Series 1

Exercise 1

A material point with mass mmm moves in the xOy plane such that its position vector is given by:

$$\vec{r} = a \cos \omega t \, \vec{\imath} + b \sin \omega t \, \vec{\jmath}$$

where a, b and ω are positive constants, and $\vec{\imath}$ and $\vec{\jmath}$ are the unit vectors.

1. Find the expression for the velocity vector and the acceleration vector as a function of time.
2. Find the equation of the trajectory and show that the force acting on the material point is directed towards the origin at every point.
3. Show that the force derives from a potential and find it.
4. Calculate the total energy and show that it is constant.
5. Calculate the angular momentum at any given time with respect to the origin.

Exercise 2

A material point with mass mmm moves along the x-axis in a force field deriving from a potential V(x). If at times t_1 and t_2, the material point is at positions x_1 and x_2 respectively, show that:

$$t2 - t1 = \sqrt{\frac{m}{2}} \int_{x_1}^{x_2} \frac{dx}{\sqrt{E - V(x)}}$$

where EEE is the total energy. If the potential is $V(x) = \frac{1}{2} k x^2$, and if the material point is at rest at $x = a$ at time t=0, find x(t) and describe the motion.

Exercise 3

A material point with mass mmm is located in space characterized by its coordinates x, y, and z. Find the expression for the velocity in spherical coordinates.

Exercise 4

Establish the expression for the gradient in polar, cylindrical, and spherical coordinates.

Exercise 5

Determine the number of degrees of freedom in the following cases:

1. A particle moves along a given curve.
2. Five particles move freely in a plane.
3. Two material points are connected by a rigid rod and move freely in a plane.