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Exercise 1 *Solve in* $\mathbb{R}$ *the following equations:*

1. $\cos(x) = \dfrac{\sqrt{2}}{2}$, $\sin(x) = \dfrac{1}{2}$, $\cos(3x + \frac{\pi}{4}) = \cos(x + \frac{\pi}{3})$ *and* $\cos(2x) = \sin(3x)$.
2. $2\cos^2(x) = 1$, $2\cos^2(x) = \sin^2(x) - 1$, $\cos^2(x) - \sin^2(2x) = 0$ *and* $\cos^2(x) - \sin^2(2x) = 0$.
3. $\sin(x) + \sin(3x) = \cos(x)$.
4. $\cos(x) + \cos(5x) = \cos(3x) + \cos(7x)$.

Exercise 2 *Solve in* $\mathbb{R}$ *and plot on a trigonometric circle the solutions within the range* 0 *to* 2π*:*

1. $\cos(x) > \frac{\sqrt{3}}{2}$, $3\tan(x) - \frac{\sqrt{3}}{3} > 0$ *and* $1 - 3\sin(x) > 0$.
2. $2\sin(3x) + 1 < 0$, $\tan\left(\frac{3\pi}{5}\right) - \tan(2x) < 0$ *and* $\sin^2(x) - \frac{3}{4} > 0$.

Determine the domain of definition and the set of roots of the functions $f : \mathbb{R} \to \mathbb{R}$.

1. $f(x) = \tan(3x + \frac{\pi}{4})$.
2. $f(x) = \dfrac{\sin^2(2x) - 1}{\tan(2x)}$.

Exercise 3 *Consider the real-valued function* f *defined by* $f(x) = 2x - \sin(x)$.
1) Show that for all real x, $2x - 1 \leq f(x) \leq 2x + 1$.
2) Deduce the limits of f *as* x *tends to* $+\infty$ *and as* x *tends to* $-\infty$.

Exercise 4 *Conversion Between Polar and Cartesian Coordinates.*

1. *Part A: Convert the following Cartesian coordinates to polar coordinates:*
   (a) $(3, 4)$, $(-2, 2\sqrt{3})$ *and* $(0, -5)$.
2. *Part B: Convert the following polar coordinates to Cartesian coordinates:*
   (a) $(6, \frac{\pi}{3})$ *and* $(-6, \frac{5\pi}{3})$.
   (b) $(2, \frac{5\pi}{6})$ *and* $(-2, -\frac{\pi}{6})$.
   (c) $(3, \pi)$ *and* $(3, -\pi)$.

Exercise 5 *Conversion Between Cylindrical and Cartesian Coordinates.*

1. *Part A:Convert the following Cartesian coordinates to cylindrical coordinates:*
   (a) $(3, 4, 5)$, $(-2, -2, 2)$ *and* $(0, 0, 7)$.
2. *Part B: Convert the following polar coordinates to Cartesian coordinates:*
   (a) $(2, \frac{\pi}{4}, 6)$,$(3, \frac{3\pi}{2}, -1)$ *and* $(4, 0, 0)$.

Exercise 6 *Conversion Between Spherical and Cartesian Coordinates*

1. *Part A: Convert the following Cartesian coordinates to spherical coordinates:*
   (a) $(3, 4, 5)$, $(-2, -2, 2)$ *and* $(0, 0, 7)$.
2. *Part B: Convert the following spherical coordinates to Cartesian coordinates:*
   (a) $(2, \frac{\pi}{4}, \frac{\pi}{3})$, $(3, \frac{3\pi}{2}, \frac{-\pi}{6})$ *and* $(4, \frac{\pi}{2}, 0)$.