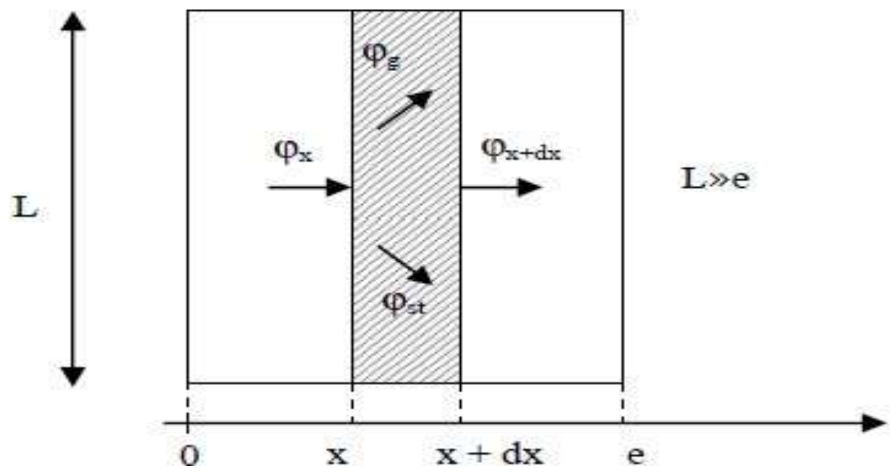


Figure 2. 2. Energy balance for an elementary system

The energy balance on this system is written:

$$\Phi_x + \Phi_g = \Phi_{x+dx} + \Phi_{st} \qquad (2.6)$$

Avec :

$$\Phi_x = -\left(\lambda S \frac{\partial T}{\partial x}\right)_x \quad \text{et} \quad \Phi_{x+dx} = -\left(\lambda S \frac{\partial T}{\partial x}\right)_{x+dx}$$

$$\Phi_g = \dot{q}.dV = \dot{q}.S.dx$$

$$\Phi_{st} = \frac{\partial Q}{\partial t} = m.c.\frac{\partial T}{\partial t} = \rho.V.c.\frac{\partial T}{\partial t} = \rho.c.S.dx.\frac{\partial T}{\partial t}$$

By reporting in the energy balance and dividing by dx, we obtain:

$$\frac{\left(\lambda S \frac{\partial T}{\partial x}\right)_{x+dx} - \left(\lambda S \frac{\partial T}{\partial x}\right)_x}{dx} + \dot{q}.S = \rho.c.S.\frac{\partial T}{\partial t} \qquad (2.7)$$

From Taylor's development:

$$\left(\lambda S \frac{\partial T}{\partial x}\right)_{x+dx} = \left(\lambda S \frac{\partial T}{\partial x}\right)_x + \frac{\partial}{\partial x}\left(\lambda S \frac{\partial T}{\partial x}\right) dx$$

Soit ;

$$\frac{\partial}{\partial x}\left(\lambda S \frac{\partial T}{\partial x}\right) + \dot{q}.S = \rho.c.S.\frac{\partial T}{\partial t} \qquad (2.8)$$

In the three-dimensional (3D) case, we obtain the following thermal equation:

$$\frac{\partial}{\partial x}\left(\lambda_x \frac{\partial T}{\partial x}\right) + \frac{\partial}{\partial y}\left(\lambda_y \frac{\partial T}{\partial y}\right) + \frac{\partial}{\partial z}\left(\lambda_z \frac{\partial T}{\partial z}\right) + \overset{*}{q} = \rho.c.\frac{\partial T}{\partial t} \qquad (2.9)$$

- Study of particular cases:

a) If the medium is isotropic and homogeneous (The conductivity only depends on the temperature of the point considered):

The heat equation (2.9) can be put in the form:

$$\lambda\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2}\right) + \frac{\partial \lambda}{\partial T}\left[\left(\frac{\partial T}{\partial x}\right)^2 + \left(\frac{\partial T}{\partial y}\right)^2 + \left(\frac{\partial T}{\partial z}\right)^2\right] + \dot{q} = \rho.c.\frac{\partial T}{\partial t} \qquad (2.10)$$

b) The temperature or presents a negligible difference:

This is particularly important in the case of a homogeneous and isotropic material with a constant λ coefficient. Consequently, the previous sentence becomes:

$$\lambda\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}+\frac{\partial^2 T}{\partial z^2}\right)+\dot{q}=\rho.c.\frac{\partial T}{\partial t} \qquad (2.11)$$

c) λ does not depend on the temperature and the internal heat release is negligible:

We have :

$$\lambda\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}+\frac{\partial^2 T}{\partial z^2}\right)=\rho.c.\frac{\partial T}{\partial t} \qquad (2.12)$$

Expression that we usually put in the form (called Poisson equation):

$$a\nabla^2 T=\frac{\partial T}{\partial t} \qquad (2.13)$$

The ratio $a = \lambda / \rho C$ is called thermal diffusivity (m2.s-1) which characterizes the speed of propagation of a heat flow through a material.

d) Constant temperature (does not depend on time):

It is the analysis of a steady state with or without heat release. If we assume that the conductivity λ is a constant and does not depend on temperature, we obtain:

- ✓ if the internal source of heat: $q \neq 0$

$$\lambda\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}+\frac{\partial^2 T}{\partial z^2}\right)+\dot{q}=0 \qquad (2.14)$$

- ✓ without internal heat release: q = 0, (we obtain the **Laplace** equation):

$$\nabla^2 T = 0$$

2.4.2. In cylindrical Cartesian coordinates:

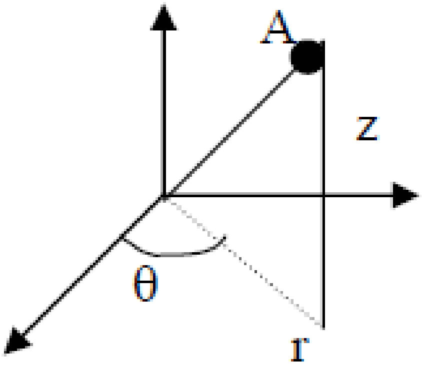


Figure 2. 3. Cylindrical coordinate system

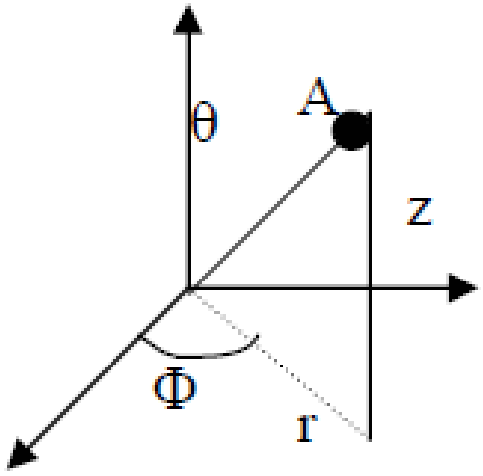
Either:

$$\begin{bmatrix} x = r.\cos\phi \\ y = r.\sin\phi \\ z = z \end{bmatrix}$$

$$\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r} + \frac{1}{r^2}\frac{\partial^2 T}{\partial \theta^2} + \frac{\partial^2 T}{\partial z^2} + \frac{\dot{q}}{\lambda} = \frac{1}{a}\frac{\partial T}{\partial t} \qquad (2.16)$$

If the case of a cylindrical symmetry problem where the temperature is only determined by r and t, equation (2.16) can be written as follows:

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) + \frac{\dot{q}}{\lambda} = \frac{1}{a}\frac{\partial T}{\partial t} \qquad (2.17)$$



2.4.3. In spherical coordinates:

Soient :
$$\begin{bmatrix} x = r.\cos\phi.\sin\theta \\ y = r.\sin\phi.\sin\theta \\ z = r.\cos\theta \end{bmatrix}$$

$$\frac{1}{r}\left(\frac{\partial^2 (rT)}{\partial r^2}\right)+\frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial T}{\partial\theta}\right)+\frac{1}{r^2\sin^2\theta}\left(\frac{\partial^2 T}{\partial\varphi^2}\right)+\frac{\dot{q}}{\lambda}=\frac{1}{a}\frac{\partial T}{\partial t} \quad (2.18)$$

2.5. Unidirectional conductive transfer:

2.5.1. Case of a simple wall

We assume that heat transfer is unidirectional and we neglect energy generation and storage.

Considering a homogeneous wall of thickness e, section S, thermal conductivity λ, whose faces are at temperatures T1 and T2, the heat flux which passes through this wall is:

$$\Phi = -\lambda.S.\frac{dT}{dx} \quad (2.19)$$

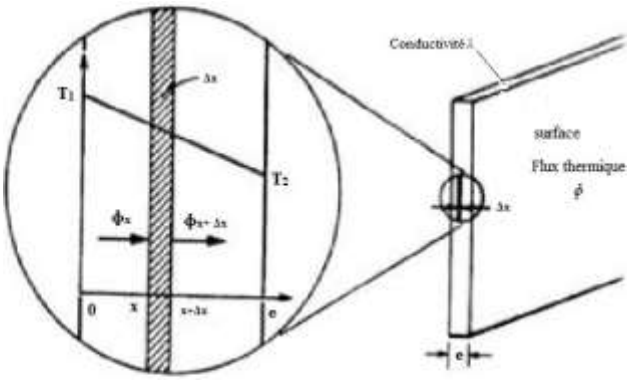


Figure 2. 4. Single wall

If we carry out an energy balance on the system (S) which contains a section of wall between the abscissa x and x +dx, it comes:

$$\Phi_x = \Phi_{x+dx} \Rightarrow -\lambda S\left(\frac{\partial T}{\partial x}\right)_x = -\lambda S\left(\frac{\partial T}{\partial x}\right)_{x+dx} \tag{2.20}$$

From where ;

$$\frac{\partial}{\partial x}\left(\frac{\partial T}{\partial x}\right) = 0$$

With the boundary conditions: T (x = 0) = T1 and T (x = e) = T2

D'où

$$T = T_1 - \frac{x}{e}(T_1 - T_2) \tag{2.21}$$

We find a linear profile for the temperature. The heat flow passing through the wall is calculated by:

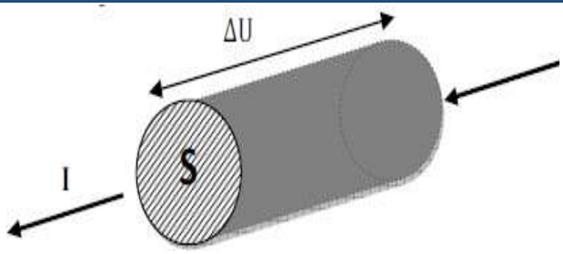
$$\Phi = -\lambda.S.\frac{dT}{dx}$$

$$\Phi = -\lambda.S.\frac{T_2 - T_1}{e} = \lambda.S.\frac{T_1 - T_2}{e} \tag{2.22}$$

- Concept of thermal resistance (electrical analogy):

An electric wire of length L, section S and thermal conductivity Ke subjected to a potential difference ΔU, lets an electric current pass such that:

$$\Delta U = U_0 - U_1 = \frac{L}{K_e \cdot S} \cdot I \text{ ou encore ; } \Delta U = R_e \cdot I$$



This equation is analogous to that giving the thermal potential:

$$\Delta T = T_1 - T_2 = \left(e\ \lambda \cdot S\right) \cdot \Phi = R_{th} \cdot \Phi \qquad (2.23)$$

We establish the following correspondence:

Table 2.2: Electrical-thermal analogy.

Thermique			électricité	
Loi de Fourier	$\Delta T = -(e/\lambda.S).\Phi$	⇔	$\Delta U = R.I$	**Loi d'Ohm**
Conductivité thermique	$\lambda(T)$	⇔	$\sigma(T)$	conductivité électrique
Température	T	⇔	V	potentiel électrique
Puissance thermique	Φ	⇔	I	intensité de courant
Résistance thermique	$R_{th} = e/\lambda.S$	⇔	R	Résistance électrique

For the thermal problem, we establish its electrical equivalent:

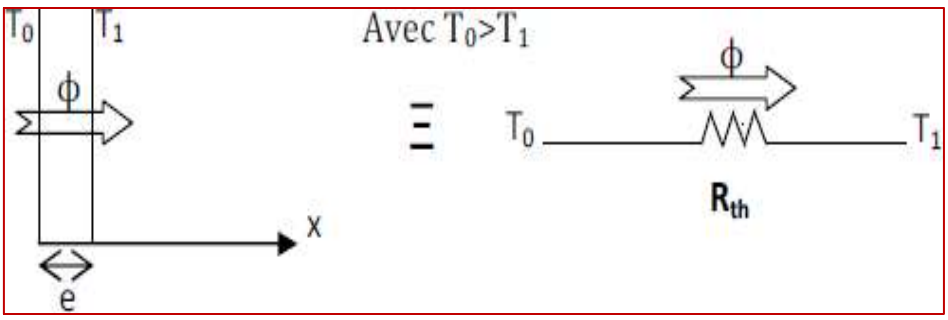


Figure 2. 5. Equivalent electrical diagram of a simple wall

In the case of a flat wall of thickness e, the thermal resistance is:

$$R_{th} = \frac{e}{\lambda.S} \quad \text{en °C/W.} \tag{2.24}$$

The total thermal resistance is determined as in electricity, we know that:

1) Two thermal resistances are added in the case where they are in series:

$$R_{eq} = R_1 + R_2$$

2) The inverse of the thermal resistances are added in the case where they are in parallel:

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} \rightarrow R_{eq} = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2}}$$

2.5.2. Case of a multi-layer wall

In steady state, the heat flow is conserved when crossing the wall and is written:

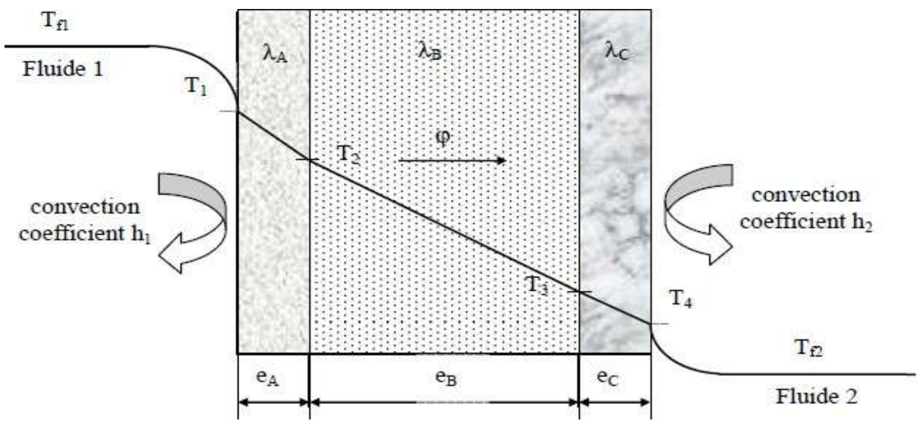


Figure 2. 6. multi-layer wall

$$\Phi = h_1 S\left(T_{f1} - T_1\right) = \frac{\lambda_A S\left(T_1 - T_2\right)}{e_A} = \frac{\lambda_B S\left(T_2 - T_3\right)}{e_B} = \frac{\lambda_C S\left(T_3 - T_4\right)}{e_C} = h_2 S\left(T_4 - T_{f2}\right)$$

$$\Phi = \frac{T_{f1} - T_{f2}}{\frac{1}{h_1 S} + \frac{e_A}{\lambda_A S} + \frac{e_B}{\lambda_B S} + \frac{e_C}{\lambda_C S} + \frac{1}{h_2 S}} \tag{2.25}$$

The thermal resistance of a multi-layer wall is:

$$R_{th} = \frac{T_{f1} - T_{f2}}{\Phi} = \frac{1}{h_1 S} + \frac{e_A}{\lambda_A S} + \frac{e_B}{\lambda_B S} + \frac{e_C}{\lambda_C S} + \frac{1}{h_2 S} \qquad (2.26)$$

The interactions between the different types of layers were considered perfect and there was no temperature discontinuity at the interfaces. Indeed, due to the roughness of the surfaces, a microlayer of air exists between the hollows of the surfaces in view, which contributes to the formation of thermal resistance (air is an insulator) called thermal contact resistance. The previous equation then becomes:

$$\Phi = \frac{T_{f1} - T_{f2}}{\frac{1}{h_1 S} + \frac{e_A}{h_A S} + R_{AB} + \frac{e_B}{h_B S} + R_{BC} + \frac{e_C}{h_C S} + \frac{1}{h_2 S}} \qquad (2.27)$$

The equivalent electrical diagram is shown in Figure 2.7.

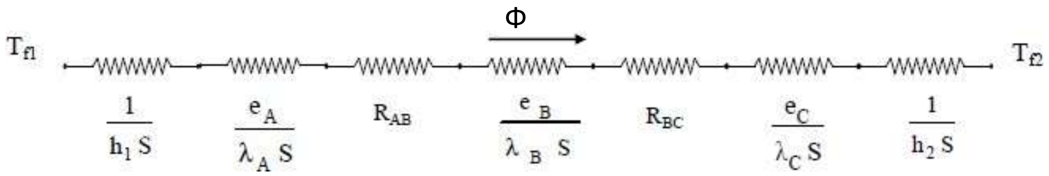


Figure 2. 7. Equivalent electrical diagram of a multi-layer wall.

2.5.3. Case of a composite wall:

The equivalent thermal resistance R of a portion of wall of width L and height using the laws of association of resistances in series and in parallel by the relationship (figure 2.8):

$$R_{th} = R_1 + R_2 + \frac{1}{\frac{1}{R_3} + \frac{1}{R_4} + \frac{1}{R_5}} + R_6 + R_7 \quad (2.28)$$

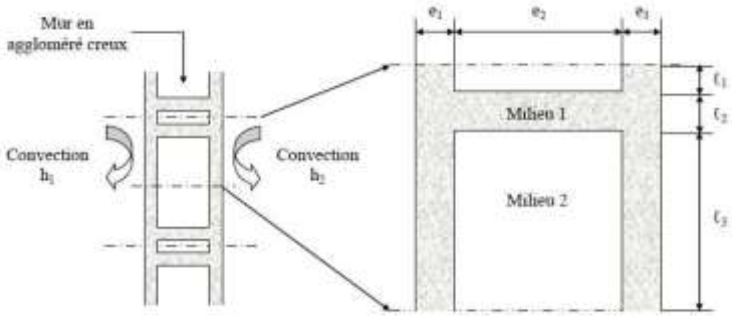


Figure 2. 8. Composite wall

With:

$$R_1 = \frac{1}{h_1 lL} \; ; \; R_2 = \frac{e_1}{\lambda_1 lL} \; ; \; R_3 = \frac{e_2}{\lambda_2 l_1 L} \; ; \; R_4 = \frac{e_2}{\lambda_1 l_2 L} \; ; \; R_5 = \frac{e_2}{\lambda_2 l_3 L} \; ; \; R_6 = \frac{e_3}{\lambda_1 lL} \; ; \; R_7 = \frac{1}{h_2 lL}$$

The equivalent electrical diagram is given in Figure 2.9.

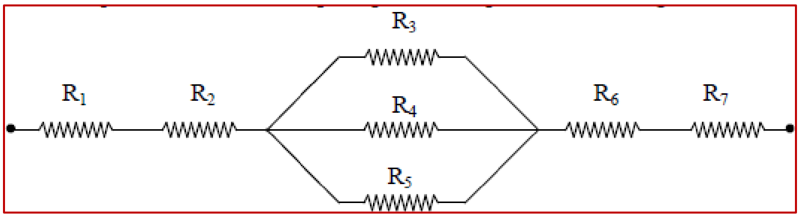


Figure 2. 9. Equivalent electrical diagram of a composite wall

2.5.4. Case of a long hollow cylinder (tube)

We assume a hollow cylinder with thermal conductivity λ interior and exterior of the cylinder, T_1 and T_2 are the temperatures of the internal and external walls (figure 2.10). The temperature gradient is considered to vary only radially.

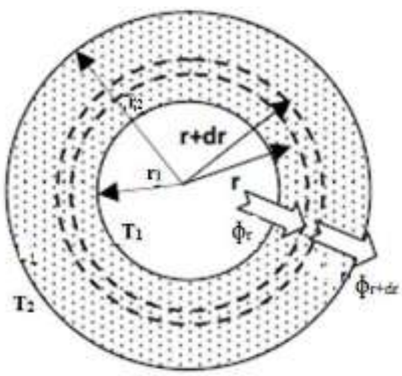


Figure 2. 10. Diagram of transfers in a hollow cylinder

Let us carry out the thermal balance of the system constituted by the part of the cylinder between the radii r and r + dr:

$$\Phi_r = \Phi_{r+dr} \qquad (2.29)$$

Avec : $\Phi_r = -\lambda 2\pi r L \left(\frac{dT}{dr}\right)_r$ et $\Phi_{r+dr} = -\lambda 2\pi (r+dr) L \left(\frac{dT}{dr}\right)_{r+dr}$

Soit : $-\lambda 2\pi r L \left(\frac{dT}{dr}\right)_r = -\lambda 2\pi (r+dr) L \left(\frac{dT}{dr}\right)_{r+dr}$ d'où $r\frac{dT}{dr} = C$

Avec les conditions aux limites : $T(r_1) = T_1$ et $T(r_2) = T_2$

D'où :

$$\frac{T(r) - T_1}{T_2 - T_1} = \frac{\ln\left(\frac{r}{r_1}\right)}{\ln\left(\frac{r_2}{r_1}\right)} \qquad (2.30)$$

And by application of the relation $\Phi = -\lambda 2\pi r L \left(\frac{dT}{dr}\right)_r$ we obtain: This relation can also be put in the form:

$$\Phi = \frac{2\pi\lambda L (T_1 - T_2)}{\ln\left(\frac{r_2}{r_1}\right)} \qquad (2.31)$$

This relationship can also be put in the form:

$$\Phi = \frac{(T_1 - T_2)}{R_{12}} \quad \text{avec} \quad R_{12} = \frac{\ln\left(\frac{r_2}{r_1}\right)}{2\pi\lambda L}$$

And be represented by the equivalent electrical diagram in Figure 2.11.

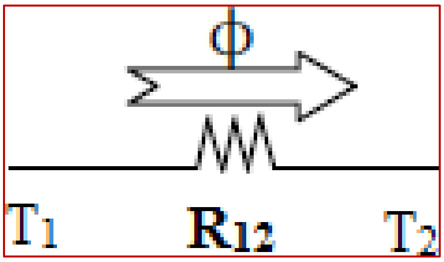


Figure 2. 11. Equivalent electrical diagram of a hollow cylinder

2.5.5. Case of a multilayer hollow cylinder

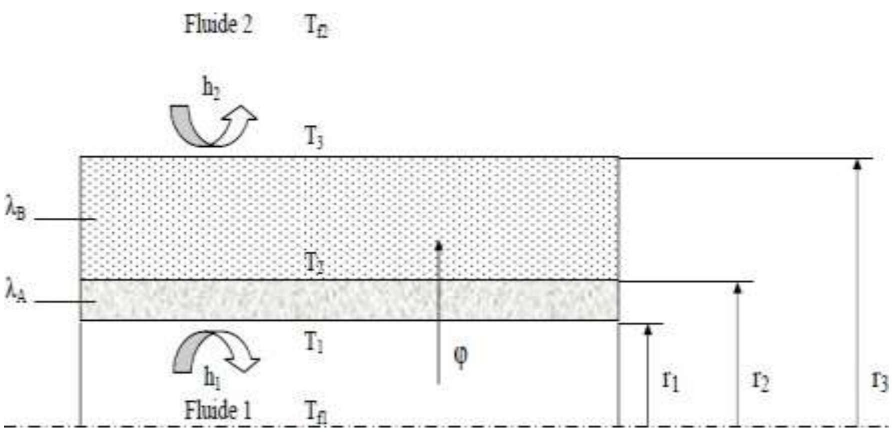


Figure 2. 12. Diagram of transfers in a multilayer hollow cylinder.

In steady state, the heat flow φ is preserved when crossing the different layers and is written (figure 2.12):

$$\Phi = h_1 2\pi r_1 L\left(T_{f1} - T_1\right) = \frac{2\pi\lambda_A L\left(T_1 - T_2\right)}{\ln\left(\frac{r_2}{r_1}\right)} = \frac{2\pi\lambda_B L\left(T_2 - T_3\right)}{\ln\left(\frac{r_3}{r_2}\right)} = h_2 2\pi r_3 L\left(T_3 - T_{f2}\right) \qquad (2.32)$$

D'où :

$$\Phi = \frac{T_{f1} - T_{f2}}{\frac{1}{h_1 2\pi r_1 L} + \frac{\ln\left(\frac{r_2}{r_1}\right)}{2\pi\lambda_A L} + \frac{\ln\left(\frac{r_3}{r_2}\right)}{2\pi\lambda_B L} + \frac{1}{h_2 2\pi r_3 L}} \qquad (2.33)$$

Which can be represented by the equivalent electrical diagram in Figure 2.13.

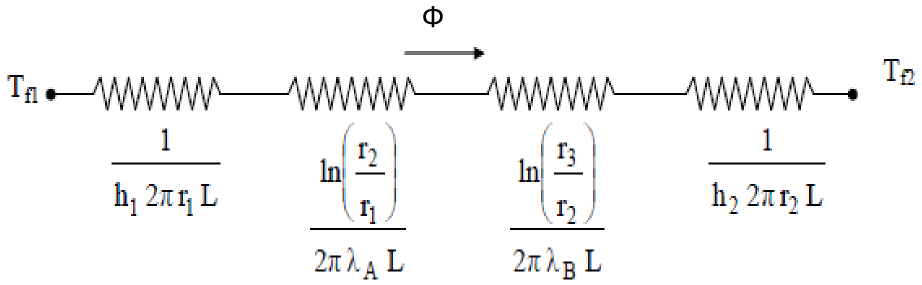


Figure 2. 13. Equivalent electrical diagram of a multilayer hollow cylinder.

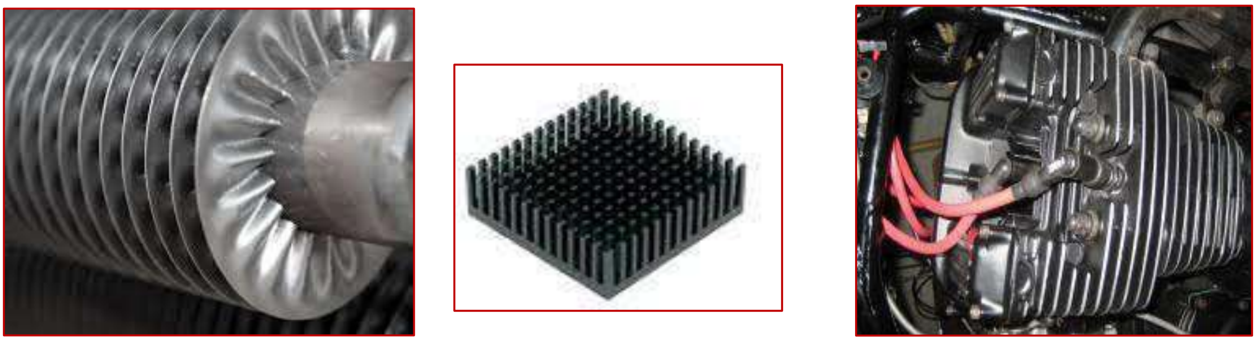
- The thermal resistance of a multilayer hollow cylinder is:

$$R_{th} = \frac{T_{f1} - T_{f2}}{\Phi} = \frac{1}{h_1 2\pi r_1 L} + \frac{\ln\left(\frac{r_2}{r_1}\right)}{2\pi\lambda_A L} + \frac{\ln\left(\frac{r_3}{r_2}\right)}{2\pi\lambda_B L} + \frac{1}{h_2 2\pi r_3 L}$$

2.5.7 Fin theory

2.5.7.1 General

The fins are good conductors of heat with a greater surface area than the rest of the body. They are used to improve heat removal from a solid-state containment system with high flux densities.



Finned tube (radiator) Heat sinks Motorcycle engine.

Figure 2. 14. Examples of finned systems used in different application sectors

2.5.7.2. The bar equation

Performing the energy balance, the temperature profile in the bar is the solution of the following differential equation, called the bar equation:

$$\frac{d^2T}{dx^2} - \frac{h.P_e}{\lambda.S}.(T - T_\infty) = 0$$

2.5.7.2.1. Flow extracted by a fin

A fin is a medium that is a good conductor of heat, one dimension of which is large compared to the others. By posing:

$$\omega^2 = \frac{h.P_e}{\lambda\, S} \quad et\ \theta = (T - T_\infty)$$

The previous equation becomes:

$$\frac{d^2\theta}{dx^2} - \omega^2.\ \theta = 0$$

2.6. Multi-directional conductive transfer

In the case where the heat diffusion does not take place in a single direction, two resolution methods can be applied:

2.6.1. Form coefficient method

In two-dimensional or three-dimensional systems where only two limiting temperatures T_1 and T_2 intervene, we show that the heat flow can be put in the form:

$$\varphi = \lambda . F(T_1 - T_2)$$

With :

ʎ : Thermal conductivity of the medium separating the surfaces S1 and S2 (W m^{-1} K^{-1})

T_1: Surface temperature S_1 (K)

T_2: Surface temperature S_2 (K)

F: Shape coefficient (m)

The shape coefficient F only depends on the shape, dimensions and relative position of the two surfaces S_1 and S_2.

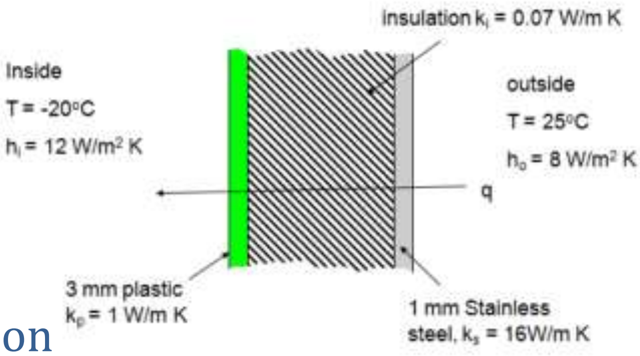
2.6.2. Numerical methods

In the case where the shape coefficient method cannot be applied (non-isothermal surfaces for example), the heat equation must be solved numerically:

- Finite difference method
- Finite element method.
- - Finite volume method…

2.1. Example

An industrial freezer is designed to operate with an internal air temperature of -20°C when the external air temperature is 25°C and the internal and external heat transfer coefficients are 12 W/m^2 K and 8 W/m^2 K, respectively. The walls of the freezer are composite construction, comprising of an inner layer of plastic (k = 1 W/m K, and thickness of 3 mm), and an outer layer of stainless steel (k = 16 W/m K, and thickness of 1 mm). Sandwiched between these two layers is a layer of insulation material with k = 0.07 W/m K. Find the width of the insulation that is required to reduce the convective heat loss to 15 W/m^2.



2.1. Solution

$q = U\Delta T$ where U is the overall heat transfer coefficient given by:

$$U = \frac{q}{\Delta T} = \frac{15}{25-(-20)} = 0.333 W/m^2K$$

$$U = \left[\frac{1}{h_i} + \frac{L_p}{k_p} + \frac{L_i}{k_i} + \frac{L_s}{k_s} + \frac{1}{h_o}\right]^{-1} = 0.333$$

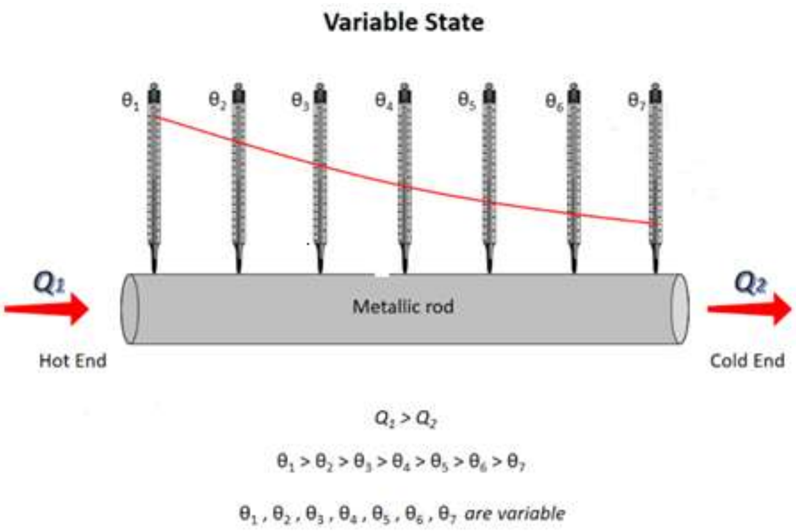
$$\left[\frac{1}{h_i} + \frac{L_p}{k_p} + \frac{L_i}{k_i} + \frac{L_s}{k_s} + \frac{1}{h_o}\right] = \frac{1}{0.333}$$

$$L_i = k_i\left\{\frac{1}{0.333} - \left[\frac{1}{h_i} + \frac{L_p}{k_p} + \frac{L_s}{k_s} + \frac{1}{h_o}\right]\right\} = 0.07\left\{\frac{1}{0.333} - \left[\frac{1}{12} + \frac{0.003}{1} + \frac{0.001}{16} + \frac{1}{8}\right]\right\}$$

$L_i = 0.195m$ (195 mm)

Chapter 3

Heat transfer by conduction in variable regime



Heat transfer by conduction in variable regime

3.1. Introduction

In this chapter, we will present thermal conduction in variable regime, that is to say that the temperature varies, not only in space but also with time.

3.1 Unidirectional conduction in variable regime without change of state:

❖ Main problem

The general equation for heat is given by:

$$div(\lambda \overrightarrow{grad}\, T) + P = \rho\, C\, \frac{\partial T}{\partial t} \tag{3.1}$$

It requires an initial condition at any point T (t=0), and two boundary conditions. In the case where the thermal conductivity λ does not depend on the temperature, we obtain the Fourrier equation:

$$\Delta T + \frac{P}{\lambda} = \frac{1}{a}\frac{\partial T}{\partial t} \tag{3.2}$$

- Dimensionless numbers:

The problem of thermal conduction (unidirectional case) is represented by the following system of equations:

$$\begin{cases} \dfrac{\partial^2 T}{\partial x^2} + \dfrac{P}{\lambda} = \dfrac{1}{\alpha}\dfrac{\partial T}{\partial t} & pour \quad 0 \prec x \prec l; \quad t \succ 0 \\ \dfrac{\partial T}{\partial x} = 0 & pour \quad x = 0; \quad t \succ 0 \\ \lambda \dfrac{\partial T}{\partial x} = -h\left(T - T_\infty\right) & pour \quad x = l; \quad t \succ 0 \\ T = T_0 & pour \quad 0 \prec x \prec l; \quad t = 0 \end{cases} \tag{3.3}$$

The number of variables in a heat transfer problem can be reduced by the introduction of dimensionless numbers:

$$x^* = \frac{x}{l}; \quad \theta = \frac{T - T_\infty}{T_0 - T_\infty}; \quad G = \frac{Pl^2}{\lambda T_0 - T_\infty}$$

System (3.3) becomes:

$$\begin{cases} \dfrac{\partial^2 \theta}{\partial x^{*2}} + G = \dfrac{\partial \theta}{\partial F_0} & pour \quad 0 \prec x^* \prec l; \quad F_0 \succ 0 \\ \dfrac{\partial \theta}{\partial x^*} = 0 & pour \quad x^* = 0; \quad F_0 \succ 0 \\ \lambda \dfrac{\partial \theta}{\partial x^*} = -B_i \theta & pour \quad x^* = l; \quad F_0 \succ 0 \\ \theta = 1 & pour \quad 0 \prec x^* \prec l; \quad F_0 = 0 \end{cases} \tag{3.4}$$

In the variable regime, two dimensionless numbers are important:

- The Biot number: is the ratio between the internal thermal resistance and the external thermal resistance, l is the characteristic languor of the medium (l = r for a sphere).

$$B_i = \frac{hl}{\lambda} \tag{3.5}$$

It calculates the thermal thickness of the domain: a medium is thermally thin if the Biot number is less than 1 (i.e. the external resistance blocks the heat flow). We can also consider that the temperature is uniform depending on the dimension of l.

- Fourier number: is the ratio of flow through l2 to the storage rate in l3, or the ratio of heat passing through to stored heat. The Fourier number describes the penetration of heat under varying conditions.

$$F_0 = \frac{\lambda \dfrac{\Delta T}{l} l^2}{\rho C l^3 \dfrac{\Delta T}{l}} = \frac{\alpha t}{l^2} \tag{3.6}$$

3.1.1 Uniform temperature environment (zero gradient method):

Heat transfer to a uniformly heated medium will be studied in this course, which is initially counterintuitive because a thermal gradient is necessary for heat transfer to occur. This proximity of medium at a constant temperature can however be justified in certain cases which we will specify.

For example, consider the quenching of a metal ball, which consists of immersing a ball initially at temperature Ti in a bath at constant temperature T0. If we assume that the temperature inside the ball is constant, which is more likely given its small size and its high thermal conductivity, we can write the heat balance of the ball between two times t and t +dt:

$$-h \cdot S \cdot (T - T_0) = \rho \cdot V \cdot c \frac{\partial T}{\partial t} \quad \text{soit ;} \quad \frac{dT}{T - T_0} = -\frac{h \cdot S}{\rho \cdot V \cdot c} dt$$

D'où :

$$\frac{T - T_0}{T_i - T_0} = \exp\left(-\frac{h \cdot S}{\rho \cdot V \cdot c} t\right) \tag{3.7}$$

We notice that the grouping $\frac{h \cdot S}{\rho \cdot V \cdot c}$ is homogeneous at one time, we will call it

The constant of system time: $\tau = \frac{\rho \cdot V \cdot c}{h \cdot S}$

This quantity is fundamental insofar as it gives the order of magnitude of time of the physical phenomenon, we have in fact:

$$\frac{T - T_0}{T_i - T_0} = \exp\left(-\frac{t}{\tau}\right) \tag{3.8}$$

The definition of these two numbers (Biot number and Fourier number) allows us to write the expression for the temperature of the ball in the form:

$$\frac{T - T_0}{T_i - T_0} = \exp(-B_i \times F_o) \tag{3.9}$$

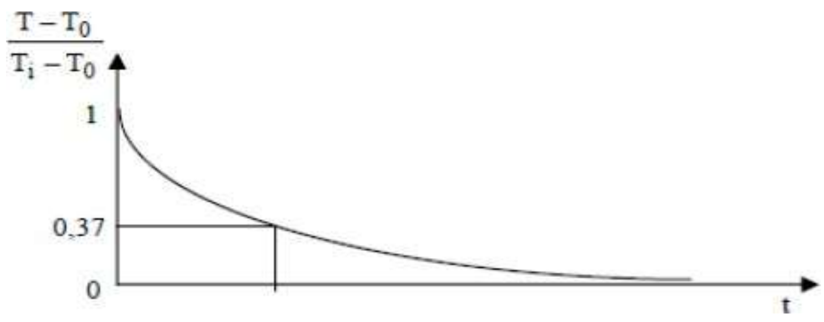


Figure 3. 1. Evolution of the temperature of a medium at uniform temperature

The product of the Biot and Fourier numbers can be used to calculate the evolution of the temperature of the sphere. The Bi < 0.1 criterion is generally called the "thermal accommodation" criterion because a device with Bi < 0.1 can be considered to be at uniform temperature.

3.1. Example

For the fin of example 4.5, a fan was used to improve the thermal performance, and as a result, the heat transfer coefficient is increased to 40 W/m^2 K. Justify the use of the lumped mass approximation to predict the rate of change of temperature with time. Using the lumped mass approximation given below, calculate the time taken, τ, for the heat sink to cool from 60°C to 30°C.

$$(T - T_f) = (T_i - T_f)\exp-\left(\frac{hA_s\tau}{mC}\right)$$

2.1 Solution

Consider a single fin (the length scale L for the Biot number is half the thickness t/2)

$$B_i = \frac{hL}{k} = \frac{h \times t/2}{k} = \frac{40 \times 0.0005}{175} \approx 10^{-4}$$

Since $B_i << 1$, we can use he "lumped mass" model approximation.

$$\frac{(T-T_f)}{(T_i-T_f)} = \exp-\left(\frac{hA_s\tau}{mC}\right)$$

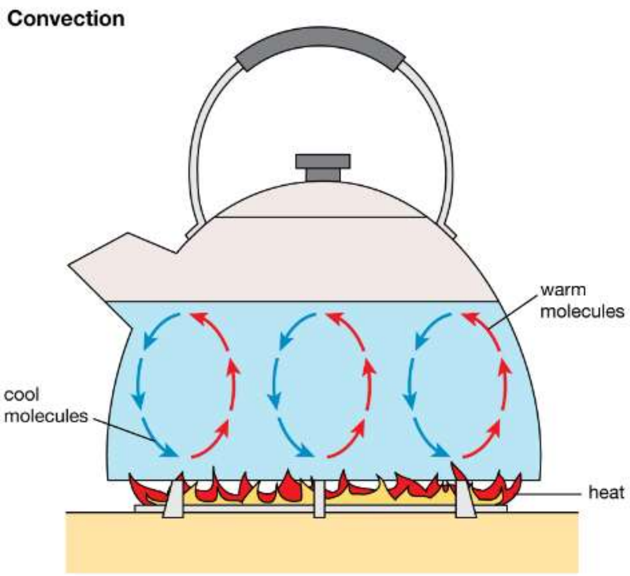
$$\tau = -\frac{mC}{hA_s}\ln\left(\frac{T-T_f}{T_i-T_f}\right)$$

$$m = \rho A_s t/2$$

$$\tau = -\frac{\rho Ct}{2h}\ln\left(\frac{T-T_f}{T_i-T_f}\right) = -\frac{2700\times900\times0.001}{2\times40}\ln\left(\frac{30-20}{60-20}\right) = 42 \text{ seconds}$$

Chapter 4

Heat transfer by convection



Heat transfer by convection

4.1. Introduction

When heat transfer is accompanied by mass transfer, it is called convective transfer. This mode of heat exchange occurs in non-isothermal fluid environments or when a fluid circulates around a solid. Convection is a mode of transport of energy by the combined action of conduction, the accumulation of energy and the movement of the fluid medium. The study of heat transfer by convection essentially makes it possible to determine the heat exchanges occurring between a fluid and a wall. The quantity of heat exchanged per unit of time depends on several parameters:
the temperature difference between the wall and the fluid;

- fluid speed;
- the heat capacity of the fluid;
- the exchange surface;
- the surface condition of the solid;
- its size
- …etc.

4.1. Natural convection-forced convection

Heat transmission by convection is designated, depending on the mode of fluid flow, by natural convection (also called free) and forced convection:

- When currents occur within the fluid simply due to temperature differences, we say that the convection is natural or free.
- On the other hand, if the movement of the fluid is caused by an external action, such as a pump or a fan, the process is called forced convection.

4.2. Newton's law

Newton's law gives the expression for the quantity of heat δQ exchanged between the surface of a solid at temperature Ts and the fluid at temperature Tf (often denoted T∞).

The study of heat transfer by convection makes it possible to determine the heat exchanges occurring between a fluid and a wall.

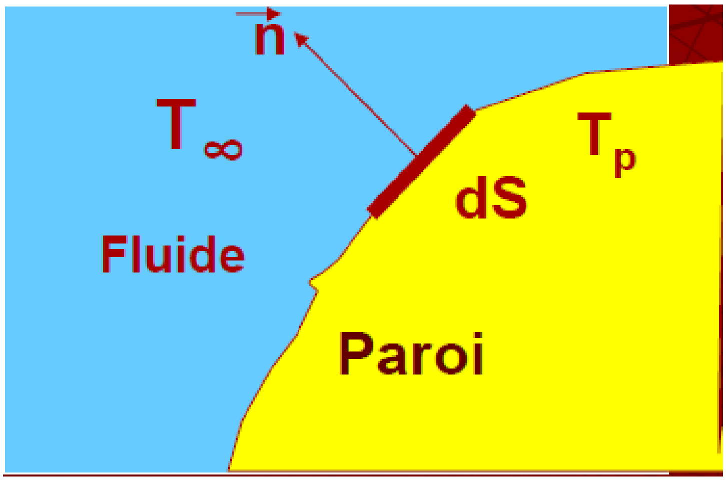


Figure 4. 1. Newton law

The quantity of heat δQ which passes through dS during the time interval dt can be written:

$$\delta Q = h\, dS\, (T_p - T_\infty)\, dt$$

- h is the convection exchange coefficient, it is expressed in W/(m2.K).
- δQ is expressed in Joules.
- δQ/dt in Watts.

Whatever the type of convection (natural or forced) and whatever the flow regime of the fluid (laminar or turbulent), the transmitted heat flow is given by the relationship known as Newton's law (empirical relationship):

$$\phi = h\, dS\, (T_p - T_\infty)$$

- ϕ is the transmitted power (W)
- h is the exchange coefficient (in W/(m^2.K))

- $T_p - T_\infty$ is the temperature difference between the body and the fluid (in K).
- dS is the exchange surface (in m^2)

4.2 Convective exchange coefficient h

The major problem to resolve before any calculation of the heat flow consists of determining the convective exchange coefficient h which depends on numerous parameters:

- Type of convection (natural or forced)
- fluid characteristics,
- nature of the flow,
- temperature,
- the shape of the exchange surface,
- ...etc.

4.3. Natural convection

4.3.1 Principle of natural convection

The hot particle starts moving and directly ensures the transfer of heat towards the colder medium: The regime becomes convective.

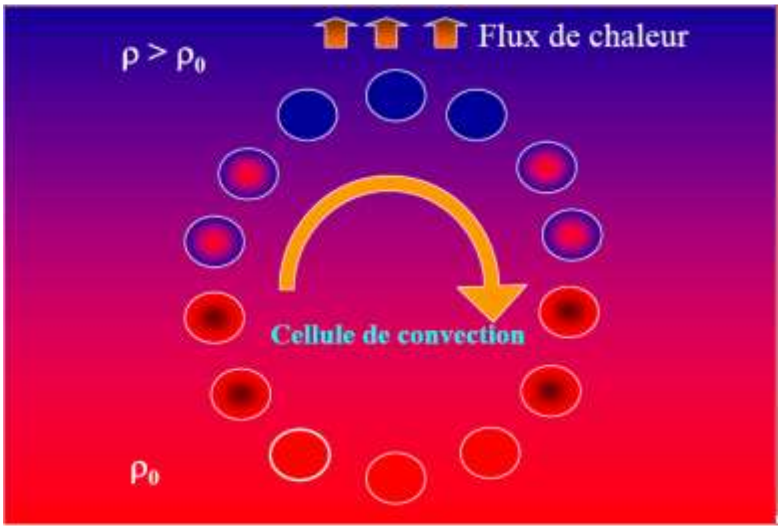


Figure 4. 2. Heat transfer by convection

4.4. Dynamic study

Consider a horizontal surface (S) at a temperature Ts in contact with a still fluid of density ρ at a temperature Tf.

A particle (P) of the fluid of volume V in contact with the surface (S) has a temperature close to Ts.

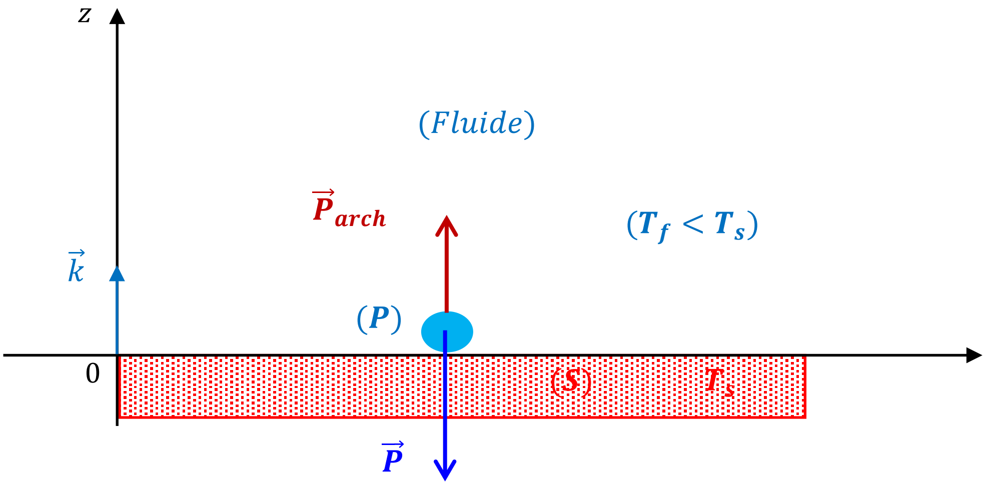


Figure 4. 3. Dynamic study

- Balance of forces acting on the particle (P):
- Archimedes' thrust: $\vec{P}_{arch} = \rho(Tf).V.g\,\vec{k}$
- The weight: $\vec{P} = -mg\vec{k} = -\rho(Ts).V.g\,\vec{k}$

As Ts>Tf, we of course have $\rho(Tf) > \rho(Ts)$ and consequently $\|\vec{P}_{arch}\| > \|\vec{P}\|$, which allows us to conclude that the densest parts are located below the least dense. Movements in the fluid will then be favored: this is the phenomenon of natural convection.

The fundamental principle of dynamics is written in this case:

$$\sum \vec{F}_{ext} = m\vec{\gamma}$$
$$\Rightarrow \vec{P}_{arch} + \vec{P} = \rho(Ts).V\,\vec{\gamma}$$
$$\Rightarrow \rho(Tf).V.g - \rho(Ts).V.g = \rho(Ts).V\,(d^2 z)/(dt^2)$$

The equation of movement of the particle (P) in the immediate vicinity of (S) is therefore written:

$$\Rightarrow (d^2 z)/(dt^2) = (\rho(Tf) - \rho(Ts))/(\rho(Ts)).g$$

4.5. Study of the convection phenomenon

4.5.1 Boundary layers

The study of flows near the walls is necessary for the determination of heat exchanges by convection between a solid and the fluid which surrounds it. Consider a fluid flowing along a surface S:

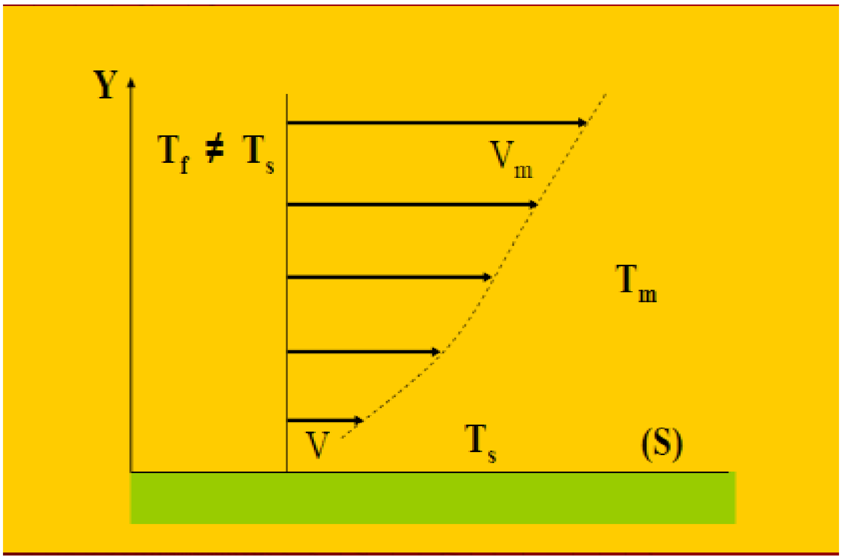


Figure 4. 4. Boundary lyers

- Far from the surface, the fluid has an average speed Vm and an average temperature Tm.
- In the immediate vicinity of the surface, the temperature of the fluid is very close to that of the surface. The speed of the fluid is almost zero.

The speed and temperature diagrams, in the direction perpendicular to the surface (the Y direction), define a layer of fluid called the “boundary layer”whose temperature and speed have the appearance of the following curves:

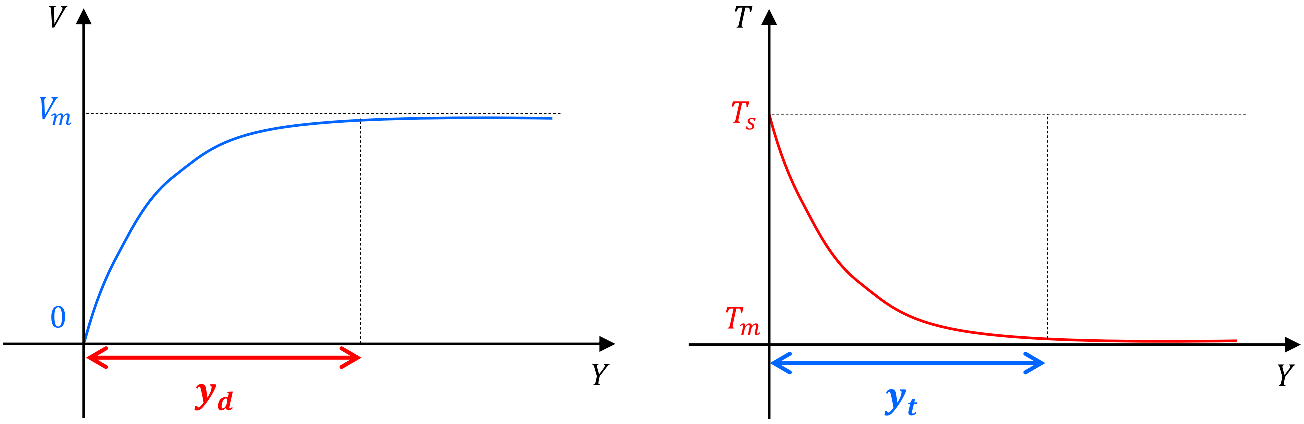


Figure 4. 5. The speed and temperature diagrams

We thus define two types of boundary layers:

- dynamic boundary layer that we note *yd*
- thermal boundary layer that we note *yt*

It should be noted that these two boundary layers generally have different thicknesses.

The velocity and temperature profiles developed within a flowing fluid can be represented like this:

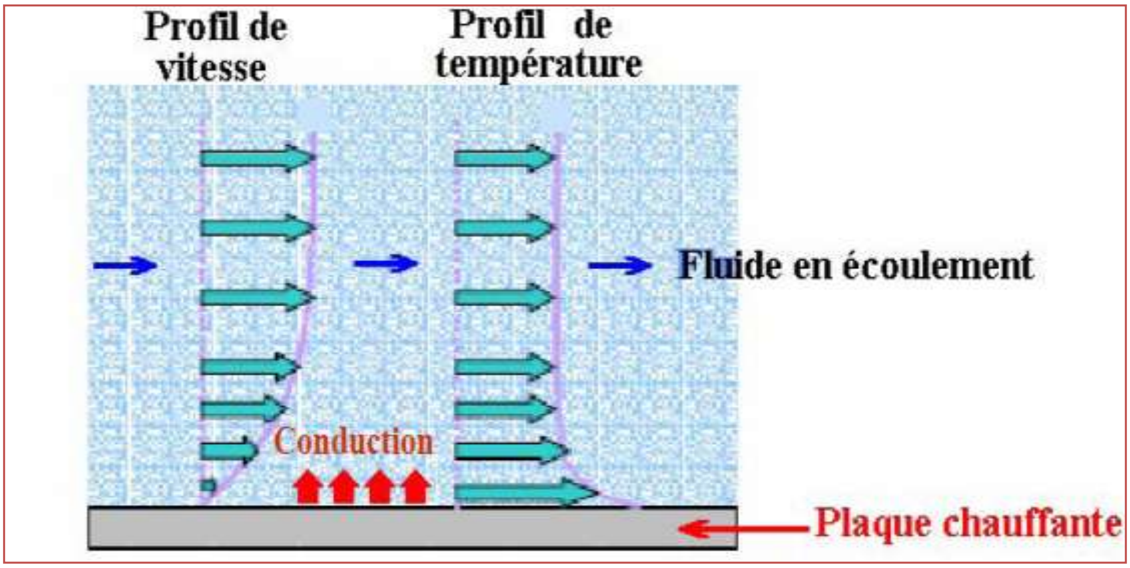


Figure 4. 6. The velocity and temperature profiles

Note that in the vicinity of the plate, dynamic and thermal boundary layers develop in which variations in speed and temperature are observed.

Heat transfer from the plate to the fluid results from two mechanisms:

- In the immediate vicinity of the surface, the transfer takes place by conduction.
- Far from the surface the transfer also results from the movement of the fluid.

In the boundary layer, if we admit that the heat transfer takes place essentially by conduction, therefore without transfer of matter in the y direction, we can write:

- The amount of heat through the surface (S):

$$\delta Q = -\lambda_s . \frac{\partial T_s}{\partial y} . dS . dt \qquad (1)$$

Ts is the surface temperature of the solid.

- The amount of heat through the boundary layer:

$$\delta Q = -\lambda_f . \frac{\partial T_f}{\partial y} . dS . dt \qquad (2)$$

Tf is the average temperature of the fluid quite far from the wall of the solid.

However, the variations of Tf and Ts as a function of y are generally not known to be able to deduce δQ from equalities (1) and (2). Newton's law allows this difficulty to be avoided by using only the temperature difference (Ts- Tf):

$$\delta Q = h.(T_s - T_f).dS.$$

4.5.2 Nature of the convective exchange coefficient h

The convective exchange coefficient, h, depends on several parameters and the heat exchange is all the more active (i.e. h greater) when:

- The fluid flow speed v is greater.
- Its specific heat c_p is greater.
- Its thermal conductivity λ (or its thermal diffusivity) is higher.
- Its kinematic viscosity ν=μ/ρ is lower.

h can also depend on the dimensions of the wall, d, its nature and its shape.We can then write: $\boldsymbol{h = h(\mathrm{v}, c_p, \lambda, \mu, d)}$

The large number of factors influencing heat transfer by convection explains the difficulty of any theoretical or even experimental study, especially if the coefficients which characterize the material vary with pressure and temperature. Hence the idea of thinking of an effective method allowing the determination of the expression of the coefficient h. The method using what is called "dimensional analysis" seems to be the easiest to implement for determining the expression of the convection coefficient h.

4.5.3 Determination of the coefficient h by the dimensional analysis method

Suppose that h is a function of the variables $\boldsymbol{v, c_p, \lambda, \mu}$ and $\boldsymbol{d}$

$$\boldsymbol{h = h(v, c_p, \lambda, \mu, d)}$$

h is also an implicit function of the temperature T since cp and λ depend on it.

Let's use the equations for the dimensions of each term:

* $[\delta Q] = [energy] = [force][displacement] = M.L^2.t^{(-2)}$

* $[\boldsymbol{\lambda}] = \frac{[\delta Q]}{L.T.t} = \frac{M.L^2.t^{-2}}{L.T.t} = M.L.T^{-1}.t^{-3}$

$$* \ [c_p] = \frac{[\delta Q]}{M.T} = \frac{M.L^2.t^{-2}}{M.T} = L^2.t^{-2}.T^{-1}$$

$$* \ [\mu] = M.L^{-1}.t^{-1}$$

$$* \ [v] = L.t^{-1}$$

$$* \ [d] = L$$

The equation with dimensions of h is obtained from Newton's law:

$$[h] = \frac{[\delta Q]}{[(T_s - T_f)].[dS].[dt]} = \frac{M.L^2.t^{-2}}{T.L^2.t} = M.T^{-1}.t^{-3}$$

By writing [h] in the form:

$$[h] = [c_p]^a.[\lambda]^b.[\mu]^c.[d]^d.[v]^e$$

or

$$[h] = (L^2.t^{-2}.T^{-1})^a.(M.L.T^{-1}.t^{-3})^b.(M.L^{-1}.t^{-1})^c.(L)^d.(L.t^{-1})^e = M.T^{-1}.t^{-3}$$

Note that the fundamental quantities involved in the calculation of h are:

The mass M, the time t, the length L and the temperature T.

Identifying the exponents in the equation with dimensions of h provides a linear system of equations for calculating a, b, c, d and e:

- the exponent of M: b+c=1
- the exponent of T: a+b=1
- the exponent of L: 2a+b-c+d+e=0
- the exponent of t: 2a+3b+c+e=3

4.5.4 Dimensionnels numbers

For convection problems, solving the dimensional equations reveals dimensionless numbers that are very useful in the study of fluid mechanics and in particular in convective phenomena. These numbers are in particular:

- The Reynolds number
- The Nusselt number
- The Prandtl number

a) The Reynolds number

The flow regime of a fluid can be laminar or turbulent. The transition from one regime to another is characterized by the Reynolds number:

$$Re = \frac{v.d}{\nu}$$

In the case of a tube, d represents the internal diameter. Experience shows that for Re less than a critical value Re $_{cr}$=2200, the flow in a pipe is always laminar:

- If $Re<Re_{cr}$: the regime is said to be laminar
- If $Re>Re_{cr}$: the regime is said to be turbulent

b) The Nusselt number

The Nusselt number characterizes the importance of convection compared to conduction. It is the ratio of the quantity of heat exchanged by convection h.S.ΔT to a quantity of heat exchanged by conduction λ.S.ΔT/d:

$$Nu = \frac{h.S.\Delta T}{\lambda.S.\frac{\Delta T}{d}} \quad \Rightarrow \quad Nu = \frac{h.d}{\lambda}$$

Nu is a direct function of h, its knowledge makes it possible to determine the value of h.c.

c) The Prandtl number

It characterizes the speed distribution in relation to the temperature distribution (characterizes the thermal properties of the fluid):

$$Pr = \frac{\mu.c_p}{\lambda}$$

d) The Grashof number:

The viscosity force of the fluid is characterized by the Grashof number:

$$Gr = \frac{g \cdot d^3 \cdot \beta_p \cdot \Delta T}{v^2}$$

d: characteristic dimension of the wall [m].

g: acceleration of gravity [m.s^{-2}].

$\Delta T = T_p - T_f$: Characteristic temperature difference [°C].

β_p : Volume expansion factor of the fluid [°C^{-1}].

pour un gaz parfait : $\beta = \frac{1}{T}$

e) The Rayleigh number:

It is identical to the Reynolds number in natural convection.

$$Ra = \Pr \cdot Gr$$

4.5.5 Thermal resistance

Consider a fluid of temperature T_1 which circulates in the vicinity of a wall of temperature T2. The exchanged heat flow is written:

$$\phi = hS(T_2 - T_1)$$

$$(T_2 - T_1) = \frac{1}{hS}\phi = R_{th}\phi$$

The analogy with Ohm's law makes it possible to define the thermal resistance R_{th} of convection

$$R_{th} = \frac{1}{hS}$$

4.6. Forced convection:

In this section of the chapter we will see how to transfer heat by convection without phase change. The most common experimental correlations in forced convection are of the following type: $Nu = f(\text{Re}, \text{Pr})$

For a given fluid temperature, the Prandtl number, which appears in

the expression of the Nusselt number and characterizes the flowing fluid, must be determined. Alternatively, if there is heat exchange, the surface temperature and the fluid temperature would be different; the fluid properties would simply be taken for the average temperature T_m.

$$T_m = \frac{T_p + T_f}{2}$$

4.6.1. Heat exchange along a flat plate (External forced convection Rec = 5*105):

- Laminar speed: $\text{Re} \leq 5.10^5$

$$Nu_L = 0.66(\text{Re}_L)^{1/2}(\text{Pr})^{1/3}$$

- Turbulent regime: $\text{Re} \succ 5.10^5$

$$Nu_L = 0.036(\text{Re}_L)^{4/5}(\text{Pr})^{1/3}$$

4.6.2. Flow inside smooth cylindrical tubes (Internal forced convection Rec =2300):

Laminar speed: $\text{Re} \leq 2300$

Valid for : $Nu_L = \text{Re}.\text{Pr} \times \frac{D}{L} \prec 100$

Turbulent regime $\text{Re} \succ 2300$

$$Nu_L = 0.023(\text{Re})^{0.8}(\text{Pr})^n$$

Valid for

$$n = 0.3 \quad si \quad T_f \succ T_p$$

$$n = 0.4 \quad si \quad T_f \prec T_p$$

4.6.3. Flow in annular spaces (Internal forced convection)

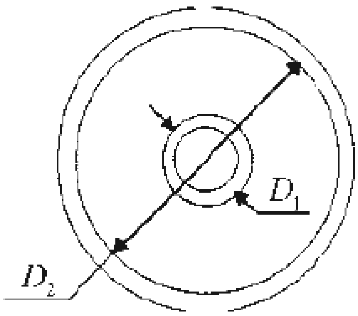


Figure 4. 7. Flow in the annular space formed by the two pipes

$$Nu_{Dh} = 0.023\left(\mathrm{Re}_{Dh}\right)^{0.8}\left(\mathrm{Pr}\right)^{n}$$

Avec ; $\mathrm{Re}_{Dh} = \dfrac{U_m.D_h}{\upsilon}$ et $Nu_{Lh} = \dfrac{h.D_h}{\lambda}$

D_h : Diamètre hydraulique. Dans ce cas: $D_h = D_1 - D_2$

$n = 0{,}4$ pour chauffage

$n = 0{,}3$ pour refroidissement

4.7. Natural convection:

a) Practical method for calculating heat flow in natural convection:

The use of dimensional analysis reveals that the relationship between the flow of heat transported by convection and the factors on which it depends is found in the form of a relationship between three dimensionless numbers: $Nu = f\left(Gr, \mathrm{Pr}\right)$

The following steps are used to calculate a heat flux transmitted by natural convection:

- The determination of the following dimensionless numbers: Gr and Pr.
- According to the result of Gr, we choose an experimental correlation corresponding to the configuration studied.
- By applying this correlation for the calculation of the number Nu.
- Calculation of $h = \dfrac{\lambda \cdot Nu}{L}$

- Using Newton's law to calculate the heat exchanged flow:

$$\Phi = h \cdot S\left(T_p - T_\infty\right)$$

Noticed :

The following should be taken into consideration when calculating h:

- The nature of the fluid (liquid or gas).
- The temperature of the fluid.
- Know the type of convection (natural or forced).
- Know the type of flow regime (laminar or turbulent).
- Know the type of contact between the fluid and surface (flat surface, or circulates between two flat surfaces, or circulates in a tube, etc.).

b) Different correlations for calculating h

We noticed that the relationships describing a natural convection problem can take the following form: $Nu = f\left(Gr, \Pr\right)$

The relationship between these three dimensionless numbers cannot be established theoretically, but must be determined experimentally. Many scientific discoveries have been compiled in the literature. They are known as "experimental correlations." In this section, we will review the most common experimental correlations in natural convection.

The most common experimental correlations in natural convection are of the following type:

$$Nu = C \cdot Ra^n \qquad \text{avec ;} \quad Ra = \Pr \cdot Gr$$

The value of the coefficient C depends on the nature of the regime and the fluids. It is determined by calculating Ra, depending on the value found we choose the suitable values of c and n which are given in the following table:

Géométrie et orientation	Dimension caractéristique	C en convection laminaire n = 1/4	C en convection turbulente n = 1/3
Plaque verticale	Hauteur	0.59 $Ra \leq 10^9$	0.10 $Ra \succ 10^9$
Plaque horizontale chauffante vers le haut	Largeur	0.54 $10^4 \leq Ra \leq 10^7$	0.15 $10^7 \prec Ra \leq 10^{11}$
Plaque horizontale chauffante vers le bas	Largeur	0.27 $10^5 \leq Ra \leq 10^{10}$	0.54 $10^{10} \prec Ra \leq 10^{13}$
Cylindre horizontal	Diamètre extérieure	$C = 1.02$ et $n = 0.148$ $10^{-2} \leq Ra \leq 10^2$	0.135 $2\times10^7 \prec Ra \leq 10^{13}$
		0.54 $5\times10^2 \leq Ra \leq 2\times10^7$	

4.1. Example

Calculate the appropriate Reynolds numbers and state if the flow is laminar or turbulent for the following:

a) A 10 m (water line length) long yacht sailing at 13 km/h in seawater ρ = 1000 kg/m^3 and μ = 1.3 x 10^{-3} kg/m s,

b) A compressor disc of radius 0.3 m rotating at 15000 rev/min in air at 5 bar and 400°C and

$$\mu = \frac{1.46\times10^{-6}T^{3/2}}{(110+T)} \text{ kg/m s}$$

c) 0.05 kg/s of carbon dioxide gas at 400 K flowing in a 20 mm diameter pipe. For the viscosity take $\mu = \dfrac{1.56\times10^{-6}T^{3/2}}{(233+T)}$ kg/m s

d) The roof of a coach 6 m long, travelling at 100 km/hr in air (ρ = 1.2 kg/m^3 and μ = 1.8 x 10^{-5} kg/m s)

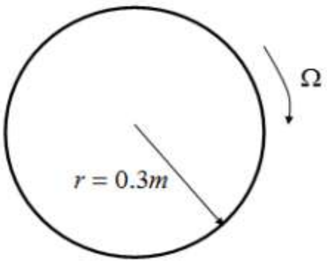
e) The flow of exhaust gas (p = 1.1 bar, T = 500°C, R = 287 J/kg K and μ = 3.56 x 10^{-5} kg/m s) over a valve guide of diameter 10 mm in a 1.6 litre, four cylinder four stroke engine running at 3000 rev/min (assume 100% volumetric efficiency an inlet density of 1.2 kg/m^3 and an exhaust port diameter of 25 mm)

4.1. Solution

a) $$\mathrm{Re} = \frac{\rho u L}{\mu} = \frac{10^3 \times \frac{13 \times 10^3}{3600} \times 10}{1.3 \times 10^{-3}} = 2.78 \times 10^7$$ (turbulent)

b) $$T = 400 + 273 = 673\ K$$

$$\mu = \frac{1.46 \times 10^{-6} \times 673^{3/2}}{(110 + 673)} = 3.26 \times 10^{-5} \quad kg/m\ s$$



$$\Omega = \frac{15000}{60} \times 2\pi = 1571\ rad/s$$

$$u = \Omega r = 1571 \times 0.3 = 471.3\ m/s$$

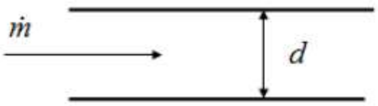
$$\rho = \frac{P}{RT} = \frac{100000}{287 \times 673} = 2.59\ kg/m^3$$

Characteristic length is r not D

$$\mathrm{Re} = \frac{\rho u D}{\mu} = \frac{2.59 \times 471.3 \times 3}{3.26 \times 10^{-5}} = 1.12 \times 10^7$$ (turbulent)

c) $$\dot{m} = \rho\, u A = \rho u \times \frac{\pi D^2}{4}$$

$$u = \frac{4\dot{m}}{\rho \pi D^2}$$



$$\mathrm{Re} = \frac{\rho u D}{\mu} = \frac{\rho \times 4\dot{m} D}{\rho \pi D^2 \mu} = \frac{4\dot{m}}{\pi D \mu}$$

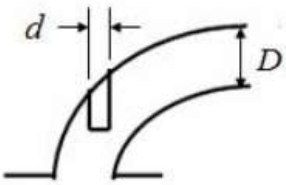
$$\mu = \frac{1.56 \times 10^{-6} \times 400^{3/2}}{(233 + 400)} = 1.97 \times 10^{-5} \quad kg/m\ s$$

$$\mathrm{Re} = \frac{4 \times 0.05}{\pi \times 0.02 \times 1.97 \times 10^{-5}} = 1.6 \times 10^5$$ (turbulent)

d) $$u = \frac{100 \times 10^3}{3600} = 27.8\ m/s$$

$$Re = \frac{\rho u L}{\mu} = \frac{1.2\times 27.8\times 6}{1.8\times 10^{-5}} = 11.1\times 10^{7}$$ (turbulent)

e) Let $\dot{m}$ be the mass flow through the exhaust port



$\dot{m}$ = inlet density X volume of air used in each cylinder per second

$$\dot{m} = 1.2\times\frac{1.6\times 10^{-3}}{4}\times\frac{3600}{60}\times\frac{1}{2} = 0.012\ kg/s$$

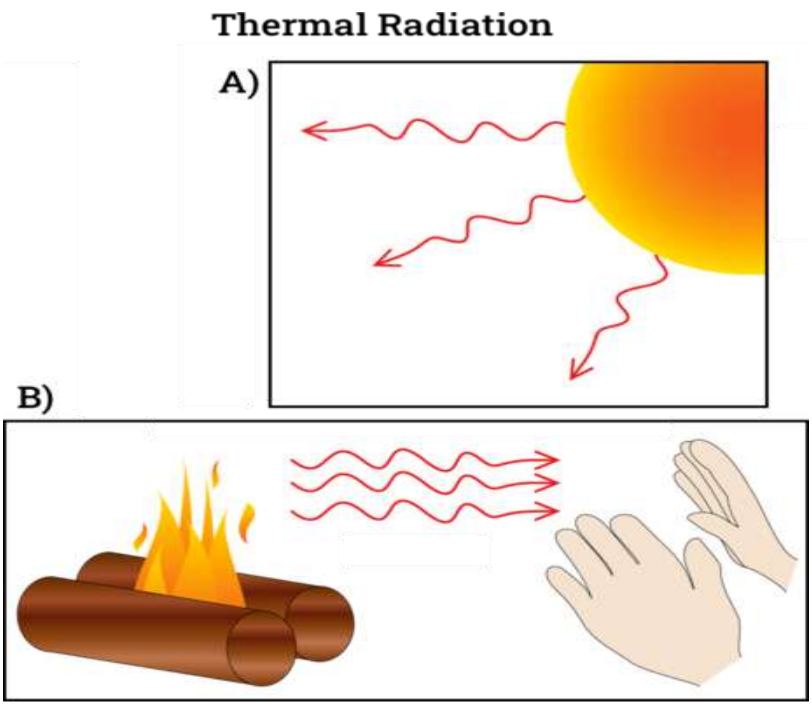
$$u = \frac{4\dot{m}}{\pi D^2 \rho}$$

$$Re_d = \frac{\rho u d}{\mu}$$

$$Re = \frac{4\times 0.01\times 0.012}{\pi\times 3.56\times 10^{-5}\times 0.025} = 6869$$ (laminar)

Chapter 5

Heat transfer by Radiation



Heat transfer by Radiation

5.1. Introduction

Thermal radiation is the energy emitted by matter that is at a finite temperature. It has a fundamental difference with respect to conduction and convection: substances that exchange heat do not need to be in contact, but can be separated by a vacuum. Radiation is a term applied generically to all kinds of phenomena related to electromagnetic waves.

We will focus our attention on radiation from solid surfaces, although this radiation can also come from liquids or gases. In thermal radiation, heat is transmitted by electromagnetic waves, like light, but of different wavelengths. Radiant energy depends on the characteristics of the surface and the temperature of the emitting body. By influencing a receptor, part of the energy passes to this other body, depending on its characteristics and its power of absorption. This energy results in an increase in the temperature of the second body. Heat transfer by radiation only involves the transport of energy, it does not need a material support, taking place even in a vacuum.

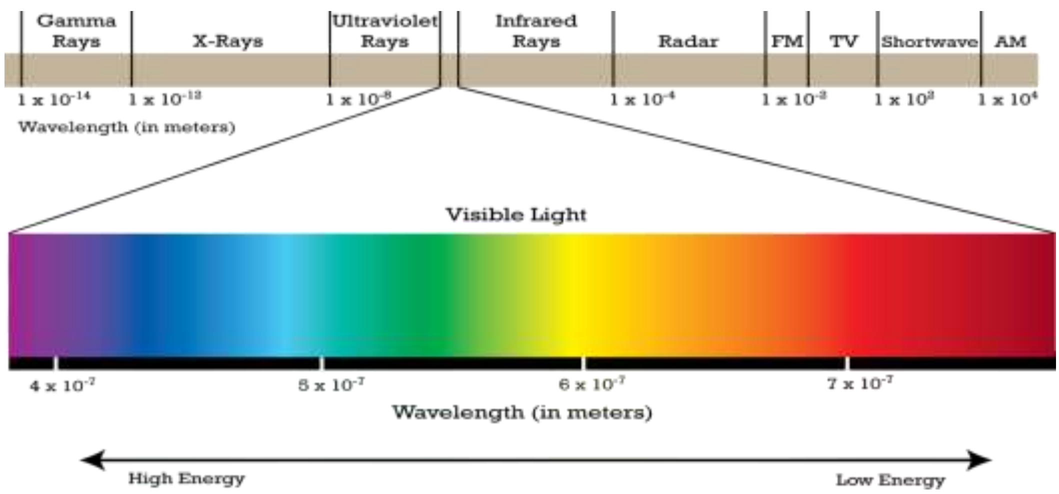


Figure 5. 1. The electromagnetic spectrum

5.2. Principle of heat transfer by radiation

It is a mode of heat (energy) exchange in the form of electromagnetic waves according to Planck's law ($E = h.\upsilon$, such that: υ is the associated wave frequency and $h = 6{,}62.10^{-34}$ J.s is Planck's constant). So it does not require any hardware support, it is analogous to the propagation of light. It propagates in a rectilinear manner at the speed of light ($c=3\times10^8$ m/s). The thermal radiation emitted by bodies is located between wavelengths of 0.1 μm to 100 μm. Practically, the three modes of heat transfer will coexist. But, this mode of transfer becomes predominant at temperatures higher than ordinary temperatures. Generally, all bodies (solid, liquid and gaseous) emit radiation of an electromagnetic nature. We can cite that, vacuum and simple gases like (O_2, H_2 and N_2) represent perfectly transparent media but, compound gases like (CO_2, H_2O, CO and CH_4) and certain liquids and solids like (glasses and polymers) are partially transparent. The majority of solids and liquids are opaque bodies since they stop the propagation of radiation just at their surfaces.

5.3. Preliminary definitions

The physical quantities will be designated according to the spectral composition or spatial distribution of the radiation:

- **Total magnitude**: it is relative to the entire spectrum;
- **Monochromatic quantity**: it only concerns a narrow spectral interval (dl), around a wavelength (l);
- **Hemispherical magnitude**: it is relative to all the directions of space;
- **Directional quantity**: it characterizes a given direction of propagation. When studying the thermal equilibrium of a system, anybody must be considered as:
- **Emitter**: if it sends radiation linked to its temperature (unless it is perfectly transparent);

- **Receiver**: If it receives, radiation emitted or reflected and diffused by the bodies that surround it.
- **Opaque body**: it is a body which does not transmit any radiation through itself, it stops the propagation of all radiation from its surface, it heats up by the absorption of the radiation;
- **Transparent body**: it is a body which transmits all the incident radiation;
- **Black body**: is the one that absorbs all the radiation it receives, it is characterized by an absorbent power ($\alpha\lambda T$= 1). All black bodies radiate in the same way the same temperature, the black body radiates more than a non-black body.
- **Gray body**: is that whose absorbent power ($\alpha\lambda T$) is independent of length wave (λ), it is characterized by ($\alpha\lambda T = \alpha T$). A gray body at high temperature for $\lambda<3\mu m$, sun), a gray body at low temperature for ($\lambda>3\mu m$, atmosphere).
- **Solid angle**: the elementary solid angle ($d\Omega$) under which the contour of a small surface (ds) is seen from a point (O) is given by:

$$d\Omega = \frac{dS \cos\alpha}{r^2} \qquad [sterad]$$

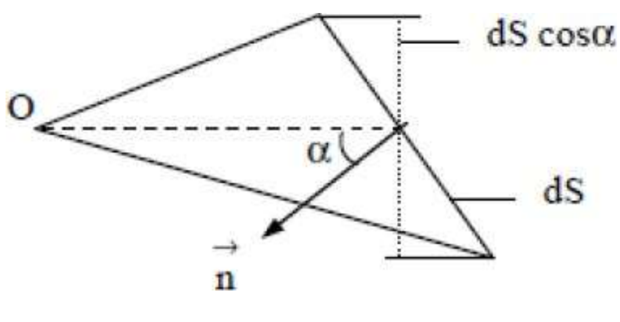


Figure 5. 2. Solid angle

Monochromatic energy emittance: the monochromatic emittance of a source at temperature (T) is given by:

$$M_{\lambda T} = d\varphi_{\lambda}^{\lambda+d\lambda} / dS.d\lambda \qquad [W/m^2]$$

(5.1)

Such as; φ: is the energy flow emitted between the two wavelengths (λ) and (λ+dλ). Total energy emittance: defined as the flux density emitted by the elementary surface (dS) over the entire wavelength spectrum:

$$M_T = \int_{\lambda=0}^{\lambda=\infty} M_{\lambda T} d\lambda = d\varphi/dS \qquad [W/m^2] \tag{5.2}$$

Energy intensity in one direction: the flux per unit of solid angle emitted by a surface (ds) under a solid angle (dΩ) surrounding the direction (Ox):

$$I_x = d^2\varphi_x/d\Omega \tag{5.3}$$

Energy luminance in one direction: the energy intensity in the direction (Ox) per unit of apparent emitting surface (the projection of the surface (S) on the plane perpendicular to Ox):

$$L_x = I_x/dS_x = I_x/dS\cos\alpha = d^2\varphi_x/d\Omega dS\cos\alpha \tag{5.4}$$

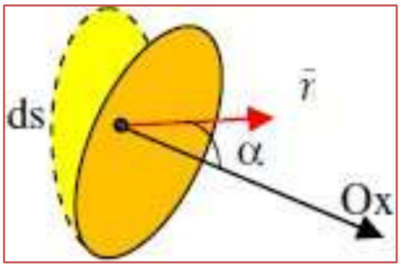
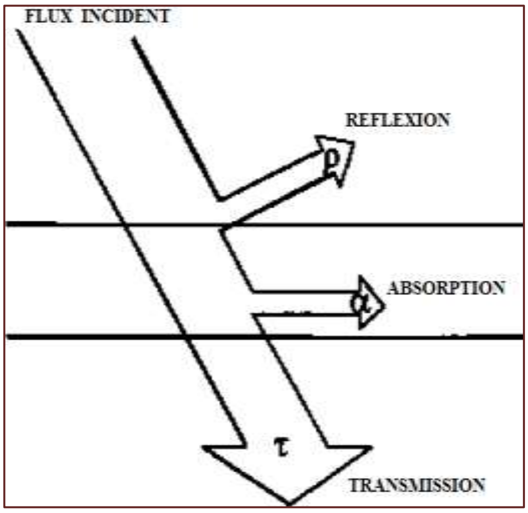


Figure 5. 3. Diagram showing the luminance of surface element ds**.**

Illumination (relative to a receiver): It is the flow received per unit of receiving surface, coming from all directions (Emittance).

5.4. Process of reception of radiation by a body

A heated equipment point emits electromagnetic radiation in all directions that are on the same side of the plane as the equipment point. When this radiation hits a body, some of the energy is reflected, some is transmitted through the body, and the rest is absorbed quantitatively as heat.When



incident radiation of energy (φλ) strikes a body (C) at temperature (T) (see figure opposite), we

Figure 5. 4 .Process of reception of radiation by a body.

Notice that:

- Part of the energy (φλ .ρλT) is reflected by the surface (S) of the body;
- Part of the energy (φλ .αλT) is absorbed by the body by heating it;
- The rest of the energy (φλ .τλT) is transmitted by continuing the path. Such as;
- From where;

$$\varphi_\lambda = \varphi_\lambda . \rho_{\lambda T} + \varphi_\lambda . \alpha_{\lambda T} + \varphi_\lambda . \tau_{\lambda T} \quad (5.5)$$

$$\rho_{\lambda T} + \alpha_{\lambda T} + \tau_{\lambda T} = 1 \quad (5.6)$$

which respectively represent; the monochromatic reflective power (ρλT), the monochromatic absorbing power (αλT) and the monochromatic transmittance power (τλT). These powers depend on the nature of the body, its thickness, its temperature (T), the wavelength (λ), the incident radiation and the angle of incidence.

5.5. Laws of radiation:

5.5.1. Planck's law

For a black body, the monochromatic Emittance only depends on the wave length (λ) and the temperature (T):

$$E_\lambda = \frac{d\varphi_\lambda}{dS} = \frac{C_1}{\lambda^5\left(e^{\frac{C_2}{\lambda T}} - 1\right)} \quad ; \quad \left[W/m^2.\mu m^{-1}\right] \tag{5.7}$$

Such as ;

T: temperature in °K

λ: The wavelength in µm

$C = 2\pi\ hc^2 = 3,74\times10^8 W\cdot\mu^4\cdot m^{-2}$: Planck constant

$C_2 = hc/k = 14400\mu K$ k: Stefan-Boltzmann constant and c the speed of light ($c = 3\times10^8\ m/s$).

Remarks

1. In the visible region (small wavelengths)

$$e^{\frac{C_2}{\lambda T}} \gg 1 \text{ ; d'où } E_\lambda = M^0_{\lambda,T} = C_1\lambda^{-5}e^{\frac{-C_2}{\lambda T}}$$

2. In the far infrared domain (long wavelengths), the development of $(e^{\frac{C_2}{\lambda T}})$ allows us to express $(M^0_{\lambda,T})$ by:

$$E_\lambda = M^0_{\lambda,T} = C_1 T / C_2\,\lambda^4$$

Planck's law makes it possible to draw isothermal curves representing the variations of E_λ as a function of wavelength for various temperatures:

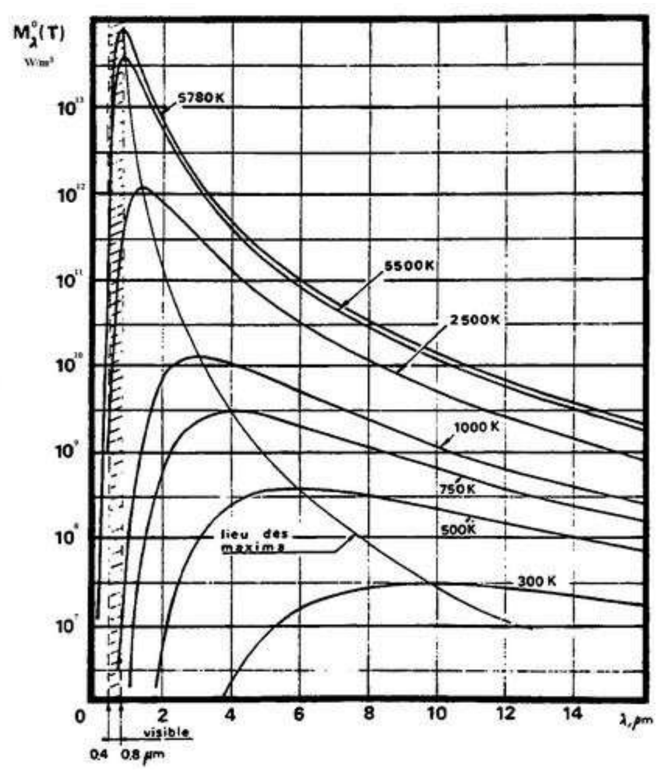


Figure 5. 5. Monochromatic emittance of the black body

5.5.2. Laws of Wien

5.5.2.1. Wien's first law

Wien's first law makes it possible to express or evaluate the wavelengths corresponding to the maximum monochromatic emittance (for which the radiation is maximum) as a function of temperature. For its derivation, it is enough to cancel the derivative of the emittance:

$$\frac{dM^0_{\lambda,T}}{d\lambda} = 0 \Leftrightarrow \quad \lambda_m = \frac{2898}{T}; \quad [\mu m] \tag{5.8}$$

Noticed

- At room temperature (T = 300 K, λm = 9.6 µm), a body emits long-wavelength infrared radiation which surrounds us but is not visible to our eye [(0.36 ÷ 0.75)] µm
- At the temperature of the sun (T = 5790 K, λm = 0.5 µm, maximum radiation), it is the visible yellow for which our eye has maximum luminous efficiency.

5.5.2.2. Wien's second law:

This law expresses the value of the maximum monochromatic emittance, it is enough to replace (λm) by its value in Planck's law to obtain:

$$M^0_{\lambda m.T} = B.T^5 \qquad (5.9)$$

With;

$B = 1.287\times10^{-5} W \cdot m^{-3} \cdot K^{-5}$ et T en °K

5.5.3 Stefan-Boltzmann law

It gives the total black body emittance, with the summation of all monochromatic emittances for all wavelengths or the integration of:

$$E = \frac{d\varphi}{dS} = \int_0^\infty E_\lambda d\lambda = M^0 = \int_0^\infty M^0_{\lambda,T} d\lambda = \sigma.T^4 \quad ; \quad [W/m^2] \qquad (5.10)$$

Such as ;

$\sigma = 5.67\times10^{-8}\, W\, m^2 \cdot K^4$ Stefan-Boltzmann constant

5.6. Radiation of real bodies

The emissive properties of real bodies are defined relative to the emissive properties of black bodies in the same temperature and wavelength range, and they are distinguished using coefficients called emissivity factors. These monochromatic or total coefficients are defined as follows;

$$\varepsilon_{\lambda T} = \frac{M_{\lambda T}}{M_{0\lambda T}} \text{ et } \varepsilon_T = \frac{M_T}{M_{0T}}$$

According to Kirchhoff's law, we show that:

$$\alpha_{\lambda T} = \varepsilon_{\lambda T} \quad (5.11)$$

5.7. Gray body radiation

Gray bodies are characterized by $\alpha_{\lambda T} = \alpha_T$ so $\varepsilon_{\lambda T} = \varepsilon_T$ or

$M_T = \varepsilon_T \times M_{0T}$ we let us deduce the gray body emittance at temperature T:

$$M_T = \varepsilon_T \times \sigma \times T^4 \quad (5.12)$$

5.1. Example

Two adjacent compressor discs (Surfaces 1 and 2) each of 0.4 m diameter are bounded at the periphery by a 0.1 wide shroud (Surface 3).

a) Given that $F_{12} = 0.6$, calculate all the other view factors for this configuration.

b) The emissivity and temperature of Surfaces 1 and 2 are $\varepsilon_1 = 0.4$, $T_1 = 800$ K, $\varepsilon_2 = 0.3$, $T_2 = 700$K and Surface 3 can be treated as radiatively black with a temperature of $T_3 = 900$ K. Apply a grey body radiation analysis to Surface 1 and to Surface 2 and show that:

$2.5\ J_1 - 0.9\ J_2 = 45545 \qquad \text{W/m}^2$

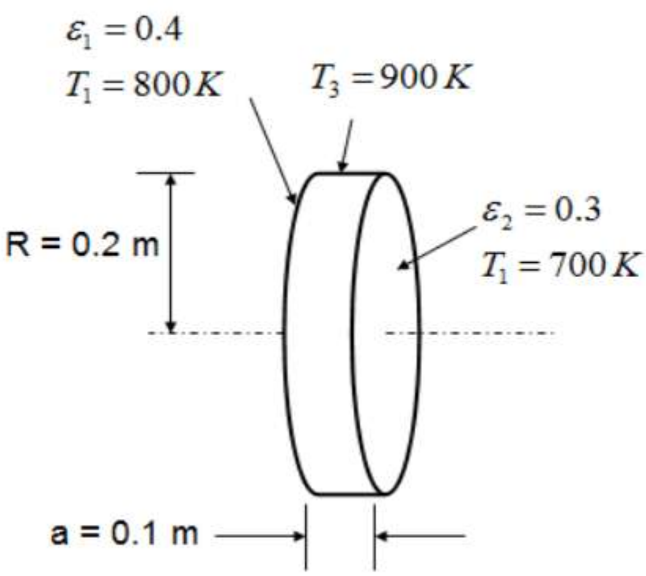
and

$3.333\ J_2 - 1.4\ J_1 = 48334 \qquad \text{W/m}^2$.

The following equation may be used without proof:

$$\frac{E_{B,i} - J_i}{\dfrac{1-\varepsilon_i}{\varepsilon_i}} = \sum_{j=1}^{N} F_{i,j}\,(J_i - J_j)$$

c) Determine the radiative heat flux to Surface 2



5.1. Solution

a) $r_1 = r_2 = r = 0.2\ m$

$a = 0.1\ m$

$\frac{r_2}{a} = \frac{0.2}{0.1} = 2$

$\frac{a}{r_1} = \frac{0.1}{0.2} = 0.5$

$F_{12} = 0.6$ (Although this is given in the question, it can be obtained from appropriate tables with the above parameters)

$F_{11} = 0$ (As surface 1 is flat, it cannot see itself)

$F_{13} = 1 - 0.6 = 0.4$ (From the relation $\sum F_{ij} = 1$ in an enclosure)

$F_{21} = 0.6$ (Symmetry)

$F_{22} = 0$

$F_{23} = 0.4$

$$F_{31} = \frac{A_1}{A_3} F_{13} = \frac{\pi \times 0.2^2}{2 \times \pi \times 0.2 \times 0.1} \times 0.4 = 0.4$$

$F_{32} = 0.4$ (Symmetry)

$F_{33} = 1 - 0.4 - 0.4 = 0.2$

b) $$\frac{E_{b,i} - J_i}{\frac{1-\varepsilon_i}{\varepsilon_i}} = \sum_{j=1}^{n} (J_i - J_j) F_{ij}$$

Apply to surface 1, (i = 1)

Let $\frac{1-\varepsilon_1}{\varepsilon_1} = \phi_1$

$$\left(E_{b,1}-J_1\right)=\phi_1\left[F_{12}\left(J_1-J_2\right)+F_{13}\left(J_1-J_3\right)\right]$$

$$E_{b,1}=J_1\left\{1+\phi_1F_{12}+\phi_1F_{13}\right\}-\phi_1F_{12}J_2-\phi_1F_{13}J_3$$

$$E_{b,1}=\sigma T_1^4$$

$J_3=\sigma T_3^4$ (Radiatively black surface)

$$\phi_1=\frac{1-\varepsilon_1}{\varepsilon_1}=\frac{1-0.4}{0.4}=1.5$$

$$\sigma T_1^4=2.5J_1-0.9J_2-0.6\sigma T_3^4$$

$$56.7\times10^{-9}\times800^4=2.5\times J_1-0.9\times J_2-0.6\times56.7\times10^{-9}\times900^4$$

$$2.5J_1-0.9J_2=45545\,W/m^2 \qquad (1)$$

Applying to surface 2 (i = 2)

$$E_{b,2}=J_2\left\{1+\phi_2F_{21}+\phi_2F_{23}\right\}-\phi_2F_{21}J_1-\phi_2F_{23}J_3$$

$$E_{b,2}=\sigma T_2^4$$

$$\phi_2=\frac{1-\varepsilon_2}{\varepsilon_2}=\frac{1-0.3}{0.3}=2.333$$

$$\sigma T_2^4=3.333J_2-1.4J_1-0.9333\sigma T_3^4$$

$$3.333J_2-1.4J_1=48334\,W/m^2 \qquad (2)$$

c) From (2):

$$J_1 = \frac{3.333 J_2 - 48334}{1.4}$$

Substituting in (1)

$$2.5 \times \frac{3.333 J_2 - 48334}{1.4} - 0.9 J_2 = 45545\ W/m^2$$

$$J_2 = 26099\ W/m^2$$

The net radiative flux to surface 2 is given by

$$q_2 = \frac{E_{b,2} - J_2}{\frac{1-\varepsilon_2}{\varepsilon_2}} = \frac{56.7 \times 10^{-9} \times 700^4 - 26099}{\frac{1-0.3}{0.3}} = -5.351 \times 10^3\ W/m^2$$

The minus sign indicates a net influx of radiative transfer as would be expected from consideration of surface temperatures.

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Série d'exercices 01

Exercice N°1 :

Une face d'une plaque de cuivre de 3 cm d'épaisseur est maintenue à 400 °C et l'autre face à 100 °C.

1. Quelle quantité de chaleur est transférée par unité de surface à travers la plaque ?

Donnée : la conductivité thermique du cuivre est de 370 W/m^2 °C.

Exercice N°2 :

L'air à 20 ° C souffle sur une plaque chauffante de 50 sur 75 cm maintenue à 250 °C. Le coefficient de transfert de chaleur par convection est de 25 W/m^2°C.

1. Calculez le transfert de chaleur transmis par convection.

Exercice N°3 :

Deux plaques noires infinies à 800 ° C et 300 ° C échangent de la chaleur par rayonnement.

1. Calculez le transfert de chaleur par unité de surface.

Exercice N°4 :

Une résistance électrique est connectée à une batterie, comme indiqué sur le schéma. Après une brève fluctuation transitoire, la résistance prend une température d'équilibre presque uniforme 95 °C, tandis que la batterie et les câbles de connexion restent à une température ambiante de 25 °C. La résistance thermique et électrique des câbles de connexion est négligeable. Si l'énergie électrique se dissipe uniformément dans la résistance de forme cylindrique de diamètre D = 60 mm et de longueur Lr = 25 mm.

1) Quelle est le flux chaleur généré par unité de volume g_{vol} en W/m^3

2) Quel est le coefficient de convection qu'il devrait avoir pour évacuer toute la chaleur si

on néglige le rayonnement de la résistance ?

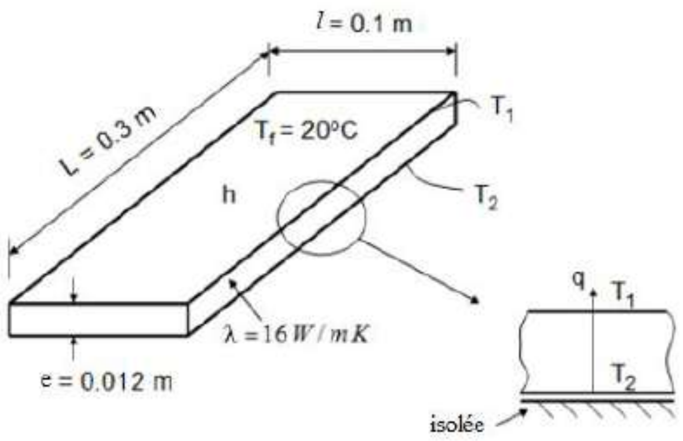
Données : V_1 = 24V, I = 6A, Lr = 25mm, Dr = 60mm, Tw = 95 °C, T∞ = 95 °C

Exercice N°5 :

Il est nécessaire de calculer de flux de chaleur perdu d'un homme dans un environnement où la température des murs est de $T_p = 27°C$ et l'environnement est de $T_\infty = 20°C$ si l'être humain a une température de surface de $T_h = 32°C$ et un coefficient de transfert de chaleur par convection entre l'homme et l'environnement et l'émissivité de $h_h = 3 W/m^2 °C$, $\varepsilon = 0{,}9$ respectivement, on sait qu'un être humain normal a une surface corporelle de $S_h = 1{,}5\ m^2$, en négligeant la résistance thermique des vêtements. Calculez également l'énergie perdue en t = 24 heures.

Exercice N°6 :

Une plaque de 0,3 m de longueur, 0,1 m de largeur et 12 mm d'épaisseur est réalisée en acier inoxydable (λ =16 W/m.K), la surface supérieure est exposée à un courant d'air de température 20oC. Dans une expérience, la plaque est chauffée par un radiateur électrique (également 0,3 m par 0,1 m) positionné sur le dessous de la plaque et la température de la plaque adjacente au radiateur est maintenue à 100°C. Un voltmètre et un ampèremètre sont connectés au radiateur et ceux-ci affichent respectivement 200 V et 0,25 A. En supposant que la plaque est parfaitement isolée de tous les côtés à l'exception de la surface supérieure, quel est le coefficient de transfert de chaleur par convection ?



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Série d'exercices 02

Exercice N°1

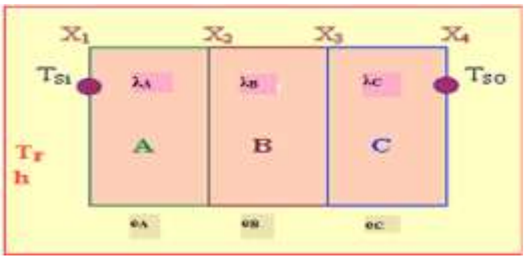
On considère un mur en béton de **3 m** de haut, **5 m** de long et **20 cm** d'épaisseur. La paroi intérieure De ce mur est à une température de **20°C**. La paroi extérieure de ce mur est à une température de **5°C**.

1. Calculer le flux de chaleur qui traverse perpendiculairement ce mur, en régime permanent.
2. On isole maintenant ce mur en ajoutant une couche de laine de verre de **8 cm** d'épaisseur et une Plaque de plâtre de **2 cm** d'épaisseur. Calculer la nouvelle valeur du flux de chaleur à travers le mur isolé.
   - Conductivité thermique du béton : **0,92 W/ (m. °C)**
   - Conductivité thermique du plâtre : **0,50 W/ (m. °C)**
   - Conductivité thermique du verre : **0,04 W/ (m. °C)**

Exercice N°2

Une paroi d'un four se compose de trois matériaux, dont deux sont de conductivité thermique connue, λ_A = 20 W/mK et 50 W/mK λ_C = et d'épaisseur connue e_A = 0.3 m et e_C = 0.15 m. Le troisième matériau B d'épaisseur e_B = 0.15 m se trouve au milieu entre les matériaux A et C, mais avec conductivité thermique inconnu λ_B. Dans des conditions de fonctionnement en régime permanent, les mesures révèlent une température de surface externe T_{S0} = 20 ºC, une température de surface interne T_{Si} = 600 ºC et un T_∞ = 800 ºC. Le coefficient de convection intérieure h = 25 W/m^2.K.

- Quelle est la valeur λ_B ?



Données :

$\lambda_A = 20\ W/mK$; $\lambda_C = 20\ W/mK$; $\lambda_B = ?$; $e_A = 0.3\ m$; $e_B = 0.15\ m$; $e_C = 0.15\ m$;

$T_F = 800\ °C$; $T_{Si} = 600\ °C$; $T_{So} = 20\ °C$

Exercice N°3

Le mur d'un four de surface **1m^2** est composé de deux couches :

- La première est en brique réfractaire : épaisseur **L_1=0,20 m**, conductivité **λ_1= 1,38W/ (m. °C)**
- La deuxième est en brique isolante : épaisseur **L_2=0,10 m**, conductivité **λ_2= 0,17W/ (m. °C)**

Calculer :

1. La résistance thermique de chaque couche.
2. La résistance totale du mur
3. La température intérieure du mur, si la température extérieure est de 30°C et les pertes Thermiques sont 1000W.

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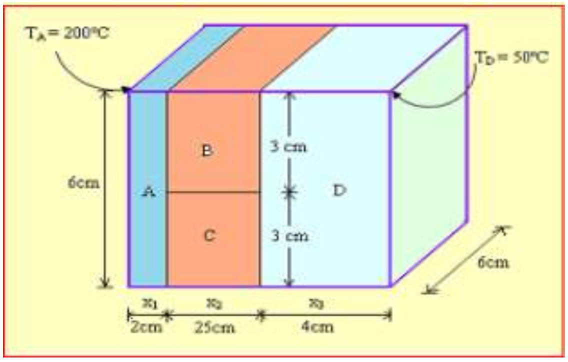
Série d'exercices 03

Exercice N°1 :

Une section de mur composite avec les dimensions indiquées sur la figure ci-dessous. Les températures uniformes de 200 °C et 50 °C dans le surfaces gauche et droite respectivement. Si les conductivités thermiques des matériaux des murs sont :

λ_A =70 W/mK ; λ_B =60 W/mK ;
λ_C = 40W/mK; λ_D = 20 W/mK.

Données : $S_A = S_D = 36*10^{-4}$ m²
$S_B = S_C = 18*10^{-4}$ m²



1) Donner le schéma électrique équivalant et calculer la résistance thermique équivalente.
2) Déterminez le flux de transfert de chaleur à travers cette section du mur et les températures aux surfaces de contact.

Exercice N°2 :

L'eau s'écoule à travers un tuyau en acier moulé (λ = 50 W/mK) avec un diamètre extérieur de 104 mm et une épaisseur de paroi de 2 mm.

1) Calculez le flux de chaleur perdu par unité de longueur, cas d'un tuyau non isolé lorsque la température de l'eau est de 15 °C, la température de l'air extérieur est de -10 °C, le coefficient de transfert de chaleur côté eau est de 30 kW/m²K et le coefficient de transfert de chaleur extérieur est 20 W/m²K.

2) Calculez le flux de chaleur perdu lorsque le tuyau est recouvert avec une isolation ayant un diamètre extérieur de 300 mm et une conductivité thermique de λ = 0,05 W/mK.

Exercice N°3 :

L'eau à 80 °C est pompée à travers 100 m de tuyaux en acier inoxydable, λ = 16 W/m.K de rayons intérieurs et extérieurs de 47 mm et 50 mm respectivement. Le coefficient de transfert de chaleur dû à l'eau est de 2000 W/m².K. La surface du tuyau perd de la chaleur par convection dans l'air à 20 °C et le coefficient de transfert de chaleur est de 200 W/m².K.

1) Calculez le flux de chaleur à travers le tuyau.

2) Calculez également le flux de chaleur à travers le tuyau lorsqu'une couche d'isolation, λ = 0,1 W/m.K et une épaisseur radiale de 50 mm est enroulée autour du tuyau.

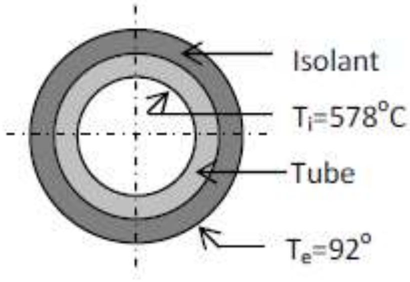
Exercice N°4 :

Un tube en acier inoxydable (λ =19W/mC) avec 3cm de diamètre intérieur et 5cm de diamètre extérieur est isolé par une couche d'amiante (λ =0,2 W/mC) de 2,5cm d'épaisseur (voir figure ci-dessous). Sachant que la température de la paroi interne de tube est maintenue à 578 °C et celle de la paroi externe de l'isolant est à 92°C.

1) Calculer le flux de chaleur perdu par mètre de longueur.

2) Donner le schéma électrique équivalent

3) Quelle est l'importance de l'utilisation de l'analogie existant entre les grandeurs thermiques et électriques?



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Série d'exercices 04

Exercice N°1 :

Calculez le temps requit pour qu'une petite plaque d'aluminium moulée à 16 °C soit chauffé à 510 °C en utilisant les gaz d'un haut fourneau à 1204 °C.

On donne : $C = V/S = 15$ cm, $h = 85$ W/m^{2}K, $\lambda = 210$ W/m.K, $\rho_{al} = 2700$ Kg/m^3 et $C_{al} = 940$ J/Kg.K.

Exercice N°2 :

Pour réchauffer le lait du bébé, la mère le verse dans une petite paroi en verre d'un diamètre de 6 cm. La hauteur du lait dans le verre est de 7 cm. Elle place ensuite le verre dans une grande cocotte remplie d'eau chaude (60 °C). Le lait est constamment agité, garantissant que sa température reste constante à tout moment. Déterminez le temps qu'il faudrait au lait pour se réchauffer de 3 °C à 38 °C si le coefficient de transfert de chaleur entre l'eau et le verre est de 120 W/m^2.°C. Suppose que les propriétés du lait sont les mêmes que celles de l'eau.

- Pourquoi le lait peut-il être traité comme un petit environnement thermique dans ces conditions ?

Donnés : La conductivité thermique, densité et la chaleur spécifique de l'eau sont
$\lambda = 0.607$ W/m.°C, $\rho = 998$ kg/m^3 et $Cp = 4.182$ kJ/kg.°C.

Exercice N°3 :

Dans un cylindre de 4 cm de diamètre circule de l'air à la vitesse moyenne de 26,5 m/s.

Calculer le coefficient de transfert de chaleur h sachant que :

$\rho = 1{,}2$ kg/m^3 , $Cp = 0{,}24$ kcal/kg.°C , $\eta = 1{,}9\ 10^{-5}$ Pa.s ,

$\lambda = 6{,}2\cdot10^{-6}$ kcal/m.s.°C, $Nu = 0{,}023\cdot Re^{0,8}\cdot Pr^{0,4}$

Exercice N°4 :

Un barreau plein de cuivre de **1cm** de diamètre et de **10cm** de long est refroidit en le balayant par un courant d'Hélium, refroidit préalablement à **77K** et qui le frappe perpendiculairement avec une vitesse moyenne d'écoulement de **54m/s**. La température de paroi du barreau de cuivre s'établit alors à **80K**. En déduire le dégagement de chaleur **(en W/g)** qui se produit dans le barreau de cuivre. Les caractéristiques de l'Hélium, à la température considérée:

• **ρ= 0,65kg/m^3** **λ = 0,06 W/ (m.K)**

• **μ=8,5.10^{-6} kg/(m.s)** **Cp = 5300 J / (kg.K)**

1. Calculer le nombre de Reynolds et le nombre de Nusselt.
2. Calculer le coefficient de convection h et le flux thermique convectif.
3. Calculer la masse du cuivre et la chaleur thermique en W/g.

BIBLIOGRAPHIE

1. Battaglia, J.-L., Kusiak, A., Pradere, C., 2020. Introduction Aux Transferts Thermiques-3e éd.: Cours et exercices corrigés. Dunod.

2. Bergman, T.L., Incropera, F.P., DeWitt, D.P., Lavine, A.S., 2011. Fundamentals Of Heat and Mass Transfer. John Wiley & Sons.

3. Cengel, Y.A., Ghajar, A.J., 2007. Heat and Mass Transfer. A Practical Approach.

4. Jannot, Y., 2012. Transferts Thermiques. Ecole des Mines Nancy.

5. Chap. N° 14 Transferts Macroscopiques d'Energie. Cours ; https://guy-chaumeton.pagesperso-orange.fr/scphysiques2010/tsch14.htm

6. Hafsaoui, Chapitre2 : Transfert de Chaleur par Conduction en Régime Permanent. 3eme Année Licence Electromécanique. http://i-hafsaoui.e-monsite.com/medias/files/chapitre2.pdf

7. Introduction Aux Transferts Thermiques ; cours et exercices corrigés ; Jean-Luc Battaglia ; Andrzej Kusiak ; Jean-Rodolphe Puiggali ; Dunod.

8. Convection Thermique ; Chapitre 3 ; Département génie des procédés ; ISET Kélibia

9. Convection ; Marcel Lacroix ; Université de Sherbrooke ; IMC 220

10. Transmission de Chaleur ; M. Bahrami ; ENSC 388 (F09)

11. Heat Transfer Principals in Electronics Cooling; Chap 8. Forced Convection Heat Transfer; MPE 635:

12. Chapiter 1; Convection Forcée Externe

13. Ecole Polytechnique Fédérale de Lausanne ; Exercices De Physique Générale ; Systèmes de Communication ; Quatrième Semestre .