

**Exercise 1** Calculate the following integrals:

1.  $\int_0^\pi x \cos(x) dx$ .
2.  $\int \frac{\cos(x) \sin(x)}{\sin(x)^2 + 1} dx$ .
3.  $\iint_D x - y dx dy$ ,  $D = \{x, y \in \mathbb{R} : 0 \leq x - y \leq 1, 1 \leq x + 2y \leq 2\}$ .
4.  $\iiint_D \sqrt{x^2 + y^2 + z^2} dx dy dz$ ,  $D = \{x, y, z \geq 0 : x^2 + y^2 + z^2 \geq 1, x^2 + y^2 + z^2 \leq 16\}$

**Exercise 2** In  $\mathbb{R}^{3 \times 3}$ , we consider the following matrices:

$$A = \begin{pmatrix} 1 & 1 & 3 \\ 1 & 3 & 0 \\ 0 & -1 & 1 \end{pmatrix}, \quad B = \begin{pmatrix} 1 & 2 & -1 \\ -1 & 2 & 1 \\ 0 & 0 & 1 \end{pmatrix}.$$

1. Determine the determinant  $\det(A)$  and  $\det(B)$ .
2. Calculate the inverse matrices  $A^{-1}$  and  $B^{-1}$ .
3. Conclude the matrices  $(AB)^{-1}$  and  $(BA)^{-1}$ .
4. Find  $x, y$  and  $z$  where

$$\begin{cases} -x + \frac{3}{2}y + 4z = 1, \\ -\frac{1}{2}x + \frac{3}{4}y + \frac{3}{2}z = 1 \\ x - y - 2z = 2 \end{cases} \quad \begin{cases} -\frac{1}{2}x + \frac{5}{2}y + 6z = 2, \\ \frac{1}{4}x - \frac{3}{4}y - 2z = 1 \\ \frac{1}{4}x - \frac{3}{4}y - z = 2 \end{cases} \quad (1)$$

Good luck