Charging and Discharging Capacitors

What you need to know:

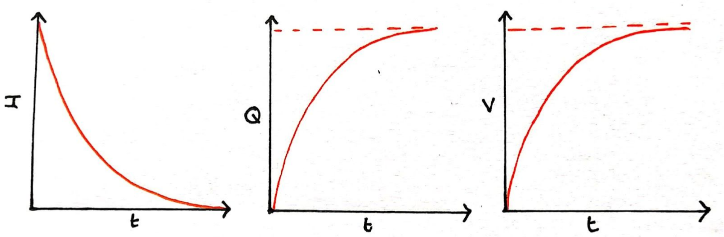
- charging and discharging capacitor through a resistor
- techniques and procedures to investigate the charge and the discharge of a capacitor using both meters and data-loggers
- time constant of a capacitor–resistor circuit; $\tau = CR$
- equations of the form $x = x_0 e^{-\frac{t}{CR}}$ and $x = x_0(1 - e^{-\frac{t}{CR}})$ for capacitor–resistor circuits
- graphical methods and spreadsheet modelling of the equation $\frac{\Delta Q}{\Delta t} = -\frac{Q}{CR}$ for a discharging capacitor
- exponential decay graph; constant-ratio property of such a graph

Explanation:

Charging graphs:

When a capacitor charges, electrons flow onto one plate and move off the other plate. This process will be continued until the potential difference across the capacitor is equal to the potential difference across the battery. Because the current changes throughout charging, the rate of flow of charge will not be linear.

At the start, the current will be at its highest but will gradually decrease to zero. The following graphs summarise capacitor charge. The potential difference and charge graphs look the same because they are proportional.

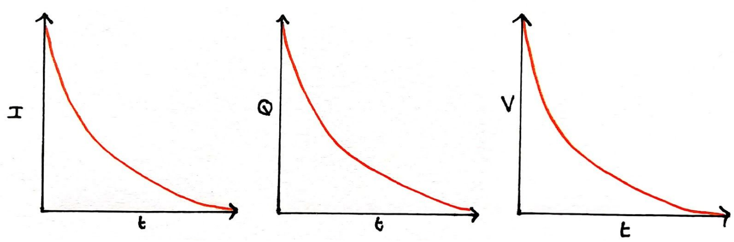


You can also see that the gradient of the charge-time graph is related to the current-time graph. This is because the gradient of the charge-time graph is $charge \div time = current$.

Discharging graphs:

When a capacitor is discharged, the current will be highest at the start. This will gradually decrease until reaching 0, when the current reaches zero, the capacitor is fully discharged as there is no charge stored across it.

The rate of decrease of the potential difference and the charge will again be proportional to the value of the current. This time all of the graphs will have the same shape:



Resistance and capacitance:

The rate at which a capacitor charges or discharges will depend on the resistance of the circuit. Resistance reduces the current which can flow through a circuit so the rate at which the charge flows will be reduced with a higher resistance. This means increasing the resistance will increase the time for the capacitor to charge or discharge. It won't affect the final pd or the total charge stored at the end.

The other factor which affects the rate of charge is the capacitance of the capacitor. A higher capacitance means that more charge can be stored, it will take longer for all this charge to flow to the capacitor.

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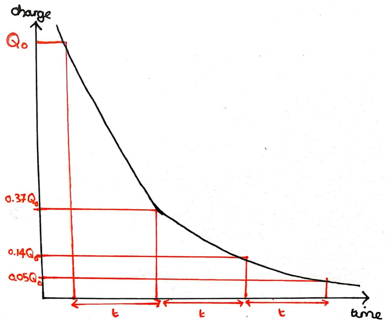
Time constant:

The time constant is the time it takes for the charge on a capacitor to decrease to $\frac{1}{e}$ (about 37%). The two factors which affect the rate at which charge flows are resistance and capacitance. This means that the following equation can be used to find the time constant:

$$\tau = CR$$

Where τ is the time constant, C is capacitance and R is resistance

The discharge of a capacitor is exponential, the rate at which charge decreases is proportional to the amount of charge which is left. Like with radioactive decay and half life, the time constant will be the same for any point on the graph:



Each time the charge on the capacitor is reduced by 37%, it takes the same amount of time. This time taken is the time constant, τ.

Example: Find the time constant for a capacitor with capacitance $5\mu F$ in a circuit with a resistance of 50Ω:

$$\tau = CR$$

$$\tau = (5 \times 10^{-6}) \times 50 = 2.5 \times 10^{-4} s$$

Equations for discharge:

The time constant we have used above can be used to make the equations we need for the discharge of a capacitor. A general equation for exponential decay is:

$x = x_0 e^{-t}$

For the equation of capacitor discharge, we put in the time constant, and then substitute x for Q, V or I:

$$Q = Q_0 e^{-\frac{t}{CR}}$$

$$V = V_0 e^{-\frac{t}{CR}}$$

$$I = I_0 e^{-\frac{t}{CR}}$$

Where:

Q/V/I is charge/pd/current at time t

$Q_0/V_0/I_0$ *is charge/pd/current at start*

C is capacitance and R is the resistance

When the time, t, is equal to the time constant the equation for charge becomes:

$$Q = Q_0 e^{-\frac{CR}{CR}}$$

$$Q = Q_0 e^{-1}$$

This means that the charge is now $\frac{1}{e}$ times the original or 37%.

Example: A capacitor with a capacitance of $100\mu F$ is fully charged, holding $10mC$ of charge. It is discharged through a $5M\Omega$ resistor. Calculate the charge after 50 seconds and the time for the potential difference to drop below 12V:

$$Q = Q_0 e^{-\frac{t}{CR}}$$

Substitute in the time 50s, C, R and the initial charge, Q_0 :

$$Q = (10 \times 10^{-3}) e^{-\frac{50}{(100 \times 10^{-6}) \times (5 \times 10^6)}}$$

$$Q = 9.05 \times 10^{-3} C$$

$Q = CV$ so $V = \frac{Q}{C}$

Initial potential difference is therefore:

$$\frac{10\times10^{-3}}{100\times10^{-6}} = 100V$$

$$V = V_0 e^{-\frac{t}{CR}}$$

Substitute in what we know, putting 12V as the pd at time t:

$$12 = 100 \times e^{-\frac{t}{100\times10^{-6}\times5\times10^{6}}}$$

Divide by 100:

$$\frac{3}{25} = e^{-\frac{t}{500}}$$

Take logs of each side:

$$ln\frac{3}{25} = -\frac{t}{500}$$

$$-t = 500ln\frac{3}{25}$$

$$t = 1060\ seconds$$

Equations for charging:

The charge after a certain time charging can be found using the following equations:

$$Q = Q_f(1 - e^{-\frac{t}{CR}})$$

$$V = V_f(1 - e^{-\frac{t}{CR}})$$

$$I = I_0 e^{-\frac{t}{CR}}$$

Where:

Q/V/I is charge/pd/current at time t

Q_f/V_f is maximum final charge/pd

C is capacitance and R is the resistance

Graphical analysis:

We can plot an exponential graph of charging and discharging a capacitor, as shown before. However, by manipulating the equation for discharging, we can produce a straight line graph:

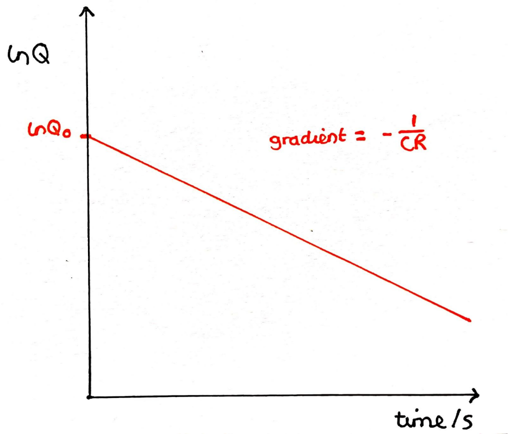
$$Q = Q_0 e^{-\frac{t}{CR}}$$

Take logs of both sides:

$$lnQ = lnQ_0 + lne^{-\frac{t}{CR}}$$

$$lnQ = lnQ_0 - \frac{t}{CR}$$

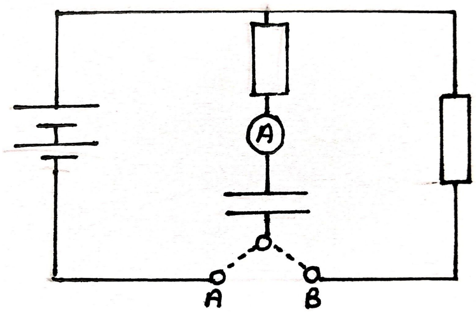
This is in the form $y = mx + c$ where y is lnQ, x is t, m is $-\frac{1}{CR}$ and c is lnQ_0



Investigating charge and discharge of capacitors:

An experiment can be carried out to investigate how the potential difference and current change as capacitors charge and discharge. The method is given below:

- A circuit is set up as shown below, using a capacitor with high capacitance and a resistor of high resistance slows down the changes (higher time constant) so it is easier to measure:



- The switch is closed at A and the capacitor begins to charge
- Record the current and pd every 20 seconds
- Once the capacitor is fully charged, close the switch at B and measure the current and pd every 20 seconds.
- Plot graphs for the current and pd as the capacitor is first charged then discharged.