Analysis 1

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Analysis 1 Tutorial

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Objectives

- Understand the properties of real functions and their behaviors.
- Discover how we define the derivative using limits.
- Learn about a bunch of very useful rules (like the power, product, and quotient rules) that help us find derivatives quickly
- The students will learn how to study the differentiability of a function, that is to say, whether a function is differentiable or not. In addition, we will see the relationship between differentiability and continuity of a function.

Chapter 2 : Elementary functions

1. Introduction

The polynomial, exponential, sine, and cosine functions are "elementary" because they are very useful and will more frequently arise naturally in an investigation (whether within math or in an application) than most other functions. So everyone needs to know them. But why are they so useful? I think fundamentally it's because they are solutions to some of the simplest differential equations you could write down. The polynomials are the functions whose nth derivative is constantly 0. The sine and cosine functions satisfy $y''+y=0$ and the exponential function satisfies$y'=y$

2. Specific Objectives

- Gives a gradual development of the material starting from the background topics related to real numbers and functions.
- Presents a strict mathematical approach to the study of elementary functions.
- Includes a large number of examples and exercises, both solved and proposed.

3. Prerequisite Test

Exercise 1

Let the function defined by:

$$f(x)=\arcsin(2x\sqrt{1-x^2})$$

1- What is the domain of definition of f.

2- By setting $t=\sin x$, simplify the writting of f with $t\in[-\frac{\pi}{2},\frac{\pi}{2}]$.

Exercise 2

Show that for every $x\in[-1,1]$

$$\arccos x+\arcsin x=\frac{\pi}{2}$$

Exercise 3

Solve the following equations

1- $\arcsin x=\arccos\frac{1}{3}-\arccos\frac{1}{4}$.

2- $\arctan 2x+\arctan 3x=\frac{\pi}{2}$.

Exercise 4

Simplify the following expressions

1- $th(Argshx)$, 2- $sh(2Argshx)$

3- $\dfrac{2ch^2x-sh2x}{x-\ln(shx)-\ln 2}$

4. Elementary functions

Summary

• function $f(x)$ is continuous at x_0 if $f(x) - f(x_0)$ is an epsilon function of $(x - x_0)$, i.e.
$f(x) = f(x_0) + \varepsilon_f(x - x_0)$.

• A function $f(x)$ is differentiable at x_0 with the derivative $f'(x_0)$ if
$f(x) = f(x_0) + f(x_0)(x - x_0) + (x - x_0)\varepsilon_f(x - x_0)$.

• If a function is differentiable at x_0, then it is also continuous at x_0. The converse does not apply.

• The derivative of a product of two functions is
$\frac{d}{dx}(f(x).\, g(x)) = f'(x) \cdot g(x) + f(x) \cdot g'(x)$.

• The derivative of a quotient of two functions is
$\frac{d}{dx}\left(\frac{f(x)}{g(x)}\right) = \frac{f'(x).g(x) - f(x) \cdot g'(x)}{g^2(x)}$

• The derivative of a composite function is
$\frac{d}{dx} f(g(x)) = f'(g(x)) \cdot g'(x)$.

5. Excercise Sheet

5.1. Excercise 1

Determine the limits of x^n when $n \to +\infty$ according to the values of x.

Solution

If $x < -1$ so x^n has no limit but $\lim n \to +\infty \mid x \mid^n = +\infty$

If $x = -1$ so $x^n (-1)^n$ has no limit.

If $|x| < 1 \Leftrightarrow -1 < x < 1$ so $\lim_{n \to +\infty} x^n = 0$

If $x = 1$ so $x^n = 1$ so $\lim_{n \to +\infty} x^n = 1$

If $x > 1$ so $\lim_{n \to +\infty} x^n = +\infty$

Let's take advantage of this little exercise to recall the following very important equivalences:

$-1 < x < 1 \Leftrightarrow |x| < 1 \Leftrightarrow x^2 < 1 \Leftrightarrow 1 - x^2 > 0$

5.2. Excercise 2

Determine the limits of $(\ln(x))_n$, with $n \in \mathbb{N}$, when $n \to +\infty$.

Solution

We have

$-1 < \ln(x) < 1 \Leftrightarrow e^{-1} < x < e \Leftrightarrow < \frac{1}{e} < x < e$

So

Obviously $x > 0$.

If $0 < x \leqslant \frac{1}{e} so \ln(x) < -1$ and $(\ln(x))_n$ has no limit.

If $\frac{1}{e} < x < e$ so $-1 < \ln(x) < 1$ and $(\ln(x))_n \to 0$

If$x = e$ so $\ln(x) = 1$and $(\ln(x))_n = 1 \to 1$

If $x > e$ so $\ln(x) > 1$and $(\ln(x))_n \to +\infty$

5.3. Excercise 3

Let f be a function defined on $\mathbb{R}$ by

$f(x) = (2x - 1)e^{x-1} + 4$

1. Study the variations of f by $\mathbb{R}$.
2. Calculate the limits of f in $\pm\infty$.
3. Sketch the graph of f

Solution[1]

1. f is obviously defined, continuous and differentiable on $\mathbb{R}$.

   As $e^{x-1} = e^x \times e^{-1} = \frac{1}{e}e^x$ we have

$f(x) = (2x - 1)e^{x-1} + 4 = \frac{1}{e}(2x - 1)e^x + 4$

$f'(x) = 2e^{x-1} + (2x - 1)e^{x-1} = e^{x-1}(2 + 2x - 1) = e^{x-1}(2x + 1)$

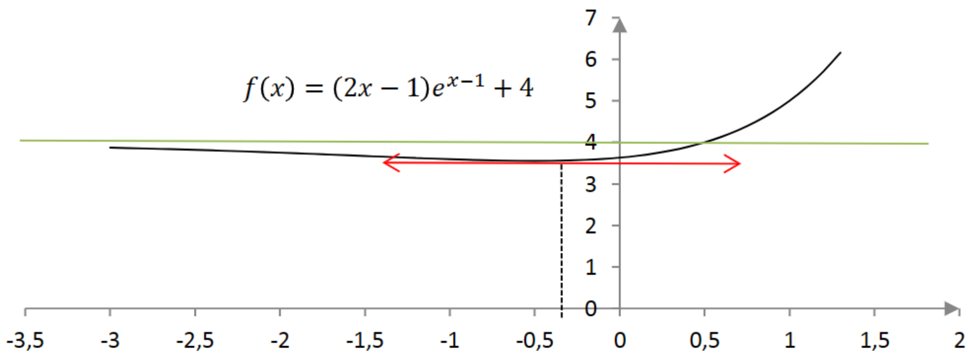
Since $e^{x-1} > 0$, the sign of $f'(x)$ is that of $2x + 1$

If $x < -\frac{1}{2}$ then $f'(x)$ is negative and f is decreasing on the interval $\left]-\infty, -\frac{1}{2}\right[$.

If $x > -\frac{1}{2}$ then $f'(x)$ is positive and f is increasing on the interval $\left]-\frac{1}{2}, +\infty\right[$.

2. The exponential prevails over the polynomial functions therefore

$\lim_{x\to-\infty} f(x) = 4$ and $\lim_{x\to+\infty} f(x) = +\infty$



For more information you can visite this channel https://www.youtube.com/@maamarben bachir

5.4. Excercise 4

Determine the domain of definition of f then simplify, its expression by using trigonometric

relations in each of the following cases :

i) $f(x) = sin(arccos(x))$, ii) $f(x) = cos(arcsin(x))$,

iii) $f(x) = cos(arctg(x))$, iv) $f(x) = sin(arctg(x))$

5.5. Excercise 5

Find the domain of definition Df of f and the domain Ef on which the function f is differentiable then calculate the derivative of f for all $x \in E_f$:

$$f\left(x\right)=\arcsin\left(\frac{1-x^2}{1+x^2}\right),\ f\left(x\right)=e^x\arcsin x.$$

$$f\left(x\right)=\ln^2 x-\ln\left(\ln x\right),\ f\left(x\right)=\frac{1+x}{1+\sqrt{x}}\exp\left\{\operatorname{arctg} x\right\}.$$

6. Quiz : Final test

Quiz

Let $f(x) = arcsin(2x), g(x) = arccos(x^2 - 1)$. We will denote D_f, D_g as the domain of definition of f, g respectively. Which statements are true?

- $D_f = [0, +\infty[$
- $D_g = \left[-2^{1/2}, 2^{1/2}\right]$
- $D_g = [-1, 1]$
- $D_f = [-1, 1]$

Quiz

Let $f(x) = (2x + 1)/(x - 1)$. Which assertions are true?

- $y = 2$ is an asymptote to the curve of f in $+\infty$.
- The curve of f admits a vertical asymptote $(x = 1)$.
- The point with coordinates $(1, 1)$ is a center of symmetry of the graph of f.
- The point with coordinates $(1, 2)$ is a center of symmetry of the graph of f.

Quiz

Let $f(x) = arcsin(cosx)$ and $g(x) = arccos(sinx)$. Which assertions are true?

- f is periodic with period π.
- g is periodic with period 2π.
- g is an odd function.
- f is an even function.

Quiz

Let $f(x) = (1 + \frac{1}{x})^x$. Which assertions are true?

- $\forall x < -1, f(x) > e$.
- $D_f =]0, +\infty[$.
- $\forall x > 0, 1 < f(x) < e$.
- $\forall x > 0, f(x) > e$.