CHAPTER 03: Gauss's Law

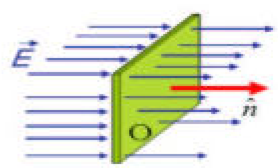
In this chapter, we describe Gauss's law to calculate the electric field of highly symmetric charge distributions and makes it possible to deal with complicated problems and verifying the properties of conductors in electrostatic equilibrium.

3.1. Electric Flux: The total number of electric field lines penetrating the surface is proportional to the product $\vec{E}$. Although the electric flux is the product of the magnitude of the electric field and surface area perpendicular to the field.

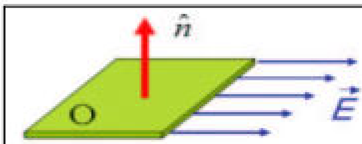
*Electric flux is proportional to the number of electric field lines that penetrate a surface. If the electric field is uniform and makes an angle θ with the normal to a surface of area S, the electric flux through the surface is: $\phi = ES\cos\theta$

* The flux Φ of the field E through the surface S is defined as: $\phi = \oint_S \vec{E}.d\vec{S}$.

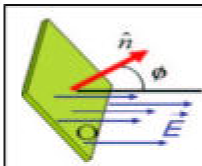
* The units for the electric flux are Nm^2/C,



$\phi = |\vec{E}| S\cos 0 = |\vec{E}| S$



$\phi = |\vec{E}| S\cos \pi/2 = 0$



$\phi = |\vec{E}| S\cos\theta$

* The meaning of flux is just the number of field lines passing through the surface.

* The value of E is constant over the spherical surface, is given by: $E = \frac{Kq}{r^2}$, Furthermore,

because the surface is spherical, $\oint dS = S = 4\pi r^2$, the net flux through the Gaussian surface is:

$$\phi = E\oint dS = \frac{Kq}{r^2} S = \frac{Kq}{r^2} 4\pi r^2 = \frac{\frac{1}{4\pi\varepsilon_0} q}{r^2} 4\pi r^2 = \frac{q}{\varepsilon_0}$$

The net flux through the spherical surface is proportional to the charge inside the surface.

3.2. Best Statement of Gauss's Law

The outward flux of the electric field through any closed surface equals the net enclosed charge divided by ε0 ($\phi = \frac{q_{int}}{\varepsilon_0}$). The net flux through any closed surface is: $\phi = \oint \vec{E}.d\vec{S} = \frac{q_{int}}{\varepsilon_0}$

where $\vec{E}$ represents the electric field at any point on the surface and q_{int} represents the net charge inside the surface.

* Gauss's law is not useful for determining the electric field if the charge distribution does not have sufficient symmetry, it is useful for determining electric fields when the charge distribution is highly symmetric, so that E can be removed from the integral.

3.3. Miscellaneous exercises

Exercise 01:

An insulating solid sphere of radius a has a uniform volume charge density ρ and carries a total positive charge Q.

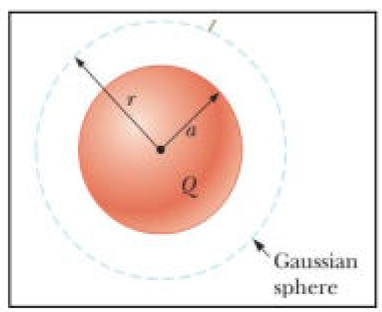
1)Calculate the magnitude of the electric field at a point outside the sphere and at a point inside the sphere?

2) Calculate the potential in both cases?

Solution:

The magnitude of the electric field at a point outside the sphere: for $r \rangle a$

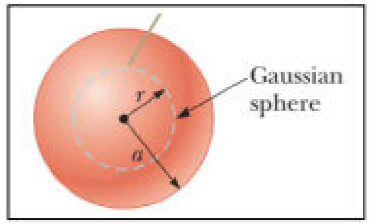
* let's choose a spherical gaussian surface of radius r, concentric with the sphere, as shown in Figure



* condition (1): E has the same value everywhere on the surface by symmetry, so we can remove E from the integral:

$$\left.\begin{array}{l}\phi=\oiint_{S_G}\vec{E}.d\vec{S}=\oiint_{S_G}E.dS=E\oiint_{S_G}dS=E.S=E4\pi r^2\\ \phi=\dfrac{q_{int}}{\varepsilon_0}=\dfrac{Q}{\varepsilon_0}=\dfrac{\rho V}{\varepsilon_0}=\dfrac{\rho\frac{4}{3}\pi a^3}{\varepsilon_0}=\dfrac{\rho 4\pi a^3}{3\varepsilon_0}\end{array}\right\}\Rightarrow\begin{cases}E4\pi r^2=\dfrac{Q}{\varepsilon_0}\Rightarrow E=\dfrac{Q}{4\pi\varepsilon_0 r^2}=K\dfrac{Q}{r^2}\\ E=K\dfrac{\rho\frac{4}{3}\pi a^3}{r^2}=\dfrac{\rho 4\pi a^3}{4\pi\varepsilon_0 3r^2}\Rightarrow E=\dfrac{\rho a^3}{3\varepsilon_0 r^2}\end{cases}$$

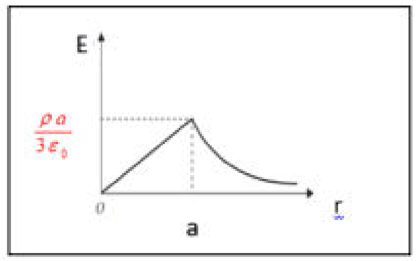
<u>*The magnitude of the electric field at a point inside the sphere:</u> for $r\langle a$



* In this case, let's choose a spherical gaussian surface having radius $r\langle a$ concentric with the insulating sphere.

*the charge q in within the gaussian surface of volume V' is less than Q

*Calculate q_{int} by using: $q_{int}=\rho V'=\rho\frac{4}{3}\pi r^3$



*The conditions (1) and (2) are satisfied everywhere on the gaussian surface.

Apply Gauss's law in the region $r\langle a$:

$$\left.\begin{array}{l}\phi=\oint\vec{E}.d\vec{S}=\oint E.dS=E\oint dS=E.S=E4\pi r^2\\ \phi=\dfrac{q_{int}}{\varepsilon_0}=\dfrac{\rho\frac{4}{3}\pi r^3}{\varepsilon_0}=\dfrac{\rho 4\pi r^3}{3\varepsilon_0}\end{array}\right\}\Rightarrow E4\pi r^2=\dfrac{\rho 4\pi r^3}{3\varepsilon_0}\Rightarrow E=\dfrac{r\rho}{3\varepsilon_0}$$

<u>Calculation of potential:</u>

The field $\vec{E}$ being radial, $dV_1=-\vec{E}d\vec{r}=-Edr\Rightarrow V_1=-\int Edr=-KQ\int\frac{dr}{r^2}=\frac{KQ}{r}+C_1$

1*for $r\rangle a$

$$V_1=\frac{1}{4\pi\varepsilon_0}\rho\frac{4}{3}\pi a^3\frac{1}{r}+C_1=\frac{\rho a^3}{3\varepsilon_0}\frac{1}{r}+C_1\begin{cases}V_1(\infty)=0\Rightarrow C_1=0\\ V_1=\dfrac{\rho a^3}{3\varepsilon_0}\dfrac{1}{r}\end{cases}$$