

Exercise 1 Calculate the following integrals:

1. $\int_0^2 \frac{x^3}{x^2 + 2} dx.$
2. $\int (\ln(x))^2 dx.$
3. $\int_1^2 \frac{1}{\frac{1}{2}x + \sqrt{x}} dx.$
4. $\int \frac{1}{x^2 + x + 1} dx.$
5. $\int \frac{1}{x + x(\ln(x))^2} dx.$
6. $\int \frac{1}{\cos(x) + 1} dx.$

Exercise 2 Calculate the area of D and the following double integrals:

Area of D :

$$\iint_D 1 dA, \quad D = \{(x, y) \in \mathbb{R}^2 : 0 \leq x \leq 2, 1 \leq y \leq 2\}$$

Double integral of $e^x + y$:

$$\iint_D (e^x + y) dx dy, \quad D = \{(x, y) \in \mathbb{R}^2 : 0 \leq x \leq 2, 1 \leq y \leq 2\}$$

Double integral of $\sqrt{x^2 + y^2}$:

$$\iint_D \sqrt{x^2 + y^2} dx dy, \quad D = \{(x, y) \in \mathbb{R}^2 : x^2 + y^2 \geq 4, x^2 + y^2 \leq 9\}$$

Double integral of $x^2 + y^2$:

$$\iint_D (x^2 + y^2) dx dy, \quad D = \{(x, y) \in \mathbb{R}^2 : x > 0, y > 0, \text{ and } x \leq x^2 + y^2 \leq 1\}$$

Exercise 3 Volume of D :

$$\iiint_D 1 dV, \quad D = \{(x, y, z) \in \mathbb{R}^3 : x > 0, y > 0, z > 0 \text{ with } z \leq 1 - y^2 \text{ and } x + y \leq 1\}$$

Triple integral of z :

$$\iiint_D z dx dy dz, \quad D = \{(x, y, z) \in \mathbb{R}^3 : 0 \leq z \leq 1 \text{ and } x^2 + y^2 \leq z^2\}$$

Triple integral of xy :

$$\iiint_D xy dx dy dz, \quad D = \{(x, y, z) \in \mathbb{R}^3 : z > 0 \text{ and } 1 \leq x^2 + y^2 + z^2 \leq 4\}$$

Triple integral of $x^2 + y^2 + z^2$:

$$\iiint_D (x^2 + y^2 + z^2) dx dy dz, \quad D = \{(x, y, z) \in \mathbb{R}^3 : z > 0 \text{ and } 1 \leq x^2 + y^2 + z^2 \leq 4\}$$