Chapter 3: Relative motion

I- Introduction

Motion is always defined with respect to an observer or reference frame. So, it is necessary to choose a reference frame in order to determine the position, velocity and acceleration of an object at each instant.

The reference frame can be stationary or moving:

- Stationnary" or "absolute" referential: is attached to the observer and it is fixe, usually with respect to the earth ($\Re$ (O,X,Y,Z)).

- Relative referential: is is itself moving ($\Re'$ (O’,X’,Y’,Z’)).

The position, velocity and acceleration depend on the frame and they can be transformed to get their equivalents in another frame.

II- Description of the motion

II.1- Motion in absolute referential

The motion is described in the absolute referential $\Re$ (O,X,Y,Z):

- The absolute position vector: $\overrightarrow{OM} = x\,\vec{\imath} + y\,\vec{\jmath} + z\,\vec{k}$
- The absolute velocity vector: $\vec{V_a} = \overrightarrow{V_{M/R}} = \left.\frac{d\overrightarrow{OM}}{dt}\right|_R = \dot{x}\,\vec{\imath} + \dot{y}\,\vec{\jmath} + \dot{z}\,\vec{k}$
- The absolute acceleration vector: $\vec{a_a} = \overrightarrow{a_{M/R}} = \left.\frac{d^2\overrightarrow{OM}}{dt^2}\right|_R = \left.\frac{d\vec{V}}{dt}\right|_R = \ddot{x}\,\vec{\imath} + \ddot{y}\,\vec{\jmath} + \ddot{z}\,\vec{k}$

II .2- Motion in relative referential

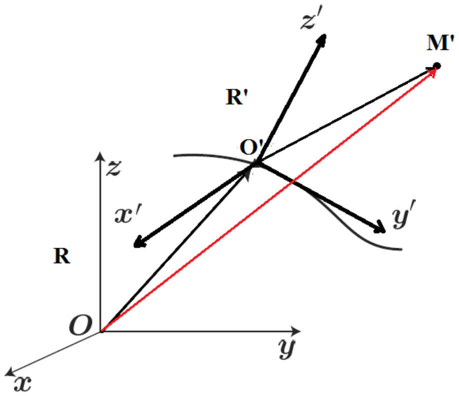
The motion is described in the relative referential $\acute{\Re}$ ($\acute{O}$,$\acute{X}$, $\acute{Y}$, $\acute{Z}$). The base of the relative referential is ($\vec{\acute{\imath}}, \vec{\acute{\jmath}}, \vec{\acute{k}}$). These vectors are fixe in $\acute{\Re}$, but they move with time in R:

- The relative position vector: $\overrightarrow{\acute{O}M} = \acute{x}\,\vec{\acute{\imath}} + \acute{y}\,\vec{\acute{\jmath}} + \acute{z}\,\vec{\acute{k}}$

- The relative velocity vector: $\vec{V_r} = \overrightarrow{V_{M/R'}} = \left.\frac{d\overrightarrow{O'M}}{dt}\right|_{R'} = \dot{x}\,\vec{i} + \dot{y}\,\vec{j} + \dot{z}\,\vec{k}$
- The relative acceleration vector: $\vec{a_r} = \overrightarrow{a_{M/R'}} = \left.\frac{d\vec{V_r}}{dt}\right|_{R'} = \left.\frac{d^2\overrightarrow{O'M}}{dt^2}\right|_{R'} = \ddot{x}\,\vec{i} + \ddot{y}\,\vec{j} + \ddot{z}\,\vec{k}$

III- Basic equations

$$\overrightarrow{OM} = \overrightarrow{OO'} + \overrightarrow{O'M}$$



III.1- Velocity vector

$$\vec{V_a} = \overrightarrow{V_{M/R}} = \left.\frac{d\overrightarrow{OM}}{dt}\right|_R = \left.\frac{d}{dt}(\overrightarrow{OO'} + \overrightarrow{O'M})\right|_R$$

$$\overrightarrow{O'M} = x'\,\vec{i} + y'\,\vec{j} + z'\,\vec{k}$$

$$\vec{V_a} = \left.\frac{d\overrightarrow{OO'}}{dt}\right|_R + \left.\frac{d}{dt}(x'\,\vec{i} + y'\,\vec{j} + z'\,\vec{k})\right|_R$$

$$\vec{V_a} = \left.\frac{d\overrightarrow{OO'}}{dt}\right|_R + \frac{dx'}{dt}\vec{i} + x'\frac{d\vec{i}}{dt} + \frac{dy'}{dt}\vec{j} + y'\frac{d\vec{j}}{dt} + \frac{dz'}{dt}\vec{i} + z'\frac{d\vec{k}}{dt}$$

$$\vec{V_a} = \left(\left.\frac{d\overrightarrow{OO'}}{dt}\right|_R + \left(x'\frac{d\vec{i}}{dt} + y'\frac{d\vec{j}}{dt} + z'\frac{d\vec{k}}{dt}\right)\right) + \left(\frac{dx'}{dt}\vec{i} + \frac{dy'}{dt}\vec{j} + \frac{dz'}{dt}\vec{k}\right)$$

$$\vec{V_e} = \left.\frac{d\overrightarrow{OO'}}{dt}\right|_R + \left(x'\frac{d\vec{i}}{dt} + y'\frac{d\vec{j}}{dt} + z'\frac{d\vec{k}}{dt}\right)$$

$$\vec{V_r} = \frac{dx'}{dt}\vec{i} + \frac{dy'}{dt}\vec{j} + \frac{dz'}{dt}\vec{k} = \dot{x}\,\vec{i} + \dot{y}\,\vec{j} + \dot{z}\,\vec{k}$$

$$\vec{V_a} = \vec{V_e} + \vec{V_r}$$

$\vec{V_e}$: Entrainment velocity, It's the velocity of the moving referential R' relative to the fixed referential R.

III.2- Acceleration vector

$$\overrightarrow{V_a} = \frac{d\overrightarrow{OO'}}{dt}\bigg|_R + \left(\acute{x}\frac{d\vec{i}}{dt} + \acute{y}\frac{d\vec{j}}{dt} + \acute{z}\frac{d\vec{k}}{dt}\right) + \left(\dot{x}\,\vec{i} + \dot{y}\,\vec{j} + \dot{z}\,\vec{k}\right)$$

$$\overrightarrow{a_a} = \overrightarrow{a_{M/R}} = \frac{d^2\overrightarrow{OM}}{dt^2}\bigg|_R = \frac{d\vec{V}}{dt}\bigg|_R$$

$$\overrightarrow{a_a} = \frac{d^2\overrightarrow{OO'}}{dt^2}\bigg|_R + \left(\dot{x}\frac{d\vec{i}}{dt} + \acute{x}\frac{d^2\vec{i}}{dt^2} + \dot{y}\frac{d\vec{j}}{dt} + \acute{y}\frac{d^2\vec{j}}{dt^2} + \dot{z}\frac{d\vec{k}}{dt} + \acute{z}\frac{d^2\vec{k}}{dt^2}\right) +$$

$$\left(\ddot{x}\,\vec{i} + \dot{x}\frac{d\vec{i}}{dt} + \ddot{y}\,\vec{j} + \dot{y}\frac{d\vec{j}}{dt} + \ddot{z}\,\vec{k} + \dot{z}\frac{d\vec{k}}{dt}\right)$$

$$\overrightarrow{a_a} = \frac{d^2\overrightarrow{OO'}}{dt^2}\bigg|_R + \left(\acute{x}\frac{d^2\vec{i}}{dt^2} + \acute{y}\frac{d^2\vec{j}}{dt^2} + \acute{z}\frac{d^2\vec{k}}{dt^2}\right) + 2\left(\dot{x}\frac{d\vec{i}}{dt} + \dot{y}\frac{d\vec{j}}{dt} + \dot{z}\frac{d\vec{k}}{dt}\right) + \left(\ddot{x}\,\vec{i} + \ddot{y}\,\vec{j} + +\ddot{z}\,\vec{k}\right)$$

$$\overrightarrow{a_e} = \frac{d^2\overrightarrow{OO'}}{dt^2}\bigg|_R + \left(\acute{x}\frac{d^2\vec{i}}{dt^2} + \acute{y}\frac{d^2\vec{j}}{dt^2} + \acute{z}\frac{d^2\vec{k}}{dt^2}\right)$$

$$\overrightarrow{a_c} = 2\left(\dot{x}\frac{d\vec{i}}{dt} + \dot{y}\frac{d\vec{j}}{dt} + \dot{z}\frac{d\vec{k}}{dt}\right)$$

$$\overrightarrow{a_r} = \ddot{x}\,\vec{i} + \ddot{y}\,\vec{j} + +\ddot{z}\,\vec{k}$$

$$\overrightarrow{a_a} = \overrightarrow{a_e} + \overrightarrow{a_c} + \overrightarrow{a_r}$$

$\overrightarrow{a_e}$: Entrainment acceleration.

$\overrightarrow{a_c}$: Coriolis acceleration or additional acceleration.

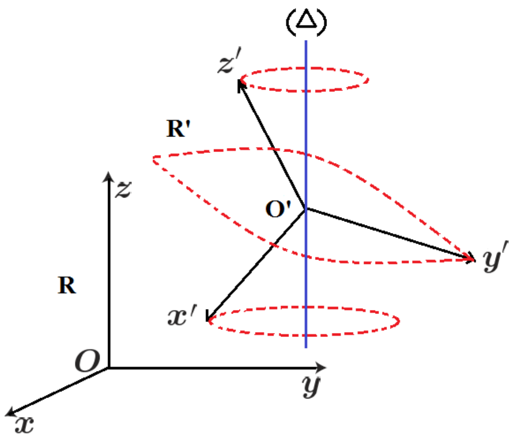
$\overrightarrow{a_r}$: Relative acceleration.

The Coriolis acceleration is the result of the rotation of the Earth on itself.

IV- Special cases of motion of R' relative to R

IV.1- Translation and rotation motion

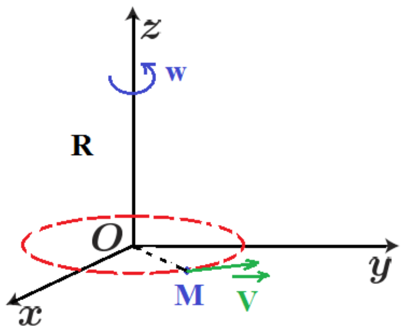
$R|\acute{R}$: Rotation & translation



IV.1.1- Velocity vector

R' rotates around a fixed axis (Δ) and the distance between o and o' is not fixed.

In the previous chapter, we showed that if an object M is rotating about (OZ):



$$\vec{V} = \frac{d\overrightarrow{OM}}{dt} = \vec{\omega} \wedge \overrightarrow{OM}$$

So, we can write: $\frac{d\vec{i}}{dt} = \vec{\omega} \wedge \vec{i}$, $\frac{d\vec{j}}{dt} = \vec{\omega} \wedge \vec{j}$, $\frac{d\vec{k}}{dt} = \vec{\omega} \wedge \vec{k}$

$$\vec{V_e} = \left.\frac{d\overrightarrow{O\acute{O}}}{dt}\right|_R + (\acute{x}\frac{d\vec{i}}{dt} + \acute{y}\frac{d\vec{j}}{dt} + \acute{z}\frac{d\vec{k}}{dt})$$

$$\vec{V_e} = \left.\frac{d\overrightarrow{O\acute{O}}}{dt}\right|_R + (\acute{x}\vec{\omega} \wedge \vec{i} + \acute{y}\vec{\omega} \wedge \vec{j} + \acute{z}\vec{\omega} \wedge \vec{k})$$

$$\vec{V_e} = \left.\frac{d\overrightarrow{O\acute{O}}}{dt}\right|_R + (\vec{\omega} \wedge \acute{x}\vec{i} + \vec{\omega} \wedge \acute{y}\,\vec{j} + \vec{\omega} \wedge \acute{z}\vec{k})$$

$$\vec{V_e} = \left.\frac{d\overrightarrow{O\acute{O}}}{dt}\right|_R + \vec{\omega} \wedge (\acute{x}\vec{i} + \acute{y}\,\vec{j} + \acute{z}\vec{k})$$

$$\vec{V_e} = \left.\frac{d\overrightarrow{O\acute{O}}}{dt}\right|_R + \vec{\omega} \wedge \overrightarrow{\acute{O}M}$$

$$\vec{V_a} = \vec{V_r} + \left.\frac{d\overrightarrow{O\acute{O}}}{dt}\right|_R + \vec{\omega} \wedge \overrightarrow{\acute{O}M}$$

IV.1.2- Acceleration vector

$$\vec{a_e} = \left.\frac{d^2\overrightarrow{O\acute{O}}}{dt^2}\right|_R + (\acute{x}\frac{d^2\vec{i}}{dt^2} + \acute{y}\frac{d^2\vec{j}}{dt^2} + \acute{z}\frac{d^2\vec{k}}{dt^2})$$

$$\vec{a_e} = \left.\frac{d^2\overrightarrow{O\acute{O}}}{dt^2}\right|_R + (\acute{x}\frac{d}{dt}\frac{d\vec{i}}{dt} + \acute{y}\frac{d}{dt}\frac{d\vec{j}}{dt} + \acute{z}\frac{d}{dt}\frac{d\vec{k}}{dt})$$

$$\vec{a_e} = \left.\frac{d^2\overrightarrow{O\acute{O}}}{dt^2}\right|_R + \acute{x}\,\frac{d}{dt}(\vec{\omega}\wedge\vec{i}) + \acute{y}\,\frac{d}{dt}(\vec{\omega}\wedge\vec{j}) + \acute{z}\,\frac{d}{dt}(\vec{\omega}\wedge\vec{k})$$

$$\vec{a_e} = \left.\frac{d^2\overrightarrow{O\acute{O}}}{dt^2}\right|_R + \acute{x}\left(\frac{d\vec{\omega}}{dt}\wedge\vec{i} + \vec{\omega}\wedge\frac{d\vec{i}}{dt}\right) + \acute{y}\left(\frac{d\vec{\omega}}{dt}\wedge\vec{j} + \vec{\omega}\wedge\frac{d\vec{j}}{dt}\right) + \acute{z}\left(\frac{d\vec{\omega}}{dt}\wedge\vec{k} + \vec{\omega}\wedge\frac{d\vec{k}}{dt}\right)$$

$$\vec{a_e} = \left.\frac{d^2\overrightarrow{O\acute{O}}}{dt^2}\right|_R + \acute{x}\left[\frac{d\vec{\omega}}{dt}\wedge\vec{i} + \vec{\omega}\wedge\left(\vec{\omega}\wedge\vec{i}\right)\right] + \acute{y}\left[\frac{d\vec{\omega}}{dt}\wedge\vec{j} + \vec{\omega}\wedge\left(\vec{\omega}\wedge\vec{j}\right)\right]$$

$$+\acute{z}\left[\frac{d\vec{\omega}}{dt}\wedge\vec{k} + \vec{\omega}\wedge\left(\vec{\omega}\wedge\vec{k}\right)\right]$$

$$\vec{a_e} = \left.\frac{d^2\overrightarrow{O\acute{O}}}{dt^2}\right|_R + \left[\frac{d\vec{\omega}}{dt}\wedge\acute{x}\vec{i} + \vec{\omega}\wedge\left(\vec{\omega}\wedge\acute{x}\,\vec{i}\right)\right] + \left[\frac{d\vec{\omega}}{dt}\wedge\acute{y}\,\vec{j} + \vec{\omega}\wedge\left(\vec{\omega}\wedge\acute{y}\,\vec{j}\right)\right]$$

$$+\left[\frac{d\vec{\omega}}{dt}\wedge\acute{z}\,\vec{k} + \vec{\omega}\wedge\left(\overrightarrow{\omega}\wedge\acute{z}\,\vec{k}\right)\right]$$

$$\vec{a_e} = \left.\frac{d^2\overrightarrow{O\acute{O}}}{dt^2}\right|_R + \left[\frac{d\vec{\omega}}{dt}\wedge\acute{x}\vec{i} + \frac{d\vec{\omega}}{dt}\wedge\acute{y}\,\vec{j} + \frac{d\vec{\omega}}{dt}\wedge\acute{z}\,\vec{k}\right]$$

$$+\left[\vec{\omega}\wedge\left(\vec{\omega}\wedge\acute{x}\,\vec{i}\right) + \vec{\omega}\wedge\left(\vec{\omega}\wedge\acute{y}\,\vec{j}\right) + \vec{\omega}\wedge\left(\overrightarrow{\omega}\wedge\acute{z}\,\vec{k}\right)\right]$$

$$\vec{a_e} = \left.\frac{d^2\overrightarrow{O\acute{O}}}{dt^2}\right|_R + \frac{d\vec{\omega}}{dt}\wedge\left[\acute{x}\vec{i} + \acute{y}\,\vec{j} + \acute{z}\,\vec{k}\right] + \vec{\omega}\wedge\left[\left(\vec{\omega}\wedge\acute{x}\,\vec{i}\right) + \left(\vec{\omega}\wedge\acute{y}\,\vec{j}\right) + \left(\overrightarrow{\omega}\wedge\acute{z}\,\vec{k}\right)\right]$$

$$\vec{a_e} = \left.\frac{d^2\overrightarrow{O\acute{O}}}{dt^2}\right|_R + \frac{d\vec{\omega}}{dt}\wedge\left[\acute{x}\vec{i} + \acute{y}\,\vec{j} + \acute{z}\,\vec{k}\right] + \vec{\omega}\wedge\left[\vec{\omega}\wedge\left(\acute{x}\,\vec{i} + \acute{y}\,\vec{j} + \acute{z}\,\vec{k}\right)\right]$$

$$\vec{a_e} = \left.\frac{d^2\overrightarrow{O\acute{O}}}{dt^2}\right|_R + \frac{d\vec{\omega}}{dt}\wedge\left[\overrightarrow{\acute{O}M}\right] + \vec{\omega}\wedge\left[\vec{\omega}\wedge\overrightarrow{\acute{O}M}\right]$$

$$\vec{a_e} = \left.\frac{d^2\overrightarrow{O\acute{O}}}{dt^2}\right|_R + \dot{\vec{\omega}}\wedge\overrightarrow{\acute{O}M} + \vec{\omega}\wedge\left(\vec{\omega}\wedge\overrightarrow{\acute{O}M}\right)$$

$$\vec{a_c} = 2\left(\dot{x}\frac{d\vec{i}}{dt} + \dot{y}\frac{d\vec{j}}{dt} + \dot{z}\frac{d\vec{k}}{dt}\right)$$

$$\vec{a_c} = 2\left(\dot{x}\,(\vec{\omega}\wedge\vec{i}) + \dot{y}(\vec{\omega}\wedge\vec{j}) + \dot{z}\,(\vec{\omega}\wedge\vec{k})\right)$$

$$\vec{a_c} = 2\left((\vec{\omega}\wedge\dot{x}\,\vec{i}) + (\overrightarrow{\omega}\wedge\dot{y}\,\vec{j}) + (\vec{\omega}\wedge\dot{z}\,\vec{k})\right)$$

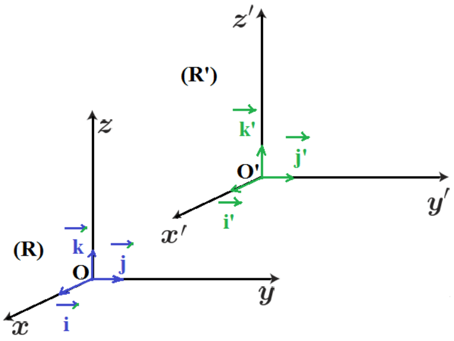
$$\vec{a_c} = 2\left(\vec{\omega}\wedge(\dot{x}\,\vec{i} + \dot{y}\,\vec{j} + \dot{z}\,\vec{k})\right)$$

$$\vec{a_c} = 2\left(\vec{\omega}\wedge\vec{V_r}\right)$$

$$\vec{a_a} = \vec{a_r} + \left.\frac{d^2\overrightarrow{O\acute{O}}}{dt^2}\right|_R + \dot{\vec{\omega}}\wedge\overrightarrow{\acute{O}M} + \vec{\omega}\wedge\left(\vec{\omega}\wedge\overrightarrow{\acute{O}M}\right) + 2\left(\vec{\omega}\wedge\vec{V_r}\right)$$

IV.2- Translation motion

IV.2.1- Velocity vector



R' is in translation with respect to R, the directions related to R' (($\vec{i}, \vec{j}, \vec{k}$)) are fixed in R ($\vec{\omega} = \vec{0}$).

$\frac{d\vec{i}}{dt} = \frac{d\vec{j}}{dt} = \frac{d\vec{k}}{dt} = \vec{0}$, donc:

$$\vec{V_e} = \left.\frac{d\overrightarrow{OO'}}{dt}\right|_R + \left(\acute{x}\frac{d\vec{i}}{dt} + \acute{y}\frac{d\vec{j}}{dt} + \acute{z}\frac{d\vec{k}}{dt}\right)$$

$$\vec{V_e} = \left.\frac{d\overrightarrow{OO'}}{dt}\right|_R$$

$$\vec{V_a} = \vec{V_r} + \left.\frac{d\overrightarrow{OO'}}{dt}\right|_R$$

IV.2.2- Acceleration vector

$$\vec{a_e} = \left.\frac{d^2\overrightarrow{OO'}}{dt^2}\right|_R + \left(\acute{x}\frac{d^2\vec{i}}{dt^2} + \acute{y}\frac{d^2\vec{j}}{dt^2} + \acute{z}\frac{d^2\vec{k}}{dt^2}\right)$$

$$\vec{a_e} = \left.\frac{d^2\overrightarrow{OO'}}{dt^2}\right|_R$$

$$\vec{a_c} = 2\left(\dot{x}\frac{d\vec{i}}{dt} + \dot{y}\frac{d\vec{j}}{dt} + \dot{z}\frac{d\vec{k}}{dt}\right)$$

$$\vec{a_c} = \vec{0}$$

$$\vec{a_a} = \vec{a_r} + \left.\frac{d^2\overrightarrow{OO'}}{dt^2}\right|_R$$

Remark

If the motion of $\grave{R}/_R$ is a uniform rectilinear motion:

$$\left.\frac{d\overrightarrow{OO'}}{dt}\right|_R = \overrightarrow{V_0} \quad (V_0 = C^{ste})$$

$$\overrightarrow{V_e} = \overrightarrow{V_0}$$

$$\overrightarrow{OO'} = \overrightarrow{V_0}\ t$$

$$\overrightarrow{V_a} = \overrightarrow{V_r} + \overrightarrow{V_0}$$

$$\overrightarrow{a_e} = \left.\frac{d^2\overrightarrow{OO'}}{dt^2}\right|_R$$

$$\overrightarrow{a_e} = \left.\frac{d\overrightarrow{V_0}}{dt}\right|_R$$

$$\overrightarrow{a_e} = \vec{0}$$

$$\overrightarrow{a_a} = \overrightarrow{a_r}$$

IV.3- Rotational motion about a fixed axis

IV.3.1- Velocity vector

In this case: $\left.\frac{d\overrightarrow{OO'}}{dt}\right|_R = \vec{0}$

$$\overrightarrow{V_e} = \left.\frac{d\overrightarrow{OO'}}{dt}\right|_R + \vec{\omega} \wedge \overrightarrow{O'M}$$

$$\overrightarrow{V_e} = \vec{\omega} \wedge \overrightarrow{O'M}$$

$$\overrightarrow{V_a} = \overrightarrow{V_r} + \vec{\omega} \wedge \overrightarrow{O'M}$$

$$\overrightarrow{a_e} = \left.\frac{d^2\overrightarrow{OO'}}{dt^2}\right|_R + \dot{\vec{\omega}} \wedge \overrightarrow{O'M} + \vec{\omega} \wedge (\vec{\omega} \wedge \overrightarrow{O'M})$$

$$\vec{a_e} = \dot{\vec{\omega}} \wedge \overrightarrow{O'M} + \vec{\omega} \wedge (\vec{\omega} \wedge \overrightarrow{O'M})$$

$$\vec{a_c} = 2(\vec{\omega} \wedge \vec{V_r})$$

$$\vec{a_a} = \vec{a_r} + \dot{\vec{\omega}} \wedge \overrightarrow{O'M} + \vec{\omega} \wedge \left(\vec{\omega} \wedge \overrightarrow{O'M}\right) + 2(\vec{\omega} \wedge \vec{V_r})$$

Remark

If the angular velocity is constant (uniform rotation):

$$\|\vec{\omega}\| = o$$

$$\vec{a_e} = \dot{\vec{\omega}} \wedge \overrightarrow{O'M} + \vec{\omega} \wedge (\vec{\omega} \wedge \overrightarrow{O'M})$$

$$\vec{a_e} = \vec{\omega} \wedge (\vec{\omega} \wedge \overrightarrow{O'M})$$

$$\vec{a_a} = \vec{a_r} + \vec{\omega} \wedge \left(\vec{\omega} \wedge \overrightarrow{O'M}\right) + 2(\vec{\omega} \wedge \vec{V_r})$$

Exercise

Consider two cars A and B moving on two lines at speeds of 70 km/h and 90 km/h respectively.

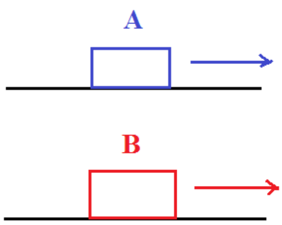
Calculate the velocity of B relative to A when the two cars are moving:

- On two parallel lines in the same direction and in the opposite direction.

- On two lines forming an angle of 60° (in the opposite direction).

Solution

- A and B are on two parallel lines in the same direction:



$$\overrightarrow{V_{B/R}} = 90\,\vec{\imath} \quad , \quad \overrightarrow{V_{A/R}} = 70\vec{\imath}$$

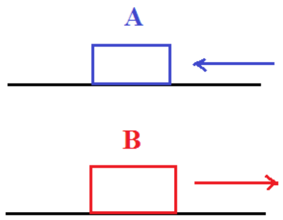
$$\vec{V_a} = \vec{V_r} + \vec{V_e}$$

$$\overrightarrow{V_{B/R}} = \overrightarrow{V_{B/A}} + \overrightarrow{V_{A/R}}$$

$$\overrightarrow{V_{B/A}} = \overrightarrow{V_{B/R}} - \overrightarrow{V_{A/R}} = 90\,\vec{i} - 70\vec{i}$$

$$\overrightarrow{V_{B/A}} = 20\,\vec{i}$$

- A and B are on two parallel lines in the opposite direction:

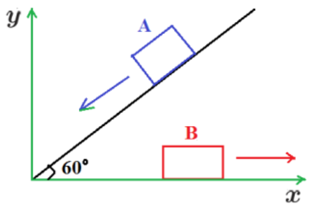


$$\overrightarrow{V_{B/R}} = 90\,\vec{i} \quad , \quad \overrightarrow{V_{A/R}} = -70\vec{i}$$

$$\overrightarrow{V_{B/A}} = \overrightarrow{V_{B/R}} - \overrightarrow{V_{A/R}} = 90\,\vec{i} + 70\vec{i}$$

$$\overrightarrow{V_{B/A}} = 160\,\vec{i}$$

- On two lines forming an angle of 60° (in the opposite direction).



$$\overrightarrow{V_{B/R}} = 90\,\vec{i} \quad , \quad \overrightarrow{V_{A/R}} = -70\cos 60\,\vec{i} - 70\sin 60\,\vec{j}$$

$$\overrightarrow{V_{A/R}} = -70\,\frac{1}{2}\vec{i} - 70\,\frac{\sqrt{3}}{2}\,\vec{j}$$

$$\overrightarrow{V_{A/R}} = -35\,\vec{i} - 35\sqrt{3}\,\vec{j}$$

$$\overrightarrow{V_{B/A}} = \overrightarrow{V_{B/R}} - \overrightarrow{V_{A/R}}$$

$$\overrightarrow{V_{B/A}} = 90\,\vec{i} - (-35\,\vec{i} - 35\sqrt{3}\,\vec{j})$$

$$\overrightarrow{V_{B/A}} = 125\,\vec{i} + 35\sqrt{3}\,\vec{j}$$

$$V_{B/A} = 138.9\ km/h$$