M'sila University	Series n° : 1 **Numerical series**	Faculty of Mathematics and Computer Science
L 2 Mathematics		Module: Analyse 3

Remark 1

The exercise marked with $(\star)$ or additional will not be corrected in the session of **TD**.

Exercise 1

Use the limit of sequences of partial sums to determine the nature of the following series of numbers:

1. $\sum_{n\geq 0}\left(\frac{2}{3}\right)^n$.
2. $\sum_{n\geq 1}\frac{1}{4n^2-1}$.
3. $\sum_{n\geq 1}\frac{1}{n(n+1)(n+2)}$ $(\star)$.
4. $\sum_{n\geq 0}\frac{1}{\sqrt{n+2}+\sqrt{n+1}}$.
5. $\sum_{n\geq 3}\ln\left(1-\frac{2}{n}\right)$.
6. $\sum_{n\geq 1}\arctan\left(\frac{1}{n^2+n+1}\right)$ $(\star)$.

Exercise 2

Show that the harmonic series $\sum_{n\geq 1}\frac{1}{n}$ is divergent.

Exercise 3

Study the convergence of the following series

1. $\sum_{n\geq 0}\frac{1}{3n+1}$.
2. $\sum_{n\geq 2}\frac{n^2+2}{n^2}$.
3. $\sum_{n\geq 1}\frac{3}{\sqrt{n}}$.
4. $\sum_{n\geq 1}\frac{(4n+1)^3}{(5n^2+1)^3}$.
5. $\sum_{n\geq 1}\left(1-\frac{1}{n}\right)^n$.
6. $\sum_{n\geq 1}\left(ne^{\frac{1}{n}}-n\right)$.

Exercise 4

Determine the nature of the series whose general terms are as follows:

1. $u_n=\left(\frac{n}{n+1}\right)^{n^2}$.
2. $u_n=\frac{1}{n\cos^2 n}$ $(\star)$.
3. $u_n=\frac{1}{(\ln n)^n}$.

Exercise 5

Give the nature of the following numerical series, using the criterion of sequences of associated partial sums.

1. $a_n=\frac{(n+1)}{3^n}$.
2. $b_n=\frac{(n-(-1)^n)}{3^n}$ $(\star)$.
3. $c_n=\frac{1+2+\ldots+n}{1^3+2^3+\ldots+n^3}$.
4. $u_n=\frac{n^4-(n+1)^3}{n!}$ $(\star)$.
5. $v_n=\frac{n}{n^4+n^2+1}$ $(\star)$.
6. $w_n=\ln\left(1-\frac{1}{n^2}\right)$.

Exercise 6

Show that the series with general term $u_n = \dfrac{(-1)^n}{\ln(2+\sqrt{n})}$ is semi-convergent.

Exercise 7

Study the convergence of the numerical series with general term u_n.

1. $u_n = (-1)^n \dfrac{n^3}{n!}$.
2. $u_n = \dfrac{a}{n!}$, $a \in \mathbb{C}$ $(\star)$.
3. $u_n = na^{n-1}$, $a \in \mathbb{C}$.
4. $u_n = \sin\left(\dfrac{n^2+1}{n}\pi\right)$
5. $u_n = \dfrac{n!}{n^n}$, $n \geq 1$ $(\star)$.
6. $u_n = (-1)^n(\sqrt{n+1} - \sqrt{n})$

Exercise 8

Study the nature of the following series with positive terms

1. $\sum\limits_{n\geq 0} \dfrac{n^2}{2^n+1}$.
2. $\sum\limits_{n\geq 1} \dfrac{2+\cos(n)}{n^3}$.
3. $\sum\limits_{n\geq 0} ne^{-n}$ $(\star)$.

Exercise 9

Using the comparison theorem, investigate the nature of the following series

1. $\sum\limits_{n\geq 1} \sin\left(\dfrac{1}{n}\right)$.
2. $\sum\limits_{n\geq 2} \dfrac{\cos(n)}{n(n-1)}$.
3. $\sum\limits_{n\geq 1} \arctan\left(\dfrac{1}{2n^2}\right)$ $(\star)$.

Exercise 10

Study the nature of the series with general term u_n (convergence and absolute convergence).

1. $u_n = \dfrac{(-1)^n}{n^2+(-1)^n}$.
2. $u_n = \dfrac{(-1)^n}{\sqrt{n}}$.
3. $u_n = \dfrac{(-1)^n}{n+(-1)^n}$.
4. $u_n = \dfrac{2+\sin(an)}{n^2}$, $a > 0$.

Supplementary exercise 1

Investigate the nature of the following Bertrand series $\sum\limits_{n\geq 2} \dfrac{1}{n^\alpha \ln^\beta(n)}$, $\quad (\alpha,\ \beta \in \mathbb{R})$.

Supplementary exercise 2

1. Find the sum of the series $\left(\sum\limits_{n\geq 0} a^{2n}\right)^2$, $a \in]0,1[$.
2. Calculate the product of series $\sum\limits_{n\geq 1} u_n$ with itself, such that $u_n = \dfrac{(-1)^n}{\sqrt{n}}$.