Summary

General introduction

This course is intended for students in the second year of the electronics and electrical engineering degree and for those who want to learn the basics of electronics in general. The latter presents a necessary basis for a future electronics engineer. This collection of basic courses is taught by all Algerian universities, it is mainly aimed at students of the second year of Electronics and Electrical Engineering, of the electronics and electrical engineering sectors.

This is fundamental electronics1, the aim of which is to provide students with a basic document that can provide significant support to students and allow them to illustrate all the parts taught in the subject.

To this end, we have set ourselves two main objectives: the first is to provide students with a useful presentation to familiarize themselves with the general concepts of electrical networks, and semiconductors ranging from the NP junction to the operational amplifier. The second objective is to allow them to have a basis that can guide them in acquiring other knowledge in the context of more in-depth studies. The course is divided into five chapters:

In the first chapter generalities on the applications of Ohm's and Kirchhoff's laws, and the methods of analysis of DC networks will be presented. In the second chapter introducing us to the concept of quadrupoles, a study of electrical networks in the form of quadrupole, followed by a study on passive filters will be presented. In the third chapter a reminder on semiconductors as a basic introduction for the PN junction and the junction diode as well as a study of some circuits based on diodes. The fourth chapter is dedicated to the study of the static and dynamic regime of bipolar transistors with these different assemblies such as common emitter, common base, and common collector. The last chapter deals with the operation of the most popular and most used integrated circuit: the operational amplifier. Finally, each chapter is crowned with examples of applications so that the student can understand the basic concepts and solve a problem in fundamental electronics in general.

I hope that placing this modest course in the hands of our students can help them to understand and assimilate the main functions of fundamental electronics.

Chapter I.

Continuous regime and fundamental theorems

1. Definitions

An electrical component can only function if it is traversed by an electric current. So it must be able to let the electric current in and let it out.

- **Terminal**

This is the part of an electrical component that can let the electric current. Figure 1. The terminals also allow connections to be made, that is to say, to connect one electrical component to another electrical component.

- **Dipole**

It is an electrical component that has two terminals Figure 1. The lamps, the switches, the generators, the batteries, THE diodes, THELED, the resistances, and the engines are dipoles.

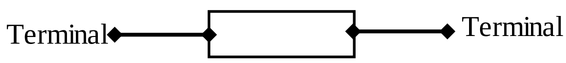


Figure I.1. Dipole

The dipole is the electrical component it has two terminals: an input terminal for the electric current and an output terminal. An electrical component cannot have fewer than two terminals. On the other hand, there are electrical components more complex than dipoles with three, four, or more terminals, we then speak of tripoles, quadrupoles, etc. Transistors, transformers, or operational amplifiers are not, for example, dipoles. Each dipole has a simplified representation called a standardized symbol. We generally distinguish two types of dipoles:

Active dipole: The generators which can produce an electric current.

Passive dipole: Receivers that receive the electric current.

The behavior of a dipole can be described by a characteristic curve either

$$I = f(U) \tag{I.1}$$

ou

$$U = f(I) \tag{I.2}$$

There are two types of dipoles: active dipoles and dipoles passive. A dipole is passive if its characteristic passes through zero. The behavior of a dipole is characterized by two dual electrical quantities: voltage and current. The voltage across a dipole represents the potential difference u(t) between the two terminals of the dipole. The voltage is expressed in Volts (V).

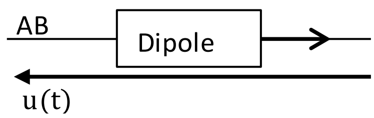


Figure I.2. Voltage across a dipole

$$u(t) = V_A - V_B \tag{I.3}$$

The current flowing through a dipole corresponds to the movement of electric charges under the effect of the electric field induced by the potential difference at the terminals of the dipole. At any time, the current entering through one terminal of a dipole is equal to the current leaving through the other terminal. The intensity i(t) is the flow rate of electric charges flowing through a section of the conductor:

$$i(t) = \frac{dq(t)}{dt} \tag{I.4}$$

Intensity is expressed in Amperes (A). Electric current is an oriented quantity. Conventionally, the positive direction corresponds to the direction of movement of positive charges.

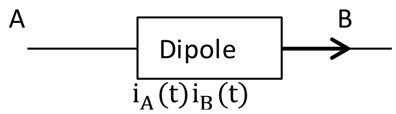


Figure I.3. The current in a dipole

$$i(t) = i_A(t) = i_B(t) \tag{I.5}$$

There are two possibilities for choosing the conventional directions of voltage and current. Depending on whether u and i are in the same direction or not, we have:

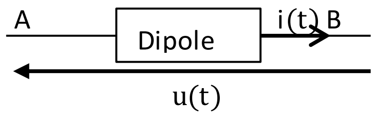


Figure I.4. Receiver

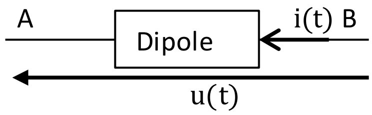


Figure I.5. Generator

In a steady state, independent of time, there is a relationship between the intensity i crossing the dipole and the voltage u between its terminals. This relationship can be put in the form of. $i = i(u) u = u(i)$

The graphs obtained are called static characteristics:

$i = i(u)$: static current-voltage characteristic of the dipole

$u = u(i)$: static voltage-current characteristic of the dipole

A dipole is passive if its short-circuit current and its open-circuit voltage are zero: its static characteristics pass through the origin. It is said to be active in the opposite case. A dipole is linear if:

$$i\ (\alpha v_1 + \beta v_2) = \alpha\ i(v_1) + \beta\ i(v_2) \tag{I.6}$$

$$u\ (\alpha i_1 + \beta i_2) = \alpha\ u(i_1) + \beta\ u(i_2) \tag{I.7}$$

- **Network**

A network is a set of dipoles connected by wires. Drivers of resistance are negligible.

- **Node**

In electricity as in electronics, a node is the electrical connection point between several components.

- **Branch**

A branch of a network is a set of dipoles connected in series.

- **Mesh**

A network mesh is a set of branches forming a closed circuit in which each node is encountered only once.

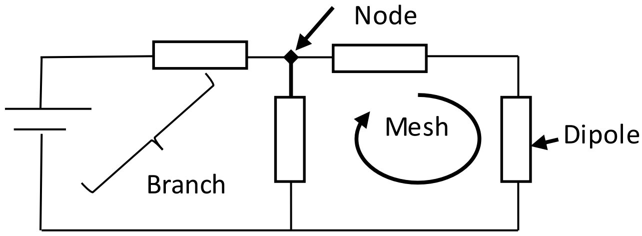


Figure I.6. Electrical network

2. Voltage

The electrical voltage between two points of a network is equal to the difference in electrical potential between these two points. The latter is an algebraic quantity, represented by an arrow shown below.

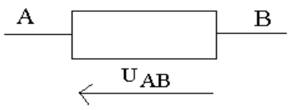


Figure I.7. Voltage across a dipole

If is the potential at point A and the potential at point B then we have: V_A V_B

$$(I.8) U_{AB} = V_A - V_B$$

$$U_{AB} > 0 \Rightarrow V_A > V_B (I.9)$$

$$U_{AB} < 0 \Rightarrow V_A < V_B (I.10)$$

Like electric potential, voltage is expressed in Volts (V).

3. Intensity

Quantity characterizing an electric current, that is to say, a movement of all mobile charges in a conductor. The intensity is expressed in Amperes (A). Relationship with other units of the International System: the intensity is linked to the charge crossing a section of the conductor by the relationship: $i(t)$ q

$$i(t) = \frac{dq}{dt} (I.11)$$

$i(t)$in A, in Coulomb (C), in second.$q t(s)$

4. Ohm's law for resistors

The electrical energy produced by the passage of a current I in a resistance is converted by the Joule effect into heat, it is expressed by the relation:

$$P = R \cdot I^2 \qquad (I.12)$$

On the other hand, the power consumed is equal to:

$$P = U \cdot I \qquad (I.13)$$

Where U denotes the potential difference "DDP" across the resistor; these two powers are equal, and we then obtain the following equality:

$$U \cdot I = R.I^2 \quad (I.14)$$

Dividing by I we get:

$$U = R.I \ (I.15)$$

This last relationship is Ohm's law.

4.1 Dipole Associations

Dipoles are said to be in series if they are traversed by the same intensity of electric current. And they are said to be in parallel if they have the same potential difference at their terminals.

4.1.1 Series connection of resistors

Let there be n resistors connected in series and carrying the same current I (figure 7).

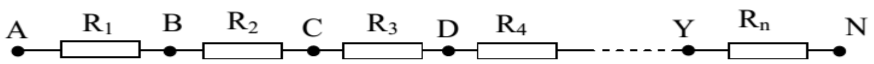


Figure I.8. Series resistors

If we consider that the resistances between A and N behave as a single resistance and as the resistances are in series; then the same current I which passes from A to N therefore we can write: R_{eq}

$$U_{AN} = R_{eq} I (I.16)$$

By applying Ohm's law to each of these resistances we can write the following relationships:

$U_{AB} = R_1 I$; $U_{BC} = R_2 I$; $U_{CD} = R_3 I$; $U_{DE} = R_4 I$; ... ; $U_{YN} = R_n I$;

The ddp between the ends A and N i.e. of the circuit is equal to the sum of the ddp between A and B, between B and C, between C and D, …, and between Y and N. $U_{AN} U_{AB} U_{BC} U_{CD} U_{YN}$

$$U_{AN} = R_1 I + R_2 I + R_3 I + R_4 I + \cdots + R_n I (I.17)$$

$$U_{AN} = (R_1 + R_2 + R_3 + R_4 + \cdots + R_n) I (I.18)$$

So by comparison we will have:

$$R_{eq} = R_1 + R_2 + R_3 + R_4 + \cdots + R_n \quad (I.19)$$

So we can conclude that the resistances of a branch (connected in series) are equivalent to a single resistance equal to the sum of these resistances of the latter.

4.1.2 Parallel or shunt connection of resistors

Let us place several resistances (for example four resistances, figure 8) between two points N and M. The current I in the circuit creates several derived currents, the intensity of which is equal to the sum of the intensities of these derived currents.

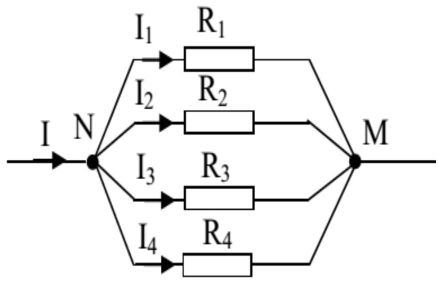


Figure I.9. Resistors in parallel.

If we consider that the resistances between M and N behave like a single resistance, we can write: R_{eq}

$$U_{MN} = R_{eq} I \quad (I.20)$$

SO

$$I = \frac{U_{MN}}{R_{eq}} = \left(\frac{1}{R_{eq}}\right) U_{MN} \quad (I.21)$$

We know that

$$I = I_1 + I_2 + I_3 + I_4 + \cdots + I_n \quad (I.22)$$

Applying Ohm's law between nodes M and N to each of the resistors, knowing that the voltage between M and N is constant, we can write the following relations: $R_1;\ R_2;\ R_3; R_4; \ldots.; R_n$

$$U_{MN} = R_1 I_1 = R_2 I_2 = R_3 I_3 = R_4 I_4 = \cdots = R_n I_n \quad (I.23)$$

$$I_1 = \frac{U_{MN}}{R_1}; I_2 = \frac{U_{MN}}{R_2}; I_3 = \frac{U_{MN}}{R_3}; I_4 = \frac{U_{MN}}{R_4}; \ldots\ldots; I_n = \frac{U_{MN}}{R_n} \quad (I.24)$$

We replace the values of in the previous equation we will have I

$$I = \frac{U_{MN}}{R_1} + \frac{U_{MN}}{R_2} + \frac{U_{MN}}{R_3} + \frac{U_{MN}}{R_4} + \cdots + \frac{U_{MN}}{R_n} \quad (I.25)$$

$$I = \left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_4} + \cdots + \frac{1}{R_n}\right) U_{MN} \quad (I.26)$$

SO

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_4} + \cdots + \frac{1}{R_n} \quad (I.27)$$

From the last relation, we can conclude that the inverse of the equivalent resistance is equal to the sum of the inverses of the resistances connected in parallel.

Rating:

The inverse of resistance is known as: conductance we can write the relationship G $\left(G = \frac{1}{R}\right)$.

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_4} + \cdots + \frac{1}{R_n} \quad (I.28)$$

In the following manner using the conductances

$$G_{eq} = G_{1+}G_2 + G_3 + G_4 + \cdots + G_{eq} \quad (I.29)$$

5. Kirchhoff's laws

An electrical circuit or network is a set of dipoles connected to each other by perfect conducting wires. A node is a point in the circuit connected to two or more dipoles. A network branch is the part of the circuit between two nodes. A mesh is a closed path of branches passing at most once through a given node. Kirchhoff's two laws allow the analysis of electrical networks.

5. 1 Law of nodes

At any node of a circuit, and any instant, the sum of the currents that arrive is equal to the sum of the currents that leave. This is a consequence of the conservation of electric charge.

$$\sum i_{\text{Entrant au noeud}} = \sum i_{\text{Sortant des noeuds}} \quad (I.30)$$

Or else

$$\sum i_{\text{Arrive at the node}} = \sum i_{\text{Leave from the node}} \quad (I.31)$$

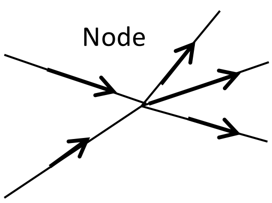


Figure I.10. Currents in a node

The law of nodes can also be written in the following form: At any node of a network, the algebraic sum of the currents is zero.

$$\sum_{k=1}^{N} \pm I_k = 0 \quad (I.32)$$

5.2 Mesh law

Along any mesh of an electrical network, at any time, the algebraic sum of the voltages is zero.

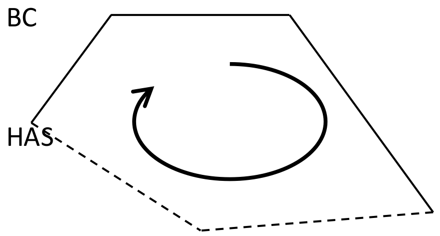


Figure I.11. Tensions in a mesh

$$(V_A - V_B) + (V_B - V_C) + (V_C - V_D) + \cdots + (V_? - V_A) = 0 \quad (I.33)$$

If we call the potential difference

$$(V_A - V_B) = V_1, ,\ldots, (I.34)\ (V_B - V_C) = V_2 (V_? - V_A) = V_k$$

So the law of meshes becomes

$$\sum_{k=1}^{N} \pm V_k = 0 \quad (I.35)$$

5.3 Divisor Rules

5.3.1 Voltage divider

It is applied to elements in series, crossed by the same current. (R_i)

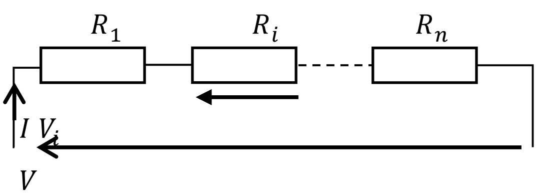


Figure I.12. Voltage divider

We have

$$V = (\sum_{i=1}^{n} R_i) I \quad (I.36)$$

And

$$V_i = R_i I \quad (I.37)$$

We can deduce the voltage at the terminal from the resistance $V_k R_k$

$$V_k = \frac{R_i}{\sum_{k=1}^{n} R_k} V \quad (I.38)$$

5.3.2 Current divider

It is applied for elements () in parallel subjected to the same voltage $G_j V$

$G_j = \frac{1}{R_j}$: is the conductance.

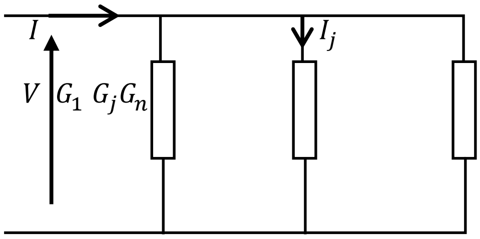


Figure I.13. Current divider

We have

$$I = V(\sum_{k=1}^{n} G_k) \quad (I.36)$$

And

$$I_j = V\, G_j \quad (I.37)$$

From the ratio of the last two equations, we can deduce the current flowing through the conductance. I_j

$$G_j I_j = \frac{G_j}{(\sum_{k=1}^{n} G_k)} I \quad (I.38)$$

Or in terms of resistance

$$I_j = \frac{\frac{1}{R_j}}{\left(\sum_{k=1}^{n} \frac{1}{R_k}\right)} I \quad (I.39)$$

6. Voltage and current sources

6.1. Ideal and real voltage sources

An ideal voltage generator delivers a voltage independent of the current supplied:

$$V_A - V_B = e = constante\ \forall i \quad (I.40)$$

i: the current delivered by the voltage source.

This voltage is the electromotive force (emf) of the generator.

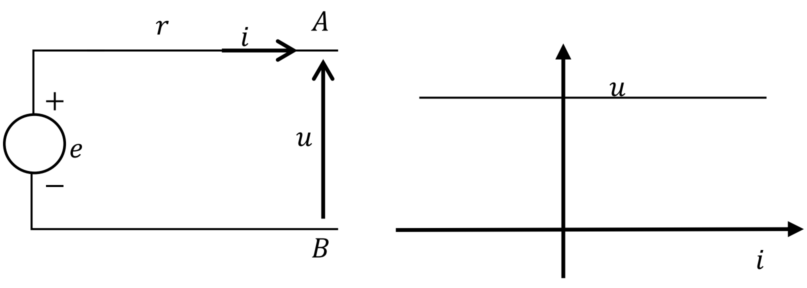


Figure I.14. Ideal generator

The internal resistance of an ideal voltage generator is zero, which is generally not the case for a real generator. A real generator is modeled by an ideal generator in series with its internal resistance. In generator convention, the static voltage-current characteristic of the real voltage generator becomes:

$$u = e - r\, i \quad (I.41)$$

Internal resistance induces a voltage drop.

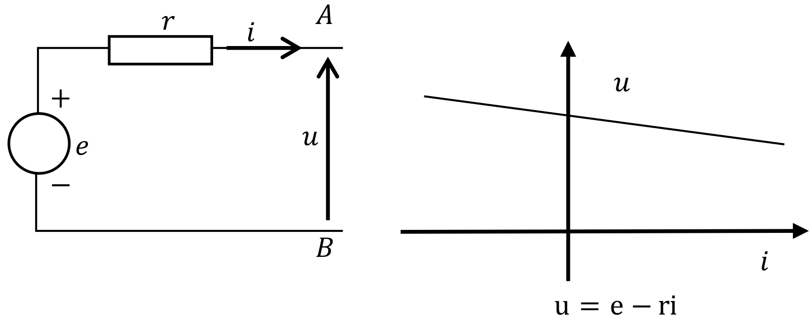


Figure I.15. Real generator

There are two types of voltage sources. An independent, or autonomous, source is a source whose emf value is constant and does not depend on the circuit. A controlled, monitored, or linked source is a source whose EMF value depends on a quantity external to the source, for example, a voltage or current of the circuit.

An ideal voltage generator is an example of a polarized dipole: the sign of the emf is independent of that of the current. Depending on the case, it functions as a generator or receiver. Indeed, in notation, the generator represents the power delivered to the rest of the circuit by the voltage source. Thus: $p = u\,i$

$$\text{If}, i > 0 \Rightarrow p > 0, \text{source} \equiv \text{generator} \qquad (I.42)$$

$$\text{If}, i < 0 \Rightarrow p < 0, \text{source} \equiv \text{receiver} \qquad (I.43)$$

6.2 Ideal and real current sources

An ideal current generator delivers a current whose intensity is independent of the voltage across the generator terminals:

$$i = i_S = \text{constante} \ \forall \ u \qquad (I.44)$$

The following figure shows the symbol of an ideal current source and its current-voltage characteristic.

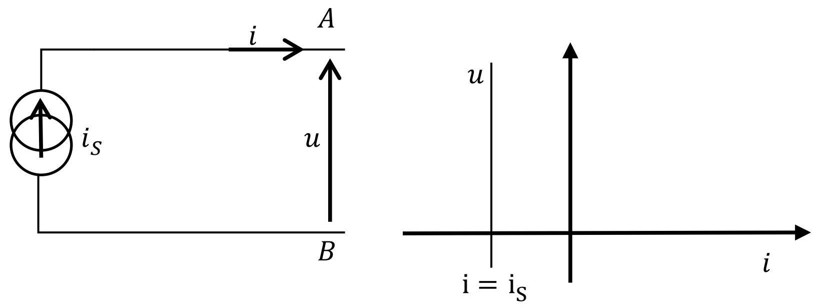


Figure I.16. Ideal current source

The internal resistance of an ideal current source is infinite. For a real generator, its internal resistance is taken into account by modeling it by an ideal current source in parallel with its internal resistance. In generator convention, the static current-voltage characteristic of the real current generator is therefore:r

$$i = i_S - \frac{u}{r} \qquad (I.45)$$

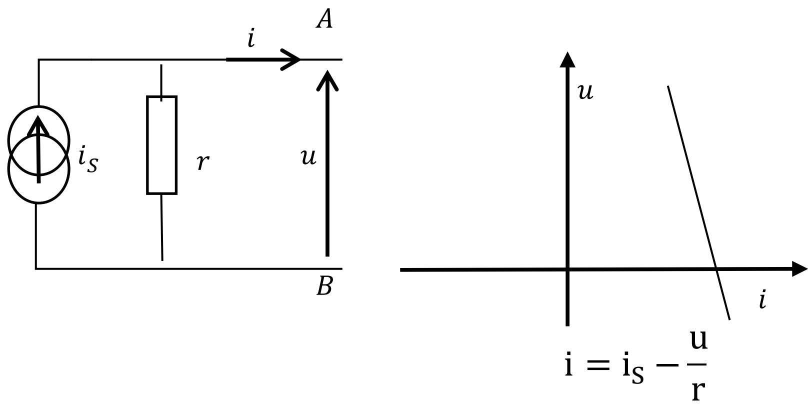


Figure I.17. Real current source

As with voltage sources, we distinguish between independent current sources and controlled current sources which depend on an electrical quantity in the circuit.

7. Fundamental theorems of electrical circuit analysis

Kirchhoff's laws are used to determine the current intensities and the potential differences (ppd) at the terminals of each branch of the electrical network. This operation is called analysis of the circuit or of the electrical network. Since all the constituent elements of the network are known, the complete calculation requires as many equations as branches. The analysis is simplified by the application of associative laws and appropriate theorems.

7.1 Mesh method

It allows to solve the problem by writing M equations to the meshes:

- We choose a system of M independent meshes.
- Each of these meshes is assigned a fictitious current flowing in an arbitrarily chosen direction.
- We apply Kirchhoff's 2nd law to each of these meshes.
- The real current of a given branch is obtained by performing the algebraic sum of the fictitious currents flowing in the branch considered.
- Branch ddps are deduced from actual currents

7.2 Knot Method

Represent by writing N equations at the nodes:

- We choose a reference node (which is most often the ground);

- Each of the remaining nodes is assigned an unknown potential V1, V2, ..., VN;
- We write for each of these N nodes Kirchhoff's 1st law.

7.3 Principle of superposition

When it contains only linear dipoles, the response (current and voltage in each branch) of a network comprising several independent sources (voltage and/or current) is equal to the sum of the responses that would be obtained by considering each of these sources separately.

For each of the independent sources, we study the response of the circuit, the other independent sources being "off". On the other hand, the controlled sources always remain active. The principle of superposition is a direct consequence of the linearity of the network.
An ideal "off" voltage source is replaced by a short circuit (). An ideal "off" current source is replaced by an open circuit ().$e = 0\ \forall i$, $i_s = 0\ \forall u$
Consider a circuit with n dipoles, including N independent voltage or current sources. The electrical state of this circuit, or its response, can be characterized by the set of current intensities flowing through each dipole and the voltages across them:

$$\{R_k = i_k V_k\}_{k=1,n} \tag{I.46}$$

We can calculate N partial states by considering each of the N sources in service only, the others being "off":

$$\{R_k = i_k V_k\}_{k=1,n} \tag{I.47}$$

Each resistance can be characterized by:
for j=1,N

$$R^j = \{i_k^i, V_k^i\}_{k=1,n} \tag{I.48}$$

The principle of superposition allows us to write the complete response from the partial states:

$$R = \sum_{j=1}^{N} R^j \tag{I.49}$$

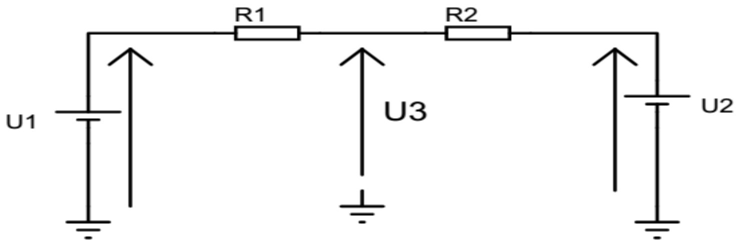
Either :

$$\begin{cases} i_k = \sum_{j=1}^{N} i_k^j \\ V_k = \sum_{j=1}^{N} V_k^j \end{cases} \forall k = 1, n \tag{I.50}$$

In another way, for a circuit of n voltage sources (E1 to En), if we want to determine the value of a potential or a potential difference of any kind in the assembly, it is sufficient:

1- To calculate the value of this potential taking into account only the source E1. The remaining n-1 sources being extinguished.
2- Repeat this operation for each voltage source (n calculations).
3- Add all the voltage values calculated in 1 and 2

Example for the circuit below



U1 and U2 known, we wish to determine the voltage U3

The value of the potential U3 can be found in two steps:

- We turn off the source U2 (replaced by a short circuit) and we calculate U31, as a function of U1, R1 and R2.

- We turn off the source U1 (replaced by a short circuit) and we calculate U32, as a function of U2, R1 and R2.

The potential difference U3 is then U31+U32.

8. Thevenin and Norton theorems

8.1 Thevenin's theorem

A linear network, comprising only independent sources of voltage, current and resistances, taken between two terminals behaves like a voltage generator E0 in series with a resistance R0. The emf E0 of the equivalent generator is equal to the voltage existing between the two terminals considered when the network is in open circuit. The resistance R0 is that of the circuit seen from the two terminals when all the sources are off.

8.2 Norton's Theorem

Similarly, any linear network, not including controlled sources, taken between two of its terminals can be replaced by a current source I0 in parallel with a resistance R0. The intensity I0 is equal to the short-circuit current, the two terminals being connected by a perfect conductor. The resistance R0 is that of the circuit seen from the two terminals when all the sources are off.

8.3 Equivalence between Thevenin and Norton representations

The respective application of Thevenin and Norton's theorems allows us to show the equivalence of the following two circuits:

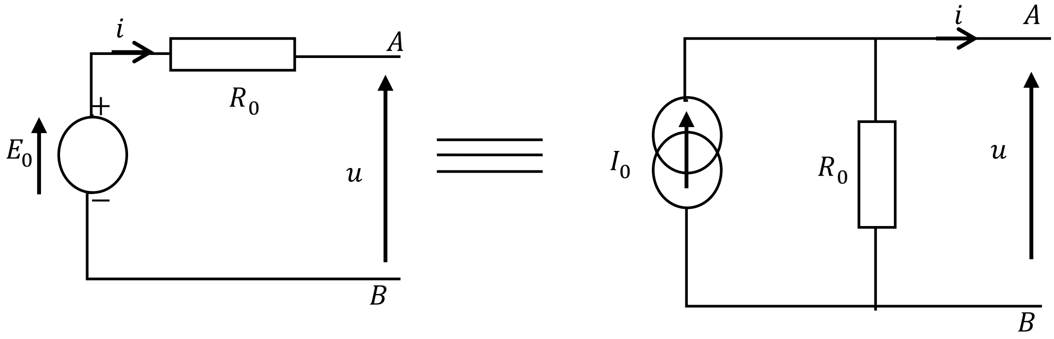


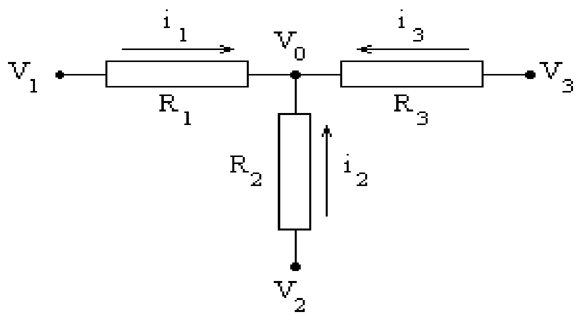
Figure I.18. Thevenin equivalence Norton

With :

$$E_0 = R_0 I_0 \tag{I.51}$$

9. Millman's Theorem

Consider the following circuit:



For each of the branches we can write:

$$\begin{cases} V_1 - V_0 = R_1 I_1 \\ V_2 - V_0 = R_2 I_2 \\ V_3 - V_0 = R_3 I_3 \end{cases} \tag{I.52}$$

Or again:

$$\begin{cases} I_1 = \frac{V_1 - V_0}{R_1} \\ I_2 = \frac{V_2 - V_0}{R_2} \\ I_3 = \frac{V_3 - V_0}{R_3} \end{cases} \tag{I.53}$$

By adding up these relations we get:

$$I_1 + I_2 + I_3 = \frac{V_1 - V_0}{R_1} + \frac{V_2 - V_0}{R_2} + \frac{V_3 - V_0}{R_3} \tag{I.54}$$

Now we have: , therefore: $I_1 + I_2 + I_3 = 0$

$$V_0 \left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} \right) = \frac{V_1}{R_1} + \frac{V_2}{R_2} + \frac{V_3}{R_3} \tag{I.55}$$

Or

$$V_0 = \frac{\frac{V_1}{R_1} + \frac{V_2}{R_2} + \frac{V_3}{R_3}}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}} \tag{I.56}$$

This result generalizes to any number of branches:

$$V_0 = \frac{\sum_1^n \frac{V_k}{R_k}}{\frac{1}{R_k}} = \frac{\sum_1^n G_k V_1}{\sum_1^n G_k} \tag{I.57}$$

The voltage at the node is the average of the voltages across all dipoles weighted by the respective conductances.

10. Kennelly's Theorem

Presentation of the assemblies in the form of a triangle (left) and a star (right). Kennelly's theorem, or triangle-star transformation, or Y-Δ transformation, or T-Π transformation, is a mathematical technique that simplifies the study of certain electrical networks.

This theorem, named in homage to Arthur Edwin Kennelly, allows you to move from a "triangle" configuration (or Δ, or Π, depending on how you draw the diagram) to a "star" configuration (or, similarly, Y or T). The diagram opposite is drawn in the "triangle-star" form; the diagrams below in the T-Π form.
This theorem is used in electrical engineering or in power electronics in order to simplify three-phase systems. It is also commonly used in electronics to simplify the calculation of filters or attenuators. The two circuits in Figure 17 are equivalent if the values of their resistances are related by the relationships shown below.

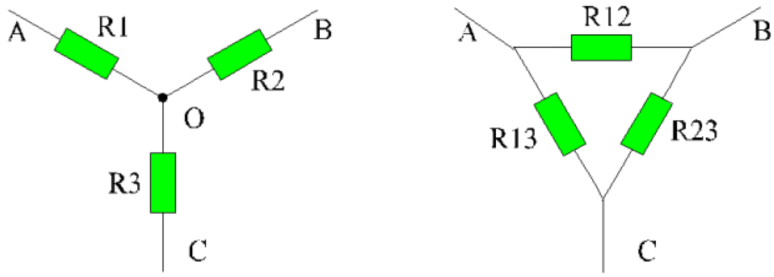


Figure I.19. Star-rod equivalence or Pi-Ti

The transition from the triangle structure (ABC) to the star structure (OABC) is obtained by the relations:

If we disconnect point A, there must be equality of impedances between B and C.

$$R_2 + R_3 = \frac{R_a(R_b+R_c)}{R_a+R_b+R_c} \tag{I.58}$$

.

We draw the following three equalities:

$$R_2 + R_3 = \frac{R_a(R_b+R_c)}{R_a+R_b+R_c} \tag{I.59}$$

$$R_2 + R_1 = \frac{R_c(R_a+R_b)}{R_a+R_b+R_c} \tag{I.60}$$

$$R_1 + R_3 = \frac{R_b(R_a+R_c)}{R_a+R_b+R_c} \tag{I.61}$$

By adding the first two equalities and subtracting the third, we deduce:

$$R_1 = \frac{R_b R_c}{R_a+R_b+R_c} \tag{I.62}$$

$$R_2 = \frac{R_a R_c}{R_a+R_b+R_c} \tag{I.63}$$

$$R_3 = \frac{R_b R_a}{R_a+R_b+R_c} \tag{I.64}$$

For the inverse transformation, we connect B and C: the conductance between A and BC is then written:

$$\frac{1}{Z_a} = \frac{1}{R_C} + \frac{1}{R_b} = \frac{R_2+R_3}{R_1R_2+R_2R_3+R_1R_3} \tag{I.65}$$

$$\frac{1}{Z_b} = \frac{1}{R_a} + \frac{1}{R_c} = \frac{R_1+R_3}{R_1R_2+R_2R_3+R_1R_3} \tag{I.66}$$

$$\frac{1}{Z_c} = \frac{1}{R_a} + \frac{1}{R_b} = \frac{R_1+R_2}{R_1R_2+R_2R_3+R_1R_3} \tag{I.67}$$

And we calculate

$$\frac{1}{Z_a} = \frac{1}{Z_b} - \frac{1}{Z_c} \tag{I.68}$$

He comes

$$\frac{2}{R_b} = \frac{2R_2}{R_1R_2+R_2R_3+R_1R_3} \tag{I.69}$$

Either

$$R_b = \frac{R_1R_2+R_2R_3+R_1R_3}{R_2} \tag{I.70}$$

Similarly we can also write that:

$$R_c = \frac{R_1R_2+R_2R_3+R_1R_3}{R_3} \tag{I.71}$$

$$R_a = \frac{R_1R_2+R_2R_3+R_1R_3}{R_1} \tag{I.72}$$

This theorem is used to transform networks in triangle form (Pi) to star form (T) and vice versa (figure 16).

1) Triangle → star transformation (Δ → Y)

$$R_1 = \frac{R_bR_c}{R_a+R_b+R_c} \tag{I.73}$$

$$R_2 = \frac{R_aR_c}{R_a+R_b+R_c} \tag{I.74}$$

$$R_3 = \frac{R_bR_a}{R_a+R_b+R_c} \tag{I.75}$$

2) Star → triangle transformation (Y → Δ)

$$R_b = \frac{R_1R_2+R_2R_3+R_1R_3}{R_2} \tag{I.76}$$

$$R_c = \frac{R_1R_2+R_2R_3+R_1R_3}{R_3} \tag{I.77}$$

$$R_a = \frac{R_1R_2+R_2R_3+R_1R_3}{R_1} \tag{I.78}$$

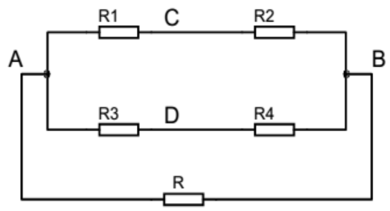
Exercise

This network represents the passive elements of a Wheatstone bridge, in the case where the internal resistance of the power supply is not negligible. Calculate the equivalent resistance to the network:

a) seen from A And B

b) seen from C And D

Digital application: $R_1 = R_2 = R_3 = R_4 = 350\Omega$, $R = 50\Omega$



Quick solution

a) Three branches connect A has B:

- a branch containing two resistance dipoles, in series. R_1 et R_2
- a branch containing two other resistance dipoles, in series. R_3 et R_4
- a branch containing a resistance dipole. R

The first branch has equivalent resistance $R_{12} = R_1 + R_2 = 700\Omega$

The second branch has equivalent resistance, the conductances are added:

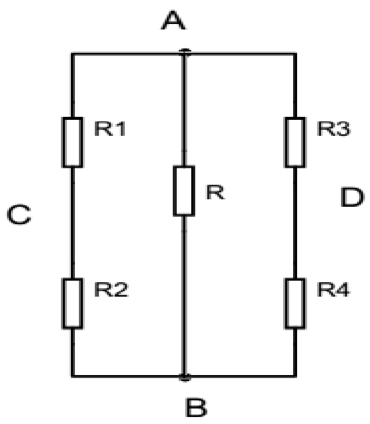
$$G_{AB} = G_{12} + G_{34} + G$$

$$\frac{1}{R_{AB}} = \frac{1}{R_{12}} + \frac{1}{R_{34}} + \frac{1}{R} = \frac{1}{700} + \frac{1}{700} + \frac{1}{50}$$

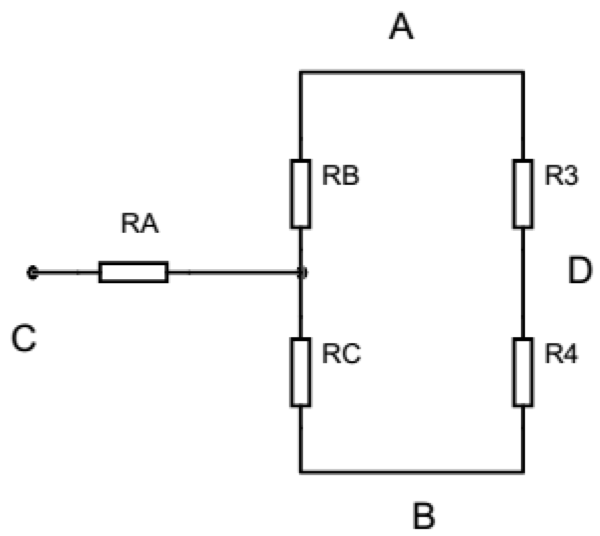
$$R_{AB} = 43{,}75\Omega$$

$$R_{AB} = 43,75$$

b) To go from C to D we can go either through A or B; A and B are connected by a branch containing a dipole of resistance R We can redraw the network as follows:



To calculate the resistance of the network seen from C and D, we transform one of the triangles into a star, for example ABC.



According to Kennelly's theorem.

$$R_C = \frac{R_1 R_2}{R_1 + R_2 + R} = 163{,}3\Omega$$

$$R_B = \frac{RR_2}{R_1 + R_2 + R} = 23{,}3\Omega$$

$$R_A = \frac{R_1 R}{R_1 + R_2 + R} = 23{,}3\Omega$$

The dipoles in series are replaced by their equivalent resistance

$$R_{A3} = R_A + R_3 = 373{,}3\Omega$$

$$R_{B4} = R_B + R_4 = 373{,}3\Omega$$

$$G_{A3B4} = G_{A3} + G_{B4}$$

$$\frac{1}{R_{A3B4}} = \frac{1}{R_{A3}} + \frac{1}{R_{B4}}$$

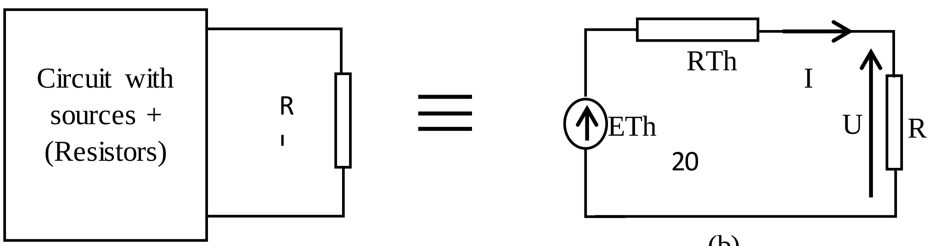
$$R_{A3B4} = 186{,}7\Omega$$

And we apply the law of serial association.

$$R_{CD} = R_C + R_{A3B4} = 350\Omega$$

11. Maximum power transfer

If we consider a circuit containing sources and passive elements (impedances); we can represent it by a voltage source which is the Thevenin generator and a resistance or Thevenin impedance as shown in the figure below.



(has (b)

Figure I.20. Thevenin equivalence

To calculate the condition for which there will be a maximum power transfer for the circuit Figure (a) we use its Thevenin equivalent; then the current absorbed by the load will be:

$$I = \frac{E_{th}}{R_{Th}+R_L} \tag{I.79}$$

So the power consumed by the load is:

$$U = R_L I \tag{I.80}$$

$$P = U.I = R_L I^2 = R_L \left(\frac{E_{th}}{R_{Th}+R_L}\right)^2 \tag{I.81}$$

To calculate when the maximum power is transferred one must calculate the maximum power i.e.:

$$\frac{dP}{dR_L} = \left(\frac{E_{th}}{R_{Th}+R_L}\right)^2 + 2R_L \left(\frac{E_{th}}{R_{Th}+R_L}\right)\left(\frac{-E_{th}}{(R_{Th}+R_L)^2}\right) \tag{I.82}$$

$$\frac{dP}{dR_L} = \left(\frac{{E_{th}}^2(R_{Th}+R_L)-2R_L{E_{th}}^2}{(R_{Th}+R_L)^3}\right) \tag{I.83}$$

$$\frac{dP}{dR_L} = 0 \Longrightarrow \left(\frac{{E_{th}}^2(R_{Th}+R_L)-2R_L{E_{th}}^2}{(R_{Th}+R_L)^3}\right) = 0 \Longrightarrow$$

$${E_{th}}^2(R_{Th}+R_L)-2R_L{E_{th}}^2 = {E_{th}}^2(R_{Th}-R_L) = 0 \Longrightarrow$$

$$R_{Th} - R_L = 0 \Longrightarrow R_{Th} = R_L \tag{I.84}$$

It can be concluded that there will be maximum power transfer to the load when .$R_{Th} = R_L$

In this case we will have

$$P_{max} = R_L \left(\frac{E_{th}}{R_{Th}+R_L}\right)^2 = R_L \left(\frac{E_{th}}{R_L+R_L}\right)^2 = R_L \left(\frac{E_{th}}{2R_L}\right)^2 = \frac{{E_{th}}^2}{4R_L} \tag{I.85}$$

$$\Longrightarrow P_{max} = \frac{{E_{th}}^2}{4R_L} \tag{I.86}$$