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Exercise 1 *In $\mathbb{R}^{2\times 2}$, consider the following matrix:*

$$A = \begin{pmatrix} 5 & -4 \\ 4 & -3 \end{pmatrix}, \quad I_2 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

- *Determine the matrix $B \in M_2(\mathbb{R})$ such that $A = I_2 + 4B$.*
- *Calculate A^2, B^2, A^T, B^T, $Tr(A, B)$ (lower and upper triagular of A, B.*
- *Calculate the matrix $-A^2 + 2A - I_2$.*
- *Conclude that the matrix A is invertible and determine its inverse A^{-1}.*

Exercise 2 *Let A and B two matrices in $\mathbb{R}^{3\times 3}$ matrix, and matrix B defined as follows:*

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}, \quad B = \begin{bmatrix} 2 & 4 & 6 \\ 1 & 2 & 3 \\ 3 & 6 & 9 \end{bmatrix}$$

1. *Calculate the determinants of matrices A and B.*
2. *Find the inverse of the invertible matrix.*

Exercise 3 *Given an upper triangular matrix A:*

$$A = \begin{bmatrix} 1+c & c & a \\ 0 & 5-2b & b \\ 0 & 0 & 9+a \end{bmatrix}.$$

1. *Calculate the determinant of matrix A.*
2. *Find the values of a, b, and c for which A is invertible.*

Now, find x, y and z where

$$\begin{cases} 2x + y + z &= 2, \\ y + 2z &= 1 \\ 10z &= 5 \end{cases} \tag{1}$$