

### 2.2.3. Electric Field of a Continuous Charge Distribution:

For calculating the electric field for continuous distribution of charge, the charge in these situations can be described as continuously distributed along some line, over some surface, or throughout some volume. To evaluating the electric field created by a continuous charge distribution, we have utilised the following procedure:

\*divide the charge distribution into small elements, each of which contains a small charge  $\Delta q$  as shown in Figure.

\*Use Equation  $\vec{E}(r) = K \sum_{i=1}^{i=n} \frac{q_i}{r_{ip}^2} \vec{r}_i$  to calculate the electric field

due to one of these elements at a point P.

\*Finally, evaluate the total electric field at P due to the charge distribution by summing the contributions of all the charge elements (that is, by applying the superposition principle)

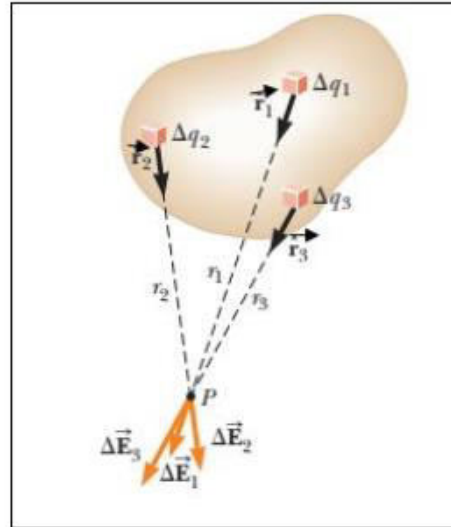
The electric field at P due to one charge element carrying charge  $\Delta q$  is:  $\Delta \vec{E} = K \cdot \frac{\Delta q}{r^2} \vec{r}$

where  $r$  is the distance from the charge element to point P and  $\vec{r}$  is a unit vector directed from the element toward P. The total electric field at P due to all elements in the charge distribution is

approximately:  $\vec{E} \approx K \cdot \sum_i \frac{\Delta q}{r_i^2} \vec{r}_i$

the total field at P in the limit  $\Delta q_i \rightarrow 0$  is: 
$$\left\{ \begin{array}{l} \vec{E} = K \cdot \lim_{\Delta q_i \rightarrow 0} \sum_i \frac{\Delta q}{r_i^2} \vec{r}_i \\ \vec{E} = K \cdot \int \frac{dq}{r^2} \vec{r} \end{array} \right.$$

When performing such calculations, it is convenient to use the concept of a charge density along with the following notations:



#### 2.2.4. Volume charge density:

\*If a charge  $Q$  is uniformly distributed throughout a volume  $V$ , the volume charge density  $\rho$  is

defined by:  $\rho \equiv \frac{Q}{V}$ .

\*If the charge is no uniformly distributed over a volume, defined by:  $dq = \rho dV$

where  $\rho$  has units of coulombs per cubic meter ( $C/m^3$ ).

#### 2.2.5. Surface charge density:

\* If a charge  $Q$  is uniformly distributed on a surface of area  $A$ , the surface charge density  $\sigma$  is

defined by:  $\sigma \equiv \frac{Q}{S}$ .

\*If the charge is no uniformly distributed over a surface, defined by:  $dq = \sigma dS$

where  $\sigma$  has units of coulombs per square meter ( $C/m^2$ ).

#### 2.2.6. linear charge density:

\* If a charge  $Q$  is uniformly distributed along a line of length  $l$ , the linear charge density  $\lambda$  is

defined by:  $\lambda \equiv \frac{Q}{l}$ .

\* If the charge is no uniformly distributed over line, the amounts of charge  $dq$  in a small volume, length element is:  $dq = \lambda dl$

where  $\lambda$  has units of coulombs per meter ( $C/m$ ).

## 2.4. Electric Potential:

Electric potential at a point in an electric field is equal to the amount of work done in bringing a unit positive charge from infinity to that point. The work done in transferring the charge placed in an electric field  $\vec{E}$  created by some source charge distribution equals the product of the force on the test charge and the parallel component of displacement.

\*The internal work done within the charge field system by the electric field on the charge for an infinitesimal displacement  $\vec{ds}$  of a point charge  $q$  immersed in an electric field is:

$$W = \vec{F}_e \cdot \vec{ds} = q\vec{E} \cdot \vec{ds}.$$

\*In a system, the internal work is equal to the negative of the change in the potential energy

$$dU = -W = -q\vec{E} \cdot \vec{ds}.$$

\*The change in electric potential energy of the system, for a finite displacement of the charge

from some point A in space to some other point B is: 
$$\Delta U = -q \int_A^B \vec{E} \cdot \vec{ds}.$$

\* Dividing potential energy by charge gives a physical quantity that depends solely on the charge distribution of the source and has a value at every point in an electric field. This quantity is called

the electric potential  $V$ : 
$$V = \frac{U}{q}$$

\* When a charge  $q$  is moved between the points A and B, the potential difference  $\Delta V = V_B - V_A$  in an electric field is defined as the change in electric potential energy of the system:

$$\Delta V = V_B - V_A = \frac{\Delta U}{q} = - \int_A^B \vec{E} \cdot \vec{ds}$$

\* The electric field is a measure of the rate of change of the electric potential with respect to position.