Exercice N° 4

2 fils : en cuivre : Cu $d_1 = 1,6\,mm$: $AB = 4\,m$

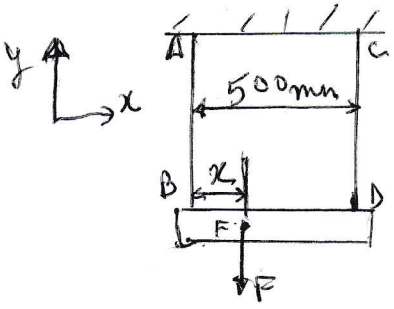
en Acier : Acié $d_2 = 0,8\,mm$: $CD = 4\,m$

$E_{AB} = 1,2.10^5\ N/mm^2$; $E_{CD} = 2.10^5\ N/mm^2$

$F = 500\,N$

sections des fils : $S_1 = \frac{\pi d_1^2}{4} = \frac{3,14 \times (1,6)^2}{4} = 2,0096\,mm^2$

$S_2 = \frac{\pi d_2^2}{4} = \frac{3,14(0,8)^2}{4} = 0,5024\,mm^2$



1) calcul de la position x de la direction de F ? Si $\Delta l_{AB} = \Delta l_{CD}$

$\sum \vec{F} = \vec{0} \Leftrightarrow \vec{F_{AB}} + \vec{F_{CD}} + \vec{F} = \vec{0}$

$\Leftrightarrow F_{AB} + F_{CD} - F = 0$

$\Leftrightarrow F_{AB} + F_{CD} = F$ (1)

$\sum \vec{M}_{/B} = \vec{0} \Leftrightarrow -xF + F_{CD}.500 = 0$

$\Rightarrow F_{CD} = \frac{xF}{500}$ (2)

$\Delta l_{AB} = \Delta l_{CD} \Leftrightarrow \frac{F_{AB}.L_{AB}}{E_{AB}.S_{AB}} = \frac{F_{CD}.L_{CD}}{E_{CD}.S_{CD}}$ (3)

$L_{AB} = L_{CD} = l = 4\,m$.

on remplace $F_{CD} = \frac{xF}{500}$ dans (1) : $F_{AB} + \frac{xF}{500} = F$

$\Rightarrow F_{AB} = F - \frac{xF}{500} = F\frac{(500 - x)}{500}$

(3) $\Rightarrow F_{AB}.(E_{CD}.S_{CD}) = F_{CD}.(E_{AB}.S_{AB})$

$\Leftrightarrow \frac{F(500-x)}{500}(E_{CD}.S_{CD}) = \frac{xF}{500}(E_{AB}.S_{AB})$

$\Rightarrow 500.(E_{CD}.S_{CD}) - x(E_{CD}.S_{CD}) = x(F_{AB}.S_{AB})$

Alors $x = \frac{500(E_{CD}.S_{CD})}{E_{AB}.S_{AB} + E_{CD}.S_{CD}} = \frac{500(2.10^5.0,5024)}{(1,2.10^5.2,0096) + (2.10^5.0,5024)}$

$x = 147,05\,mm$.

2) calcul de Δl_{AB} et Δl_{AC} si $F = 500N$ et $x = 250mm$

$\Rightarrow F_{AB} = 250\,N$ et $\Delta l_{AB} = 4,14\,mm$

$F_{CD} = 250\,N$ $\Delta l_{CD} = 9,96\,mm$