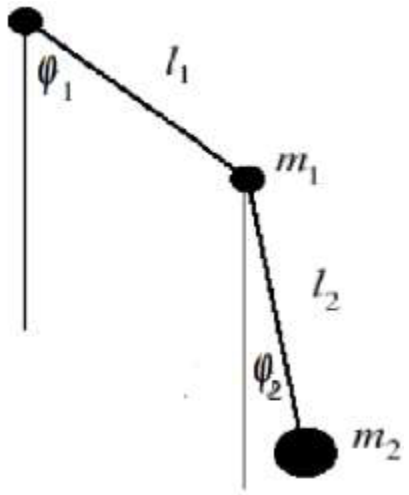
Department of Physics
University of M'sila
Module: Analytical Mechanics
Series 2
Exercise 1

Consider a mass m_1 suspended from a point P, which we take as the origin, by a rigid rod of length l_1 and negligible mass. Let's add a second mass m_2, suspended from mass m_1 by another rigid rod of negligible mass and length l_2 , also constrained to pivot only in the (x, y) plane. Let φ_2 be the angle that the second rod makes with the vertical (as opposed to φ_1 for the first rod, of length l_1). Let (x_1, y_1, z_1) and (x_2, y_2, z_2) be the Cartesian coordinates of the two masses.

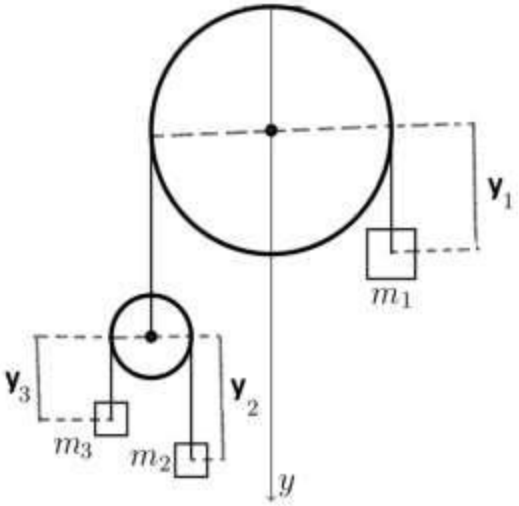


1. Write the constraint equations and define the generalized coordinates of this system.
2. Write the kinetic and potential energy of this system.
3. Write the Lagrangian and the Lagrange equations.
4. Write the previous equations assuming that the angles φ_1 and φ_2 are small, and deduce the expression for the equations of motion.

Exercise 2
Write the Lagrangian of a particle of mass mmm in spherical coordinates in the potential field $V(r, \theta)$

Application: $V(r, \theta) = \frac{cos\theta}{r^2}$

Exercise 3: Generalized Atwood Machine: Consider the system shown in which a mass m_1 is connected, via pulley A, to a second pulley B with mass M. Pulley B, in turn, connects two other masses m_2 and m_3. The masses of the ropes and pulleys are negligible, and gravity g acts downward. The vertical positions of the three masses are y_1, y_2, and y_3, and movement in any direction other than vertical is ignored. Pulley A is fixed, and the ropes have constant lengths.



1. Show that this problem has two degrees of freedom.
2. Write the Lagrangian of this system, using are $y_1$and y_2 as generalized coordinates.
3. Find an explicit expression for the accelerations $\ddot{y}_1$ and $\ddot{y}_2$ in terms of g and the three masses. Under what condition is $\ddot{y}_1$ zero?

Exercise 4

Two masses m_1 and m_2 arranged on a perfectly smooth double inclined plane are connected by an inextensible string of negligible mass that passes without friction over a pulley of negligible mass. Let α_1 and α_2 be the angles of the inclined planes.

1. Show that this problem has one degree of freedom.
2. Write the Lagrangian and the Lagrange equations.
3. Deduce the conditions for equilibrium.