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Presented by

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Mathematics

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L’objectif

L’objectif de l’enseignement des mathématiques est de recentrer le rôle de cette science, plus particulièrement de la géométrie, dans la formation de l’étudiant en architecture. Le programme permet à l’étudiant d’acquérir les outils de base lui permettant de formuler, représenter et calculer les formes et/ou les espaces qu’il est à même d’imaginer

The objective of mathematics education is to reposition the role of this science, particularly geometry, in the training of architecture students. The program enables students to acquire the fundamental tools that allow them to formulate, represent, and calculate the shapes and/or spaces they are capable of imagining.

REMILI WALID

Contents

Chapter 1

Geometry

Section 1.1 Euclidean Geometry

Euclidean geometry is a branch of mathematics that focuses on the study of geometric shapes and properties in a two-dimensional space. In this section, we will introduce the fundamental tools and primary theorems essential to the study of Euclidean Geometry.

1.1.1 Element in the plan

In this subsection, we are going to present some geometric Vocabulary.

Point: A point, in geometry and mathematics, is a fundamental concept that represents a precise location in space. It is dimensionless, meaning it has no length, width, or height. Points are typically denoted by capital letters such as A, B, C.

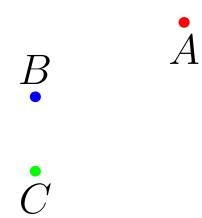


Figure 1.1: Figure of ponts.

Line: A line is a one-dimensional geometric object that consists of an infinite number of points arranged in a straight and continuous path. We can be denoted by (AB) or (D).

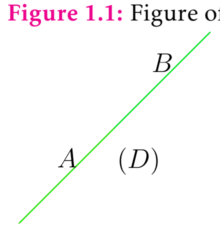


Figure 1.2: Figure of Line (D).

Half-line: A half-line is a one-dimensional geometric object that starts at a particular point, called the endpoint, and extends infinitely in one direction along a straight path. We can be denoted by $[AB)$.

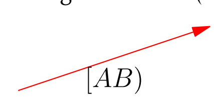


Figure 1.3: Figure of half Line $[AB)$.

Segment: A line segment is a line defined by two endpoints. A line segment is denoted by square brackets: $[AB]$..

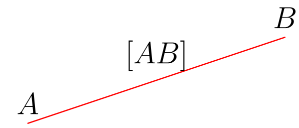


Figure 1.4: Figure of sgment $[AB]$.

Angle: An angle corresponds to a sector of the plane delimited by two half-lines. If the angle is delimited by the half-lines $[AB)$ and $[AC)$, it is denoted as angle $\widehat{BAC}$ with $0° \leq \widehat{BAC} \leq 180°$.

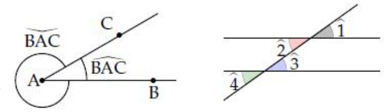


Figure 1.5: Figure of the angle $\widehat{BAC}$

Triangle: A triangle ABC is a polygon with three sides and three angles where the sum of the angles is equal 180°. It is one of the simplest and most fundamental geometric shapes.

1. If $AB = AC = BC$, the tringle ABC is Equilateral Triangle.
2. If $\widehat{BAC} = 90°$, the tringle ABC is rectangle Triangle.

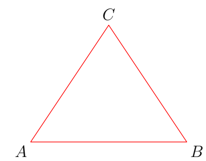


Figure 1.6: Figure of the triangle ABC

Circle: A circle is the set of points equidistant from a center. A circle C is defined by its center O and its radius r.

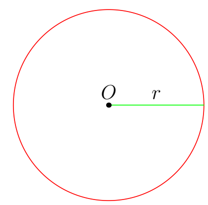


Figure 1.7: Figure of the Circle with center O and its radius r

1.1.2 Principle theorems

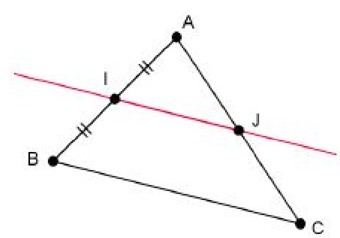
There are numerous theorems in Euclidean geometry, and many of them are foundational principles. Here are some of the usual theorems in Euclidean geometry:

The midpoint theorem

Dirict theorem

Theoreme 1.1 *In a triangle ABC, the line passing through the midpoint of one side and parallel to a second side intersects the third side at its midpoint.*

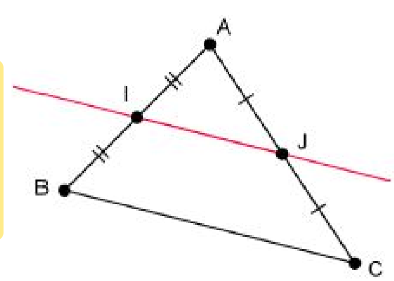
$$\text{If } I = m[AB] \text{ and } (IJ)//(BC) \text{ Then } J = m[AC] \text{ and } IJ = \frac{1}{2}BC.$$



Converse of the midpoint theorem

Theoreme 1.2 *In a triangle, the line that passes through the midpoints of two sides is parallel to the third side.*

$$\text{If } I = m[AB] \text{ and } J = m[AC] \text{ Then } (IJ)//(BC) \text{ and } IJ = \frac{1}{2}BC.$$

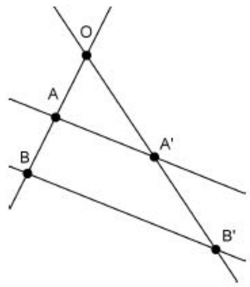


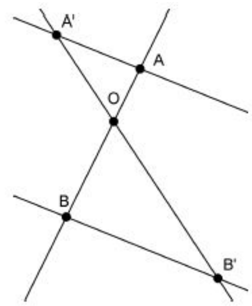
Thales's theorem

Dirict theorem

Theoreme 1.3 *Let two lines (AB) and $(A'B')$ intersect at O.*

$$\text{If } (AA')//(BB') \text{ Then, we have } \frac{OA}{OB} = \frac{OA'}{OB'} = \frac{AA'}{BB'}.$$





Converse of the midpoint Thales's theorem

Theoreme 1.4 *Let O, A, B on one hand, and O, A', B' on the other hand, be aligned in this order*

$$\text{If } \frac{OA}{OB} = \frac{OA'}{OB'} \text{ Then, we have } (AA')//(BB').$$

The Pythagorean theorem

Dirict theorem

Theoreme 1.5 *In a right triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides. So, if ABC is a right triangle at A, we have :*

$$BC^2 = AB^2 + AC^2.$$

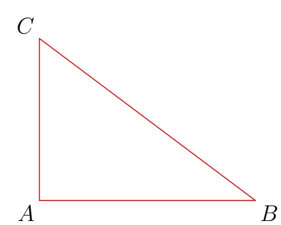


Figure 1.8: Figure of the Circle with center O and its radius r

Converse of the midpoint theorem

Theoreme 1.6 *If in a triangle, the square of the longest side is equal to the sum of the squares of the other two sides, then this triangle is a right triangle. If triangle ABC is such that:*

$$BC^2 = AB^2 + AC^2.$$

Then triangle ABC is right-angled at A.

Trigonometry In a right triangle, the following ratios are defined:

$$\sin(\widehat{ABC}) = \frac{\text{opposite side}}{\text{hypotenuse}} = \frac{AC}{BC}, \quad \sin(\widehat{ABC}) = \frac{\text{adjacent side}}{\text{hypotenuse}} = \frac{AB}{BC}$$

$$\tan(\widehat{ABC}) = \frac{\text{opposite side}}{\text{adjacent side}} = \frac{\sin(\widehat{ABC})}{\cos(\widehat{ABC})} = \frac{AC}{AB}$$

1.1.3 Application

Exercise *In a tringle ABC , D and E are points on the sides AB and AC respectively such that DE//BC*

- *If $AD/DB = 3/4$ and $AC = 15cm$ find AE.*
- *If $AD = 8x-7, DB = 5x-3, AE = 4x-3$ and $EC = 3x-1$, find the value of x.* ∎

Exercise *In a tringle ABC , D and E are points on the sides AB and AC respectively. For each of the following cases show that DE//BC*

- *$AB = 12cm, AD = 8cm$, $AE = 12cm$ and $AC = 18cm$.*
- *$AB = 5.6cm$, $AD = 1.4cm$, $AC = 7.2$ cm and $AE = 1.8cm$.* ∎

Exercise *In figure 1.10 DE//BC and CD//EF . Prove that $AD^2 = AB \times AF$.*

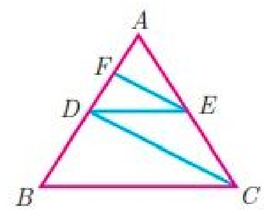


Figure 1.9: Triangle

Exercise *Use the Pythagorean Theorem to find out if these are right triangles and Justify your answers*

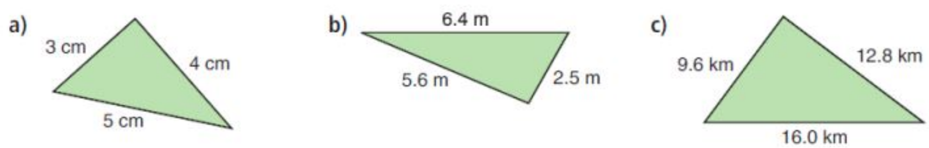
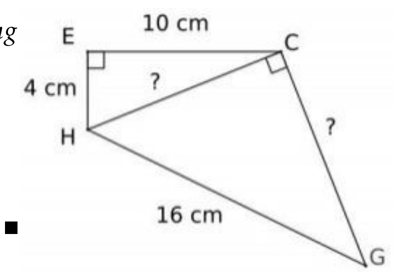


Figure 1.10: Triangles

Exercise *Consider two right triangles, HEC and HCG, as shown in the following figure.*



- *Determine the length HC.*
- *Find the value of HC*
- *Find $\cos(\widehat{HGC})$, $\sin(\widehat{HGC})$ and $\tan(\widehat{HGC})$.* ∎

Exercise *Let ABC is a right triangle with $\widehat{ABC} = 30°$ and $AB = 2$.*

- *Find $\cos(\widehat{ABC})$.*
- *Determine the lengths AC and BC.*
- *Find $\cos(\widehat{ACB})$, $\sin(\widehat{ACB})$ and $\tan(\widehat{ACB})$.* ∎

Section 1.2 Trigonometry and coordinate systems

In this section, we will discuss the properties of trigonometric functions, including cosine, sine, and tangent. In the second part, we will explore various coordinate systems in both 2D and 3D, including Cartesian, polar, cylindrical, and spherical coordinates.

1.2.1 Reminders of trigonometry

The word “Trigonometry” comes from the Greek “trigonon” (meaning triangle) and “metron” (meaning measure). So, simply put, Trigonometry is the study of the measures of triangles. This includes the lengths of the sides, the measures of the angles and the relationships between the sides and angles.

Radians and Degrees: Angles in Trigonometry can be measured in either radians or degrees:

- There are 360 degrees (*i.e.*, $360°$) in one rotation around a circle.
- There are 2π(6.283) radians in one rotation around a circle.

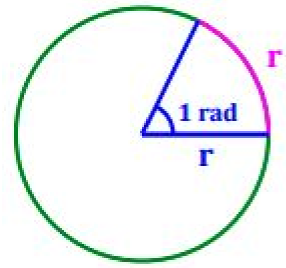


Figure 1.11: Figure of the Circle with center O and its radius r

> **DEFINITION 1.1** *Given a real number θ, let P be the point at θ radians on the unit circle, as indicated on the right. The functions sin and cos are defined as*
>
> - $\cos(\theta)$ = *x-coordinate of the point P,*
> - $\sin(\theta)$ = *y-coordinate of the point P.*
>
> *As θ can be any real number, functions sin and cos both have domain $\mathbb{R}$.*

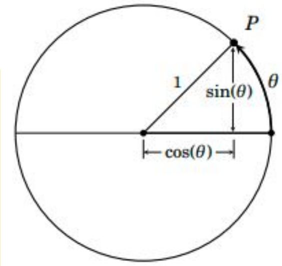


Figure 1.12: Figure of the Circle with center O and its radius r

Trigonometric Function: Here are some properties of trigonometric functions :

1. For all $x \in \mathbb{R}$, we have $-1 \leq \cos(x) \leq 1$ and $-1 \leq \sin(x) \leq 1$.
2. For all $x \in \mathbb{R}$, we have $\cos(x)^2 + \sin(x)^2 = 1$.
3. $\tan(x) = \frac{\sin(x)}{\cos(x)}$, for the signs of functions see Figure 1.13.

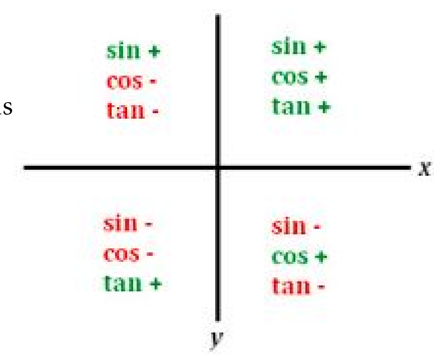


Figure 1.13: Signs of Trig functions.

The Unit Circle diagram below provides x- and y-values on a circle of radius 1 at key angles. At any point on the unit circle, the x-coordinate is equal to the cosine of the angle and the y-coordinate isequal to the sine of the angle. Using this diagram, it is easy to identify the sines and cosines of angles that recur frequently in the study of Trigonometry.

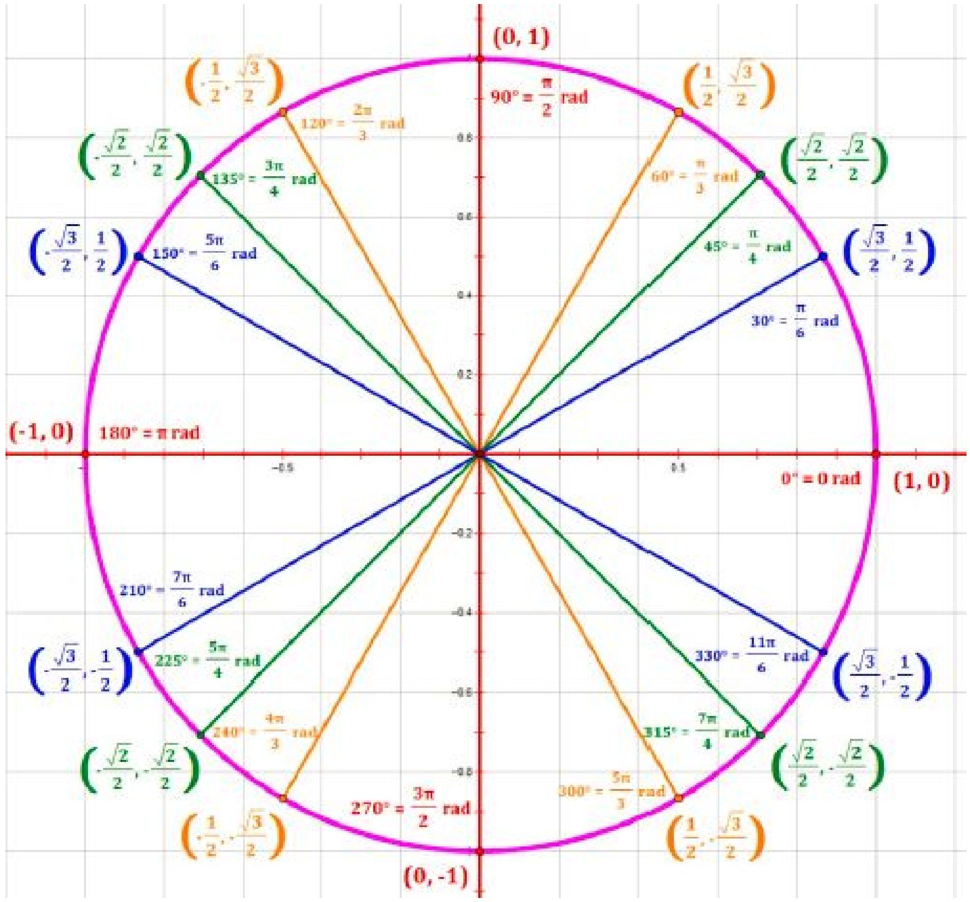


Figure 1.14: Signs of Trig functions.

Trig Functions of Special Angles (θ)				
Radians	Degrees	$\sin\theta$	$\cos\theta$	$\tan\theta$
0	0°	$\frac{\sqrt{0}}{2} = 0$	$\frac{\sqrt{4}}{2} = 1$	$\frac{\sqrt{0}}{\sqrt{4}} = 0$
$\pi/6$	30°	$\frac{\sqrt{1}}{2} = \frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{1}}{\sqrt{3}} = \frac{\sqrt{3}}{3}$
$\pi/4$	45°	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{\sqrt{2}} = 1$
$\pi/3$	60°	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{1}}{2} = \frac{1}{2}$	$\frac{\sqrt{3}}{\sqrt{1}} = \sqrt{3}$
$\pi/2$	90°	$\frac{\sqrt{4}}{2} = 1$	$\frac{\sqrt{0}}{2} = 0$	*undefined*

Figure 1.15: Trig Functions of Special Angles (θ).

In trigonometry, we have several important identities that relate the cosine (cos) and sine (sin) functions.

Properties For any real number x, the following identities hold:

1. $\cos(-x) = \cos(x)$ and $\sin(-x) = -\sin(x)$.
2. $\cos(\pi - x) = -\cos(x)$ and $\sin(\pi - x) = \sin(x)$.
3. $\cos(\pi + x) = -\cos(x)$ and $\sin(\pi + x) = -\sin(x)$.
4. $\cos\left(\frac{\pi}{2} - x\right) = \sin(x)$ and $\sin\left(\frac{\pi}{2} - x\right) = \cos(x)$
5. $\cos\left(\frac{\pi}{2} + x\right) = -\sin(x)$ and $\sin\left(\frac{\pi}{2} + x\right) = \cos(x)$.

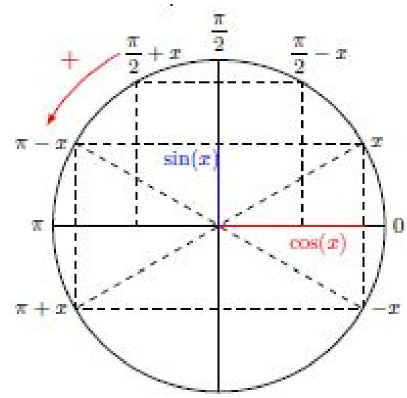


Figure 1.16: Signs of Trig functions.

In trigonometry, we give several important formulas for addition and duplication. Let's explore these concepts:

Addition Formulas For any real numbers a and b, the following trigonometric identities hold:

1. $\cos(a+b) = \cos(a)\cos(b) - \sin(a)\sin(b)$.
2. $\sin(a+b) = \sin(a)\cos(b) + \cos(a)\sin(b)$.
3. $\cos(a-b) = \cos(a)\cos(b) + \sin(a)\sin(b)$.
4. $\sin(a-b) = \sin(a)\cos(b) - \cos(a)\sin(b)$.

If $a = b$, we have the duplication formules

1. $\cos(2a) = \cos^2(a) - \sin^2(a)$.
2. $\sin(2a) = 2\sin(a)\cos(a)$.

Example 1.1 *For the equations* $\cos(x) = 0$ *and* $\sin(x) = 0$, *the solutions in* $\mathbb{R}$ *can be expressed as follows:*

$$x = \frac{\pi}{2} + k\pi, \quad k \in \mathbb{Z} \tag{1.1}$$

$$x = \pi + k\pi, \quad k \in \mathbb{Z}, \tag{1.2}$$

respectively.

1.2.2 Coordinate systems

Here, we will first state the general definition of a unit vector, and then extend this definition into 2D polar coordinates and 3D spherical coordinates.

2D Coordinates

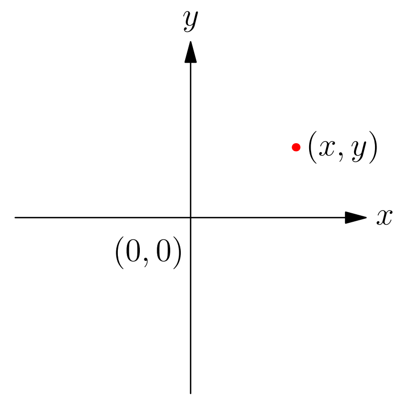


Figure 1.17: Figure of the Circle with center O and its radius r

Cartesian Coordinates In the 2D Cartesian coordinate system, every point is uniquely identified by two numerical values, usually written as (x, y), where:

1. x represents the horizontal position or the distance from the vertical axis (often called the x-axis).
2. y represents the vertical position or the distance from the horizontal axis (often called the y-axis).

The intersection point of the x-axis and y-axis is known as the origin, denoted as (0, 0).

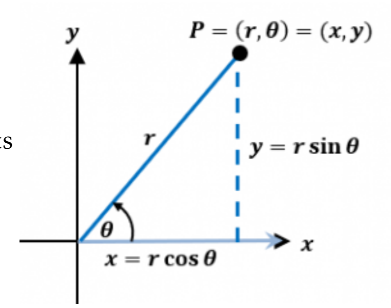


Figure 1.18: Figure of the Circle with center O and its radius r

Polar Coordinates
The polar coordinates, denoted as (r, θ), are a system used to represent points in a two-dimensional plane. The coordinates consist of two components:

1. r: The radial distance from the origin to the point.
2. θ: The polar angle.

Converting between polar and Cartesian coordinates

From polar to cartesian coordinates:

1. $x = r \cdot \cos(\theta)$.
2. $y = r \cdot \sin(\theta)$.

From cartesian to polar coordinates:

1. $r = \sqrt{x^2 + y^2}$.
2. $\tan(\theta) = \frac{y}{x}$.

3D Coordinates

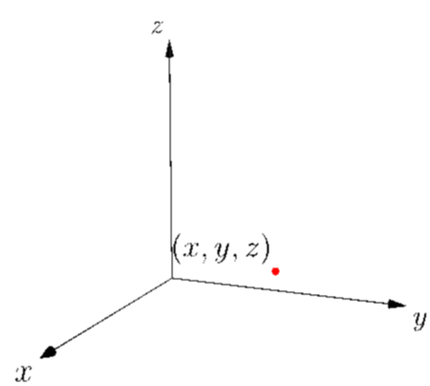


Figure 1.19: Figure of the Circle with center O and its radius r

Cartesian Coordinates in 3D: Cartesian coordinates in three-dimensional space are used to represent the position of a point in a three-dimensional Cartesian coordinate system. These coordinates are denoted as (x, y, z) and consist of three components:

1. x: The horizontal position or distance along the x-axis.
2. y: The vertical position or distance along the y-axis.
3. z: The position along the third axis, often referred to as the z-axis.

Cylindrical Coordinates: Cylindrical coordinates are a three-dimensional coordinate system used to specify the location of a point in space. Cylindrical coordinates are represented as (r,θ,z) and consist of three components:

1. r: The radial distance from the origin to the point's projection onto the xy-plane where $r=\sqrt{x^2+y^2}$.
2. θ: The angle measured in the xy-plane where $\tan(\theta)=\frac{y}{x}$.
3. z: The position along the third axis, often referred to as the z-axis.

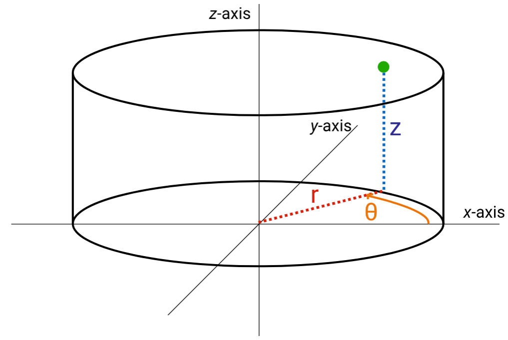


Figure 1.20: Figure of the Circle with center O and its radius r

Spherical Coordinates: Spherical coordinates are a three-dimensional coordinate system used to represent a point's position in space. These coordinates are denoted as (p,θ,ϕ) and include three components:

1. p: Radial distance from the origin to the point, $p^2=x^2+y^2+z^2$.
2. θ: Polar angle, measured from a reference direction (usually the positive x-axis), $\tan(\theta)=\dfrac{y}{x}$.
3. ϕ: The angle measured in the zy-plan, $\cos(\phi)=\dfrac{z}{p}$.

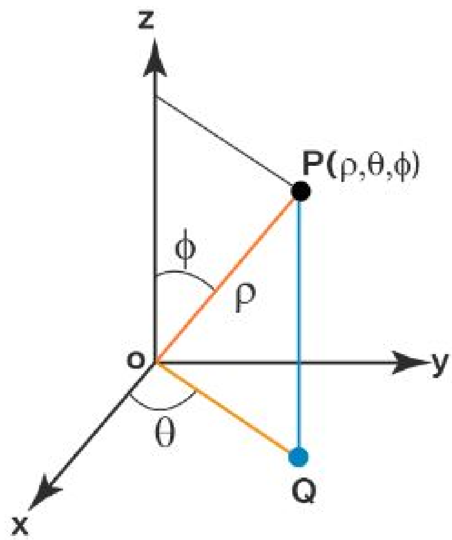


Figure 1.21: Figure of the Circle with center O and its radius r

Converting between Spherical and Cartesian coordinates

From spherical to cartesian coordinates:

1. $p^2=x^2+y^2+z^2$.
2. $x=p\cdot\sin(\phi)\cos(\theta)$.
3. $y=p\cdot\sin(\phi)\sin(\theta)$.
4. $z=p\cdot\cos(\phi)$.

From cartesian to spherica coordinates:

1. $p^2=x^2+y^2+z^2$.
2. $\tan(\theta)=\dfrac{y}{x}$.
3. $\cos(\phi)=\dfrac{z}{p}$.

1.2.3 Application

Exercise *Solve in $\mathbb{R}$ the following equations:*

1. $\cos(x)=\dfrac{\sqrt{2}}{2}$, $\sin(x)=\dfrac{1}{2}$, $\cos(3x+\frac{\pi}{4})=\cos(x+\frac{\pi}{3})$ *and* $\cos(2x)=\sin(3x)$.
2. $2\cos^2(x)=1$, $2\cos^2(x)=\sin^2(x)-1$, $\cos^2(x)-\sin^2(2x)=0$ *and* $\cos^2(x)-\sin^2(2x)=0$.
3. $\sin(x)+\sin(3x)=\cos(x)$.
4. $\cos(x)+\cos(5x)=\cos(3x)+\cos(7x)$. ■

Exercise *Solve in $\mathbb{R}$ and plot on a trigonometric circle the solutions within the range 0 to 2π:*

1. $\cos(x) > \frac{\sqrt{3}}{2}$, $3\tan(x) - \frac{\sqrt{3}}{3} > 0$ *and* $1 - 3\sin(x) > 0$.
2. $2\sin(3x) + 1 < 0$, $\tan\left(\frac{3\pi}{5}\right) - \tan(2x) < 0$ *and* $\sin^2(x) - \frac{3}{4} > 0$.

Determine the domain of definition and the set of roots of the functions $f : \mathbb{R} \to \mathbb{R}$.

1. $f(x) = \tan(3x + \frac{\pi}{4})$.
2. $f(x) = \dfrac{\sin^2(2x) - 1}{\tan(2x)}$. ■

Exercise *Consider the real-valued function f defined by $f(x) = 2x - \sin(x)$.*

1) Show that for all real x, $2x - 1 \leq f(x) \leq 2x + 1$.

2) Deduce the limits of f as x tends to $+\infty$ and as x tends to $-\infty$. ■

Exercise *Conversion Between Polar and Cartesian Coordinates.*

1. *Part A: Convert the following Cartesian coordinates to polar coordinates:*
   (a) $(3, 4)$, $(-2, 2\sqrt{3})$ *and* $(0, -5)$.
2. *Part B: Convert the following polar coordinates to Cartesian coordinates:*
   (a) $(6, \frac{\pi}{3})$ *and* $(-6, \frac{5\pi}{3})$.
   (b) $(2, \frac{5\pi}{6})$ *and* $(-2, -\frac{\pi}{6})$.
   (c) $(3, \pi)$ *and* $(3, -\pi)$. ■

Exercise *Conversion Between Cylindrical and Cartesian Coordinates.*

1. *Part A:Convert the following Cartesian coordinates to cylindrical coordinates:*
   (a) $(3, 4, 5)$, $(-2, -2, 2)$ *and* $(0, 0, 7)$.
2. *Part B: Convert the following polar coordinates to Cartesian coordinates:*
   (a) $(2, \frac{\pi}{4}, 6)$,$(3, \frac{3\pi}{2}, -1)$ *and* $(4, 0, 0)$. ■

Exercise *Conversion Between Spherical and Cartesian Coordinates*

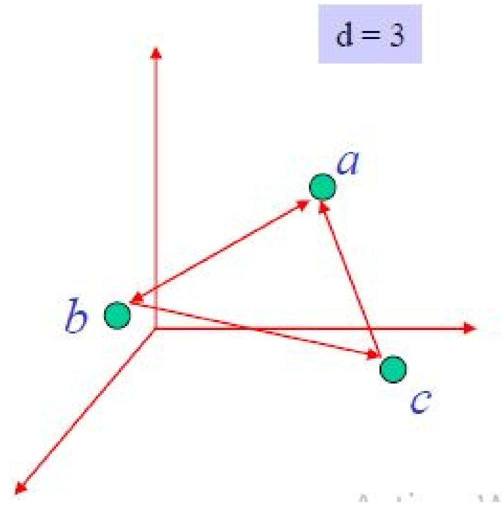
1. *Part A: Convert the following Cartesian coordinates to spherical coordinates:*
   (a) $(3, 4, 5)$, $(-2, -2, 2)$ *and* $(0, 0, 7)$.
2. *Part B: Convert the following spherical coordinates to Cartesian coordinates:*
   (a) $(2, \frac{\pi}{4}, \frac{\pi}{3})$, $(3, \frac{3\pi}{2}, \frac{-\pi}{6})$ *and* $(4, \frac{\pi}{2}, 0)$. ■

Section 1.3 Notion of Distances

In this section, we first provide a definition of the term ’metric’ or mathematics metric.

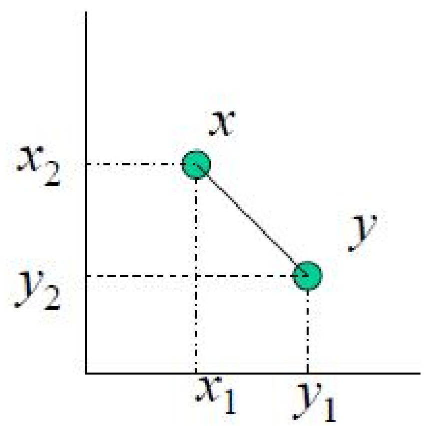
Metric: In mathematics, a metric typically refers to a metric space or a metric function **D** with three properties:

1. Non-negativity: $\mathbf{D}(a,b) \geq 0$
2. Reflexivity: $\mathbf{D}(a,b) = 0$ If and only if $a = b$.
3. Symmetry: $\mathbf{D}(a,b) = \mathbf{D}(b,a)$.
4. Triangle inequality: $\mathbf{D}(a,c) \leq \mathbf{D}(a,b) + \mathbf{D}(b,c)$.



Distance: A distance is a particular case from metric there are three distance metrics used in various mathematical and computational contexts. Here’s an overview of each:

1. Euclidean Distance: $\mathbf{D}(x,y) = \sqrt{\sum_{i=1}^{N} |x_i - y_i|^2}$
2. Maximum (Chebyshev) Distance: $\mathbf{D}(x,y) = \max_{i=1,\dots,n} |x_i - y_i|$.
3. Manhattan Distance: $\mathbf{D}(x,y) = \sum_{i=1}^{N} |x_i - y_i|$.

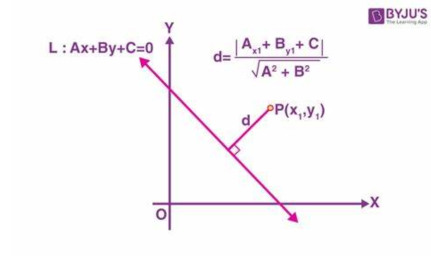


1.3.1 Distance in 2D

In this subsection, we provide a rules metric of distances between points, lines, and circles in two-dimensional (2D) space.

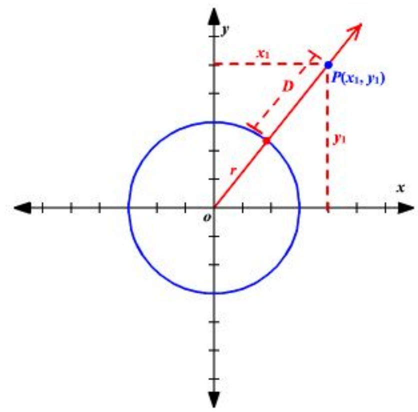
Point and line: The distance d between a point $P(x_1, y_1)$ and a line (D) where $Ax + By + C = 0$ in two-dimensional space is given by:

$$D(P,(D)) = \frac{|Ax_1 + By_1 + C|}{\sqrt{A^2 + B^2}}.$$



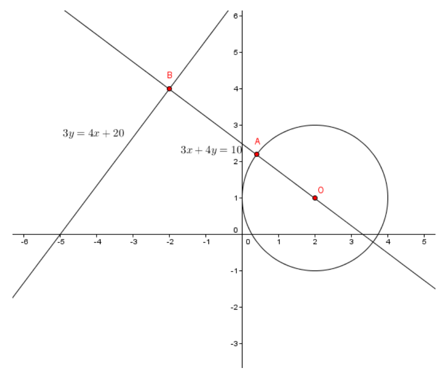
Point and Circle: Let $P(x_1, y_1)$ be a point and C be a circle with center $O(a, b)$ and radius r. The equation of the circle is $(x-a)^2 + (y-b)^2 = r^2$. The distance d between the point P and the circle C in two-dimensional space is given by:

$$D(P,C) = \sqrt{(x_1 - a)^2 + (y_1 - b)^2} - r.$$



Line and Circle: Let C be a circle with center (a, b) and radius r where $(x-a)^2 + (y-b)^2 = r^2$, and let (D) be a line with the equation $Ax + By + C = 0$. The distance d between the circle C and the line (D) in two-dimensional space is given by:

$$D((D),C) = \frac{|Aa + Bb + C|}{\sqrt{A^2 + B^2}} - r.$$

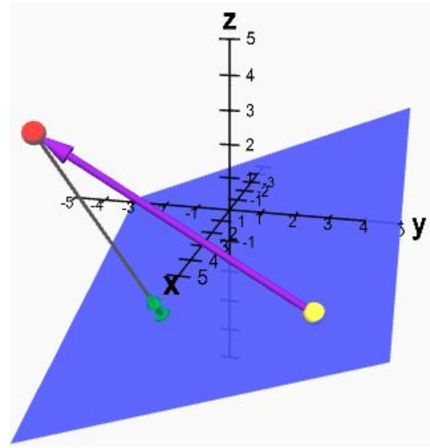


1.3.2 Distance in 3D

In this section, we will provide the same definitions as in the previous subsection, but we will present them in a three-dimensional (3D) context, enhancing our understanding by adding depth and dimension to the concepts.

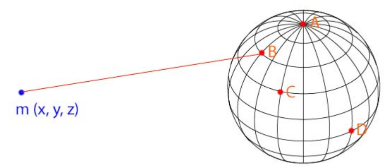
Point and Plan: The distance d between a point $P(x_1, y_1, z_1)$ and a plan (D) where $Ax + By + Cz + d = 0$ in three-dimensional space is given by:

$$D(P,(D)) = \frac{|Ax_1 + By_1 + Cz_1 + d|}{\sqrt{A^2 + B^2 + C^2}}.$$



Point and Sphere: Let $P(x_1, y_1, z_1)$ be a point and C be a Sphere with center $O(a,b,c)$ and radius r. The equation of the Sphere is $(x-a)^2 + (y-b)^2 + (z-c)^2 = r^2$. The distance d between the point P and the ball C in three-dimensional space is given by:

$$D(P,C) = \sqrt{(x_1-a)^2 + (y_1-b)^2 + (z-c)^2} - r.$$



Plan and Sphere: Let C be a Sphere with center (a,b,c) and radius r where $(x-a)^2 + (y-b)^2 + (z-c)^2 = r^2$, and let (D) be a plan with the equation $Ax + By + Cz + d = 0$. The distance d between the Sphere C and the plan (D) in three-dimensional space is given by:

$$D((D),C) = \frac{|Aa + Bb + Cc + d|}{\sqrt{A^2 + B^2 + C^2}} - r.$$