

Practice Sheets 01(Analysis 1)

Part 1: Real number system

Exercise 01

Answer "True" or "False" to the statements below. If the statement is False, explain why.

- 1.) -5 is a rational number. _____
- 2.) $\sqrt{8}$ is rational. _____
- 3.) $\sqrt{16}$ is a natural number _____
- 4.) $\sqrt{2.25}$ is rational.. _____
- 5.) $\frac{22}{7}$ is a rational number _____
- 6.) π is a rational number. _____
- 7.) $\sqrt[3]{9}$ is rational. _____
- 8.) $\sqrt{16}$ is an irrational number _____
- 9.) $10\frac{3}{4}$ is rational.. _____
- 10.) $\frac{\pi}{2}$ is a rational number _____

Exercise 02

1. Let $n \in \mathbb{N}$. Prove that if n^2 is even then n is even
- 2 Prove that $\sqrt{2}$ is irrational.

Part 2: Supremum and Infimum

Exercise 03

Find the infimum and the supremum, whenever they exist, of the following sets

1. $A_1 = [-2, 0[$
2. $B_1 =]-\infty, 2[$
3. $B_2 = [-1, 0] \cup [3, 4]$
4. $A_2 = \left\{ 1 - \frac{(-1)^n}{n}, n \in \mathbb{N} \right\}$
5. $A_3 = \left\{ x \in \mathbb{R}^*, x < \frac{1}{x} \right\}$
6. $A_4 = \{x \in \mathbb{R}, x + 2 \geq x^2\}$
7. $A_5 = \left\{ \frac{1}{n} - \frac{1}{m}, n, m \in \mathbb{N} \right\}$
8. $A_6 = \{x \in \mathbb{Q}, x^2 \leq 2\}$
9. $A_7 = \{x \in \mathbb{R} \setminus \mathbb{Q}, x^2 \leq 2\}$

Exercise 04

Given nonempty subsets A and B of \mathbb{R} and $k \in \mathbb{R}$, we define the following subsets of \mathbb{R} :

$$\begin{aligned} kA &:= \{k.a, a \in A\} \\ k + a &:= \{k + a, a \in A\} \\ A + B &:= \{a + b, a \in A, b \in B\} \end{aligned}$$

Assume that A and B are nonempty bounded subsets of \mathbb{R} . Prove (any two of) the following

1. If $k > 0$, then $\inf(kA) = k \inf(A)$, $\sup(kA) = k \sup(A)$.
2. If $k < 0$, then $\inf(kA) = k \sup(A)$, $\sup(kA) = k \inf(A)$.
3. $\sup(A + B) = \sup A + \sup B$, $\inf(A + B) = \inf A + \inf B$
4. $\sup(A \cup B) = \sup \{\sup A, \sup B\}$, $\inf(A \cup B) = \inf \{\inf A, \inf B\}$

Part 3: The Modulus (Absolute value)**Exercise 5**

Let $a, b \in \mathbb{R}$

1. Prove that $|a + b| \leq |a| + |b|$ (Triangle Inequality)
2. Prove that $|a + b| \geq ||a| - |b||$ (Reverse Triangle Inequality)

Exercise 6

Solve the following equations

1. $|2x + 5| - 2 = 2$
2. $\frac{1}{4}|2x - 6| + 1 = 2$
3. $|6 - x^2| + 1 = 3$
4. $|x + 3| = x^2 - 4x - 3$
5. $|x^2 + 1| = 2x$

Exercise 7

Find which $x \in \mathbb{R}$ meet

1. $|x - 4| \leq 2$
2. $|x + 3| > 4$
3. $|x + 2| \leq -1$
4. $3|2x - 5| \geq -6$
5. $|x^2 - 1| > 2x$
6. $|x^2 - 3| < 2$

Part 4: Integer Part

Exercise 08

Find the integer part and fractional part of the following numbers

$$14, \quad 4.1, \quad \frac{35}{4}, \quad \frac{-22}{3}, \quad \sqrt{14}, \quad \pi, \quad \frac{\pi}{2}, \quad \frac{\pi}{3}$$

Exercise 09

Find the smallest positive $x \in \mathbb{R}$ such that $[x^2] - x[x] = 2019$