# Chapter 2: Statics (Equilibrium of rigid body)

The basic of statics deals with the necessary and sufficient conditions for the external forces acting on a structure to maintain its equilibrium. Physical bodies are typically three-dimensional, but some can be considered two-dimensional if external forces are in a single plane or can be projected onto a single plane. In the following, only the equilibrium of bodies subjected to two-dimensional force systems is taken into account.
As shown in Fig. 2.1, the movement of a rigid body differs from that of a particle in that we also have to consider the rotation of the body.

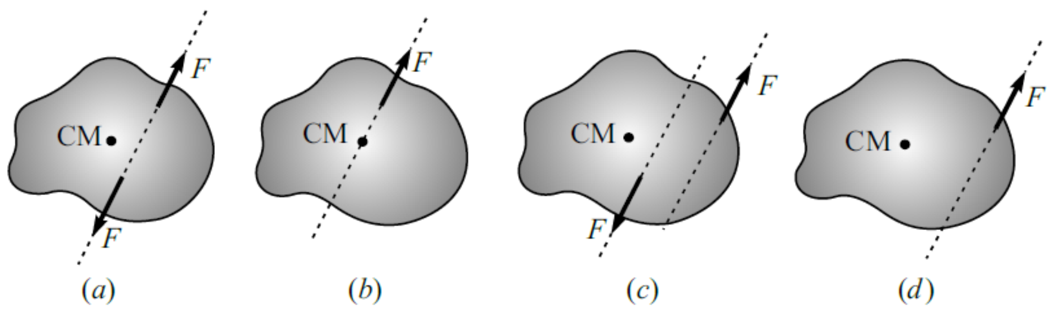



**Fig.2.1:** Equilibrium or motion of a rigid body subject to forces: (a) equilibrium, (b) translation, (c) rotation about centre of mass (CM) and (d) rotation and translation.

Hence, the rigid body is said to be in equilibrium when resultant force and couple moment are both equal to zero. Mathematically, the equilibrium of a body is expressed as:

$$\sum F_x = 0, \qquad \sum F_y = 0, \qquad \sum M_o = 0 \tag{2.1}$$

## 2.1. Axioms of statics

The axioms (concept) of statics apply to the equilibrium of rigid bodies.
**Axiom N°1:** A free rigid body subjected to the action of two forces can be in equilibrium if, and only, if the two forces are equal in magnitude, collinear, and opposite in direction.

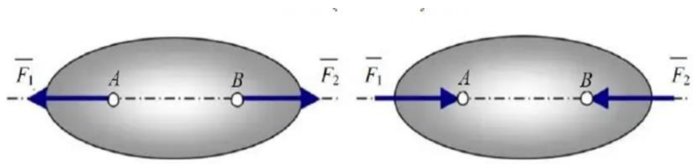



Fig. 2.2.

**Axiom N°2:** The action of given force system on a rigid body remains unchanged if another balanced force system is added to, or subtracted from, the original system. This indicates that the point of application of a force, acting on a rigid body, can be transferred to any other point on the line of action of the force without altering its.

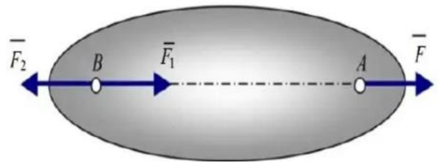



Fig.2.3.

**Axiom N°3:** The resultant of two forces acting at the same material point is equal to the vector sum of the two forces. The line of the resulting force's action contains the material point. This axiom obeys the principle of vector summation (parallelogram law).

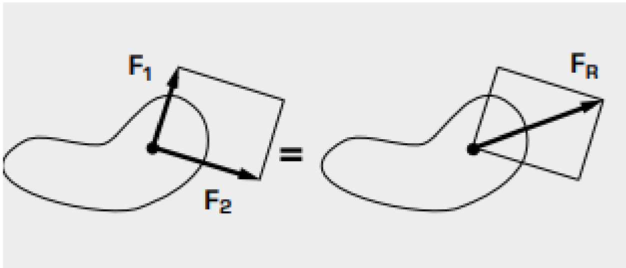



Fig.2.4.

**Axiom N°4:** If forces arise in pairs when two rigid bodies act upon each other, the pair of forces have the same line of action, the same magnitude and opposite sign. This axiom is also known as the principle of action and reaction.

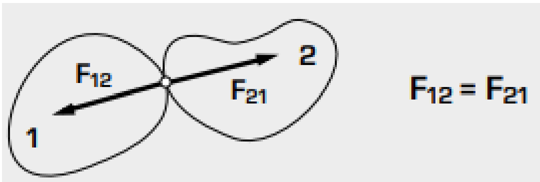



Fig.2.5.

**Axiom N°5:** If a deformable body subjected to the action of forces is in static equilibrium, the state of equilibrium will not be disturbed if the body solidifies (becomes rigid). This axiom is also known as the principle of solidification.

**2.2. Type of forces acting on a body**

Forces that act on a body can be divided into three general categories:

**Applied forces:** These are the forces of push or pull applied externally to the body. Each force has a point of contact with the body.

**Sel-weight:** The weight (W = mg) of the body acts vertically downward. It can be assumed to be acting through the centre of gravity of the body. The self-weight can be negligible if its magnitude is very small in comparison with the other forces acting on the body.

**Reactive forces (**or, simply, reactions)**:** Reactions are those forces that are exerted on a body by the supports to which it is attached.

An example of the three types of forces is illustrated in Fig. 2.6.

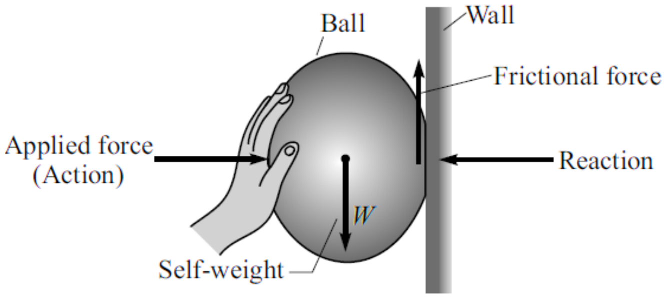



**Fig. 2.6:** A ball being pressed against a wall.

## 2.3. Free body diagram

The first step in equilibrium analysis is to identify all the forces that act on the body. This is accomplished by means of a free-body diagram (FBD).

The FBD of a body is a sketch of the body showing all forces that act on it. The term free implies that all supports have been removed and replaced by the forces (reactions) that they exert on the body.

The following is the general procedure for constructing a free-body diagram:

1. A sketch of the body is drawn assuming that all supports (surfaces of contact, supporting cables, etc.) have been removed.
2. All applied forces are drawn and labelled on the sketch. The weight of the body is considered to be an applied force acting at the center of gravity.
3. The support reactions are drawn and labelled on the sketch. If the sense of a reaction is unknown, it should be assumed. The solution will determine the correct sense: A positive result indicates that the assumed sense is correct, whereas a negative result means that the correct sense is opposite to the assumed sense.
4. All relevant angles and dimensions are shown on the sketch.

Ones this process is completed, a drawing containing all of the information required to write the equilibrium equations for the body is obtained. Some examples of FBD for frequent cases are shown in Fig. 2.7.

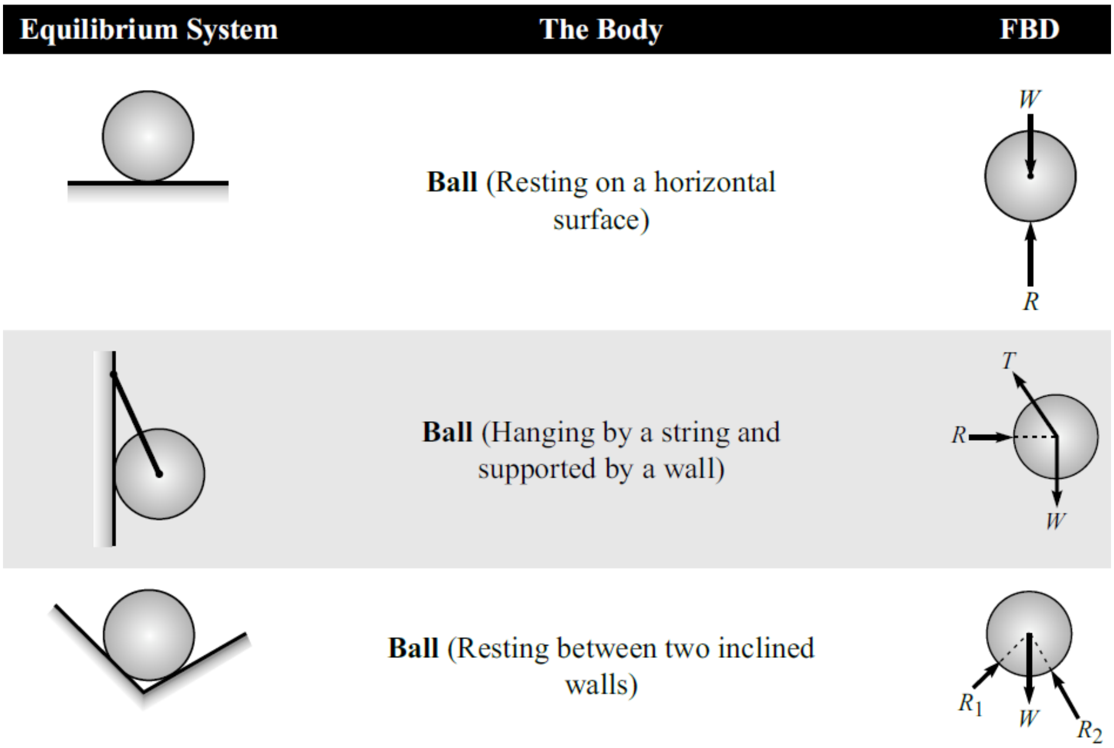



**Fig. 2.7:** FBD of some equilibrium system.

The determination of the support reactions is the most challenging step in the construction of FBDs. Table 2.1 shows the reactions exerted by various coplanar supports; it also lists the number of unknowns that are introduced on an FBD by the removal of each support.

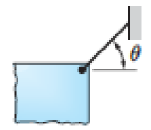
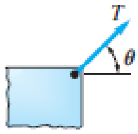
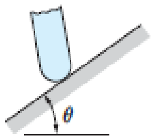
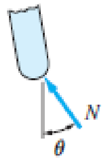
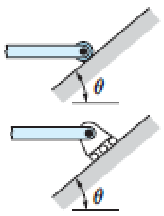
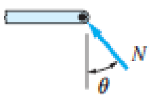
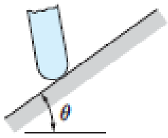
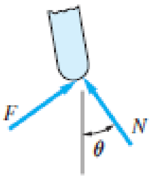
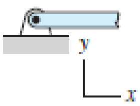
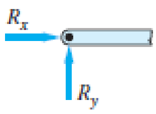
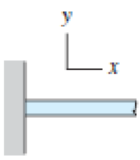
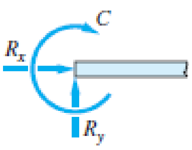
(a) **Flexible cable (negligible Weight)**: A flexible cable exerts a pull, or tensile force, in the direction of the cable. With the weight of the cable neglected, the cable forms a straight line. If its direction is known, removal of the cable introduces one unknown in a free-body diagram—the magnitude of the force exerted by the cable.

(b) **Frictionless surface (single point of contact)**: When a body is in contact with a frictionless surface at only one point, the reaction is a force that is perpendicular to the surface, acting at the point of contact. This reaction is often referred to simply as the normal force. Therefore, removing such a surface introduces one unknown in a free-body diagram—the magnitude of the normal force.

(c) **Roller Support**: A roller support is equivalent to a frictionless surface: It can only exert a force that is perpendicular to the supporting surface. The magnitude of the force is thus the only unknown introduced in a free-body diagram when the support is removed.

(d) **Surface with Friction (single point of contact):** A friction surface can exert a force that acts at an angle to the surface. The unknowns may be taken to be the magnitude and direction of the force. However, it is usually advantageous to represent the unknowns as N and F, the components that are perpendicular and parallel to the surface, respectively. The component N is called the normal force, and F is known as the friction force.

**Table 2.1.** Reactions of Coplanar Supports

<table>
<tr><th>Support</th><th>Reaction(s)</th><th>Description of reaction(s)</th><th>Number of unknowns</th></tr>
<tr><td>(a)<br><br>Flexible cable of negligible weight</td><td></td><td>Tension of unknown magnitude T in the direction of the cable</td><td>One</td></tr>
<tr><td>(b)<br><br>Frictionless surface (single point of contact)</td><td></td><td>Force of unknown magnitude N directed normal to the surface</td><td>One</td></tr>
<tr><td>(c)<br><br>Roller support</td><td></td><td>Force of unknown magnitude N normal to the surface supporting the roller</td><td>One</td></tr>
<tr><td>(d)<br><br>Surface with friction (single point of contact)</td><td></td><td>Force of unknown magnitude N normal to the surface and a friction force of unknown magnitude F parallel to the surface</td><td>Two</td></tr>
<tr><td>(e)<br><br>Pin support</td><td></td><td>Unknown force R</td><td>Two</td></tr>
<tr><td>(f)<br><br>Built-in (cantilever) support</td><td></td><td>Unknown force R and a couple of unknown magnitude C</td><td>Three</td></tr>
</table>

(e) **Pin Support:** A pin is a cylinder that is slightly smaller than the hole into which it is inserted. Neglecting friction, the pin can only exert a force R that is normal to the contact surface. A pin support introduces two unknowns chosen to be perpendicular components of R, such as Rx and Ry.

(f) **Built-in support:** A built-in support, also known as a cantilever support, prevents all motion of the body at the support. Translation (horizontal or vertical movement) is prevented by a force, and a couple prohibits rotation. Therefore, a built-in support introduces three unknowns in a free-body diagram: the magnitude and direction of the reactive force R (these unknowns are commonly chosen to be two components of R, such as Rx and Ry) and the magnitude C of the reactive couple.

**Example 1:** The homogeneous 6-m bar AB in Fig. 2.8 is supported in the vertical plane by rollers at A and B and by a cable at C. The mass of the bar is 50 kg. Draw the FBD of bar AB. Determine the number of unknowns on the FBD.

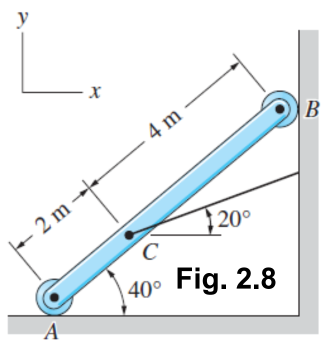



Fig. 2.8

**Example 2:** Two smooth pipes, each having a mass of 300 kg, are supported by the forked tines of the tractor, see Fig. 2.9. Draw the free-body diagrams for each pipe and both pipes together.

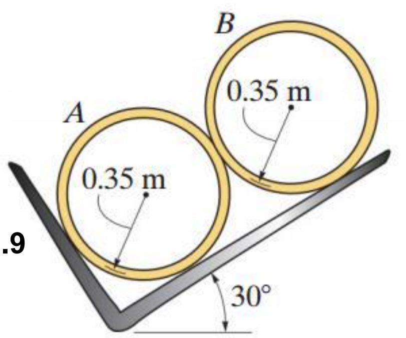



Fig. 2.9

The three steps in the equilibrium analysis of a body are:

Step 1: Draw a free-body diagram (FBD) of the body that shows all of the forces and couples that act on the body.

Step 2: Write the equilibrium equations in terms of the forces and couples that appear on the free-body diagram.

Step 3: Solve the equilibrium equations for the unknowns.

**Example 1:** Determine the force F required to keep the 200-kg crate in equilibrium position as shown in Fig. 2.10.

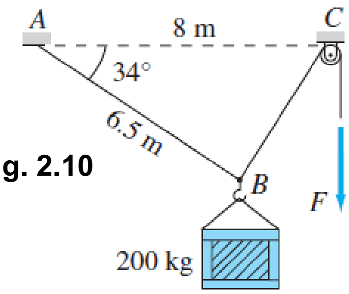



Fig. 2.10

**Example 2:** A beam having a pin connection at its end A is supported by a roller at its end B to keep it horizontal. It is loaded as shown in Fig. 2.11. Determine the reactions on the beam, neglecting its thickness and mass.

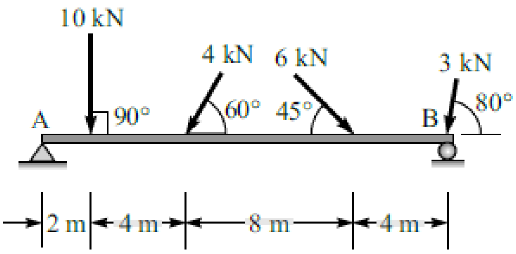



Fig. 2.11

**Example 3:** Two cylinders A and B of diameters 6 cm and 3 cm and weighing 80 kN and 20 kN, respectively are placed as shown in Fig 4.154. Assuming all the contact surfaces to be smooth, find the reactions at the walls.

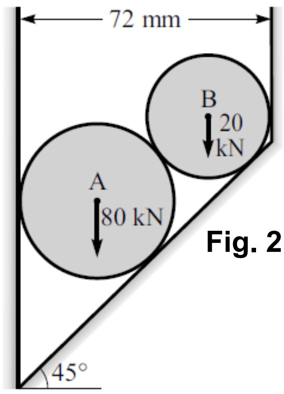



Fig. 2.12

## 2.4. Friction

### 2.4.1. Introduction

In most of the equilibrium problems that we have analyzed up to this point, the surfaces of contact have been frictionless. The reactive forces were, therefore, normal to the contact surfaces. The concept of a frictionless surface is, of course, an idealization. All real surfaces also provide a force component that is tangent to the surface, called the friction force, that resists sliding.

Friction is undesirable in most engineering systems because it causes a loss of energy. But it is essential for common activities like walking, driving, maintaining equilibrium, etc. It is a common experience that one cannot walk on a smooth oily surface without slipping.

Dry friction refers to the friction force that exists between two unlubricated solid surfaces. Fluid friction acts between moving surfaces that are separated by a layer of fluid. The friction in a lubricated journal bearing is classified as fluid friction, because the two halves of the bearing are not in direct contact but are separated by a thin layer of liquid lubricant. In this chapter, we consider only dry friction.

### 2.4.2. Classification of friction

Friction can be divided into two categories:

1. **Dry friction:** refers to the friction force that exists between two unlubricated solid surfaces. It is also called ***Coulomb friction*** since its characteristics were studied extensively by the French physicist Charles-Augustin de Coulomb in 1781. It is further subdivided into two categories:

(i) Static friction: it is the friction between two stationary surfaces which have a tendency of relative motion due to external forces.

(ii) Kinetic Friction (Dynamic Friction): It is the friction between two moving surfaces in contact. It is of two types **sliding friction** experienced when a body slides over another body or surface, and **rolling friction** experienced when a body rolls on another body or surface.

2. **Fluid friction**, also called lubricated friction, resists the relative lateral motion of two solid surfaces separated by a layer of gas or liquid. Fluid friction depends on the relative velocity of fluid layers and the viscosity of fluids in contact.

In this chapter, only dry friction is considered.

Consider a block of weight W resting on a horizontal rough surface. Its weight W acts downward through its centre of gravity and is balanced by the normal reaction N, which acts perpendicular to the plane of contact (Fig 2.13a). There is no other force acting on the block. The friction is zero. The block is in equilibrium.

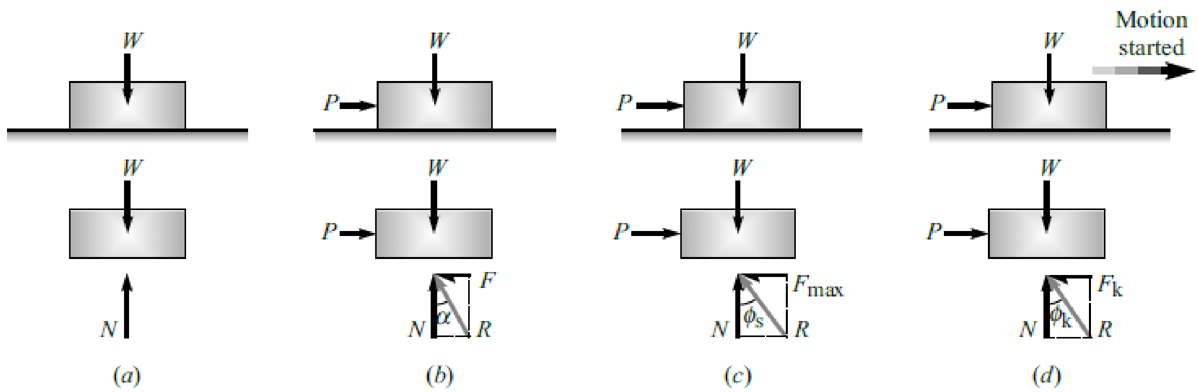



**Fig. 2.13:** Development of frictional force.

Now, if a small force P gently pushes the block, it does not move (Fig 2.13b). It means an opposing force must have developed that balances the applied force. This developed force is known as the force of friction or simply friction that always opposes or tends to oppose the motion. If we push the block a little harder, it still does not move (Fig. 2.13c). The friction works like a demon, which adjusts itself to just balance the applied force. It is our common experience that ultimately the applied force wins and the block starts moving (Fig 2.13d). After all, there is a limit to which frictional force can increase. Beyond this limit, it gives up the fight, but not completely. It becomes a little weaker but continues to oppose the applied force. This self-adjusting nature of the frictional force is its main characteristic, as shown in Fig. 2.14.

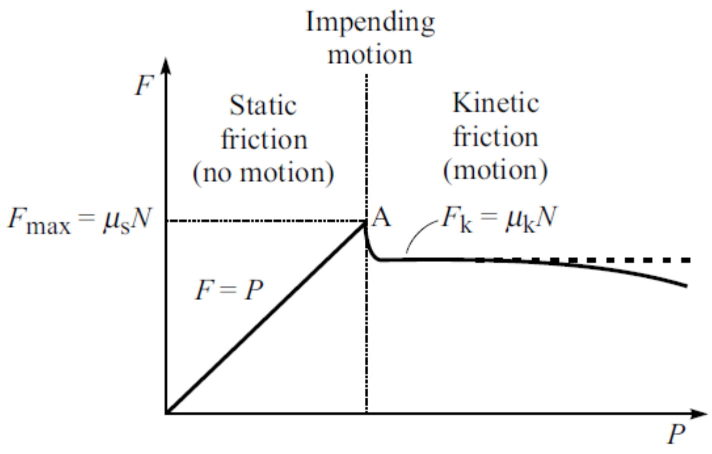



**Fig. 2.14:** Characteristics of frictional force.

As the applied force P is increased, the frictional force F increases at the same rate. At impending motion (point A), the frictional force assumes its limiting or maximum value, $F_{max}$ (corresponding to Fig. 2.13c). After this, the block just starts moving and the frictional force decreases a bit from $F_{max}$ to a value $F_k$ (corresponding to Fig. 2.13d). The net horizontal force ($P-F_k$) moves the body. When the applied force P is increased till point A, the motion of the block becomes imminent. Thus, the point A represents the *impending motion* of the block. If P is further increased, the velocity of the block increases, and the frictional force $F_k$ (under kinetic condition) becomes even weaker.

**Impending Motion:** In cases where the surfaces of contact are rather "slippery," the frictional force F may not be great enough to balance P, and consequently the block will tend to slip.

$$F_{max} = \mu_s N \qquad (2.2)$$

Where Fmax is the limiting static frictional force, $\mu_s$ is the coefficient of static friction, and N the normal force.

The angle $\phi_s$ that R makes with N is called the angle of static friction. It is given by:

$$\tan \phi_s = \frac{F_{max}}{N} \qquad (2.3)$$

**Motion:** If the magnitude of P acting on the block is increased so that it becomes slightly greater than $F_{max}$, the frictional force at the contacting surface will drop to a smaller value $F_k$, called the kinetic frictional force.

$$F_k = \mu_k N \quad (2.4)$$

where $\mu_k$, is called the coefficient of kinetic friction. The angle of kinetic friction is given by:

$$\tan \phi_k = \frac{F_k}{N} \quad (2.5)$$

Table 2.2 gives approximate values of $\mu_s$ and $\mu_k$ for various surfaces.

Table 2.2. Approximate values of coefficients of friction for various surfaces.

| Mating surfaces | $\mu_s$ | $\mu_k$ |
|---|---|---|
| Steel on steel (dry) | 0.74 | 0.57 |
| Steel on steel (greasy) | 0.10 | 0.05 |
| Aluminium on steel | 0.61 | 0.47 |
| Copper on steel | 0.53 | 0.36 |
| Teflon on steel | 0.041 | 0.04 |
| Wire rope on iron pulley (dry) | 0.20 | 0.15 |
| Steel on wet grindstone | – | 0.70 |
| Rubber or leather on wood | 0.40 | 0.30 |
| Rubber on concrete | 1.0 | 0.80 |
| Rubber tyre on dry pavement | 0.90 | 0.80 |
| Asbestos brake lining on cast iron | 0.40 | 0.30 |
| Wood on wood | 0.25-0.50 | 0.20 |
| Waxed wood on wet snow | 0.14 | 0.10 |
| Waxed wood on dry snow | – | 0.04 |
| Glass on glass | 0.94 | 0.40 |
| Ice on ice | 0.10 | 0.03 |
| Teflon on teflon | 0.04 | 0.04 |
| Copper on copper | 1.21 | – |

### 2.4.3. Law of friction

In 1781, Coulomb summed his research on friction and proposed the laws of friction:

1. Friction always acts in a direction opposite to the motion or impending motion.
2. The limiting friction is directly proportional to the normal reaction.
3. Until the motion starts, the static frictional force adjusts itself to just balance the force tending to produce motion.
4. Friction is independent of the area of contact between the two surfaces but depends on the roughness of the surface.

5. Kinetic friction also bears a constant ratio with normal reaction but this ratio is slightly less than that in case of limiting friction.
6. For moderate speeds, friction remains constant. But it decreases slightly for higher speeds.

**Example 1:** The uniform 100 KN ladder in Fig. 2.15 rests against the smooth wall at B, and the end A rests on the rough horizontal plane for which the coefficient of static friction is $\mu_s = 0.3$. Determine the angle of inclination θ of the ladder and the normal reaction at B if the ladder is on the verge of slipping.

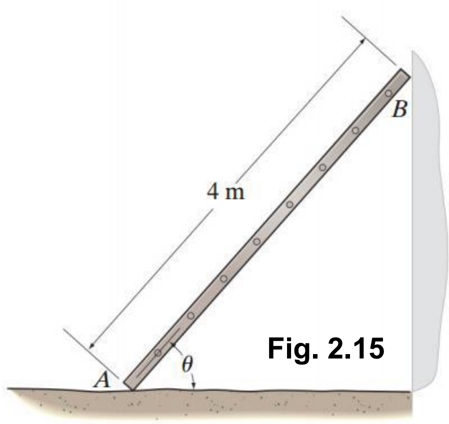


Fig. 2.15

**Example 2:** Determine the largest and smallest values of the force P for which the system in Fig. 2.16 will be in static equilibrium. The homogeneous bars AB and BC are identical, each having a mass of 100 kg. The coefficient of static friction between the bar at C and the horizontal plane is 0.5.

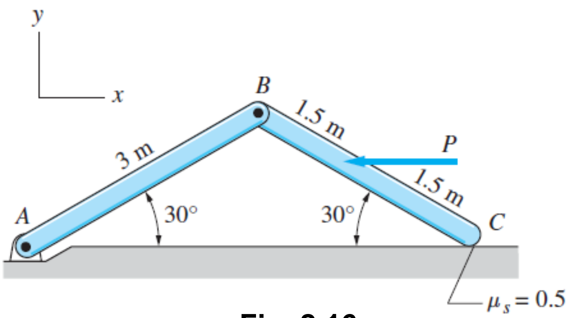


Fig. 2.16