A- Errors, measurements and representation

1- Unit

- All physical quantities are quantified, these quantities are characterized by units that are suitable for their measurements.
- In the international system (MKSA), we have 7 main units, the rest follows from that.

- M $\longrightarrow$ meter (length).
- K $\longrightarrow$ kilogram (mass).
- S $\longrightarrow$ second (time).
- A $\longrightarrow$ Ampère (electrical intensity).
- N $\longrightarrow$ mole (number of particles)
- K $\longrightarrow$ Kelvin (temperature)
- Cd $\longrightarrow$ Candela (luminous Intensity)

2- Scientific notation

When quantifying physical quantities, some of them are very large or too small, for this, notation is used to write them. which is called scientific notation

$$\boldsymbol{v.10^n}\begin{cases} *\ \boldsymbol{v: real\ number} \quad \boldsymbol{1 \le v \le 9} \\ *\ \boldsymbol{n: Integer\ number} \end{cases}$$

Example:

- The earth mass: " 6 followed by 24 zeros " $\longrightarrow$ $\boldsymbol{m = 6\ \ 10^{24} kg}$
- The electron mass: " 9.11 preceded by 30 zero " $\longrightarrow$ $\boldsymbol{m = 9.11\ \ 10^{-31} kg}$

3- Measurement, errors and significant figures

3.1 Measurements

There are two types of measures

a- Direct measurements

This is the operation of reading or sampling directly from the measuring instrument (length, time, current, ...).

b- Indirect measurements

The desired quantity is expressed mathematically as a function of other quantities measured directly (area, volume, density, ...)

3.2 Errors

a- Notions of error and uncertainty

- Error:

Is the difference between the real and measured value of the physical quantity. This difference can be positive or negative.

There are two types of errors:

- *systematic errors*:

Those repeated each time in the same way (error of the instrument, ...)

- *Incidental errors*:

Those that appear each time but in a random or unpredictable way (reading, temperature change, ...)

- Uncertainty: is the maximum absolute value that the error can take.

b- Determination of uncertainty

- If "$\boldsymbol{x}$" is the real value of the physical quantity, while the measured value of the same quantity is "$\boldsymbol{x_0}$", then the error is:

$$\boldsymbol{e} = \boldsymbol{x} - \boldsymbol{x_0}$$

Note: The error may be negative or positive ($\boldsymbol{e} < 0 \ ou \ e > 0$)

- The absolute value of the error is: the absolute error

$$\boldsymbol{\delta x} = |\boldsymbol{e}| = |\boldsymbol{x} - \boldsymbol{x_0}|$$

- The absolute uncertainty is given by:

$$\Delta x = \boldsymbol{max}(\boldsymbol{\delta x})$$

Note: we always have $\Delta \boldsymbol{x} \geq \boldsymbol{\delta x}$

- If the error is positive ($\boldsymbol{e} > 0$) :

  $|\boldsymbol{x} - \boldsymbol{x_0}| = \boldsymbol{x} - \boldsymbol{x_0} \Longrightarrow \Delta \boldsymbol{x} \geq \boldsymbol{\delta x} = \boldsymbol{x} - \boldsymbol{x_0} \Longrightarrow \boldsymbol{x} \leq \boldsymbol{x_0} + \Delta \boldsymbol{x}$
- If the error is negative ($\boldsymbol{e} < 0$) :

  $|\boldsymbol{x} - \boldsymbol{x_0}| = -(\boldsymbol{x} - \boldsymbol{x_0}) \Longrightarrow \Delta \boldsymbol{x} \geq \boldsymbol{\delta x} = \boldsymbol{x_0} - \boldsymbol{x} \Longrightarrow \boldsymbol{x} \geq \boldsymbol{x_0} - \Delta \boldsymbol{x}$
- The real value can finally be written:

$$\boxed{\boldsymbol{x} = \boldsymbol{x_0} \pm \Delta \boldsymbol{x}}$$

- Determination of uncertainty
  - If the quantity is measured directly, the error made is on the smallest digit of the instrument. (Graduated rule in millimeters: the error made is in the mm).
  - If the quantity is given by indirect measurement, the error is expressed as a function of the errors of the quantities measured directly ($x = F(a, b, c..)$)

* <u>Sum</u>:

$$\boldsymbol{x} = \boldsymbol{a} + \boldsymbol{b} + \boldsymbol{c} + \cdots$$

$$\Delta \boldsymbol{x} = \Delta \boldsymbol{a} + \Delta \boldsymbol{b} + \Delta \boldsymbol{c} + \cdots$$

* <u>Product</u>:

$$\boldsymbol{x_0} = \boldsymbol{a}.\boldsymbol{b}.\boldsymbol{c}$$

$$\Delta \boldsymbol{x} = (\boldsymbol{b}.\boldsymbol{c})\Delta \boldsymbol{a} + (\boldsymbol{a}.\boldsymbol{c})\Delta \boldsymbol{b} + (\boldsymbol{a}.\boldsymbol{b})\Delta \boldsymbol{c}$$

And $\frac{\Delta \boldsymbol{x}}{\boldsymbol{x_0}} = \frac{\Delta \boldsymbol{a}}{\boldsymbol{a}} + \frac{\Delta \boldsymbol{b}}{\boldsymbol{b}} + \frac{\Delta \boldsymbol{c}}{\boldsymbol{c}} \Rightarrow \Delta \boldsymbol{x} = \left(\frac{\Delta \boldsymbol{a}}{\boldsymbol{a}} + \frac{\Delta \boldsymbol{b}}{\boldsymbol{b}} + \frac{\Delta \boldsymbol{c}}{\boldsymbol{c}}\right) \boldsymbol{x_0}$

finally: $\boldsymbol{x} = \boldsymbol{x_0} \pm \Delta \boldsymbol{x}$

Examples:

1°- Perimeter of a rectangle: $\boldsymbol{L}$ is the length and $\boldsymbol{l}$ is the width

$$\boldsymbol{P} = \boldsymbol{2}.(\boldsymbol{L} + \boldsymbol{l}) \Rightarrow \Delta \boldsymbol{P} = \boldsymbol{2}(\Delta \boldsymbol{L} + \Delta \boldsymbol{l})$$

2°- Surface of this rectangle:

$$S = L.l \quad \Rightarrow \quad \Delta S = l.\Delta L + L.\Delta l \quad \Rightarrow \quad \frac{\Delta S}{S} = \frac{\Delta L}{L} + \frac{\Delta l}{l}$$

$$\Rightarrow \Delta S = \left(\frac{\Delta L}{L} + \frac{\Delta l}{l}\right) S$$

4- Signifiant figures

During the measurement, we write the quantified quantity in scientific notation, the figures that express this quantity are said to be *significant*.

Note: "13" and "13.0" have the same value, but their meanings are different i.e., the error of the second is 10 times less than the first

Generally:

- Non-zero figures are always significant (3.1415 $\longrightarrow$ 5 significant digits).
- All zeros that come at the end are significant (0.4500 $\longrightarrow$ 4 significant digits).
- The zeros between the significant digits are significant (0.104 $\longrightarrow$ 3 significant digits).
- The zeros used to move the comma are not meaningful (0.00125=1.25 10^{-5} $\longrightarrow$ 3 significant digits).

Some rules on significant numbers

5- Data and graphs

5.1- Data

These are the values that a physical quantity can take in different states

5.2- Graphs

The dependence that exists between two or more physical quantities is expressed by a function that can be represented by a curve or a graph.

There are several types of functions:

- Linear functions:

$\boldsymbol{y = ax + b}$, express the dependence between y et x.

- Quadratic functions:

$y = ax^2 + bx + c$ (Parabola of the 2nd order as well as that of the 3rd order and so on)

- Inverse functions:

$$y = \frac{k}{x}$$

- Exponential and logarithmic functions:

$y = ae^{u(x)}$, $y = \ln(v(x))$ $où\ u(x)\ et\ v(x)$ are any numeric functions

- Circular or trigonometric functions:

$y = a.sin[u(x)]$, $y = b.cos[u(x)]$, $y = tg[u(x)]$ ...

- Hyperbolic functions:

$y = a.sinh[u(x)]$, $y = b.cosh[u(x)]$, $y = tgh[u(x)]$ ...

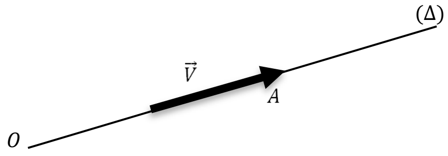
- Special functions.

B- <u>Vectors</u>

1- Notion of vector

1.1- Definition:

A vector is a mathematical entity that represents an element of a vector space $\mathbb{E}^3$ associated with an affine space (point), $\mathbb{R}^3$ where a direction, modulus, and point of application are defined.



- "$\boldsymbol{O}$" point of application

- "Δ" line of action

- In the orthonormal basis$(\vec{\boldsymbol{\imath}}, \vec{\boldsymbol{\jmath}}, \vec{\boldsymbol{k}})$,) and in Euclidean geometry:

The modulus of the vector $\vec{\boldsymbol{V}}$ is:

$$\vec{\boldsymbol{V}} = \overrightarrow{|\boldsymbol{OA}|} = \sqrt{x^2 + y^2 + z^2}$$

- From <u>O to A</u> is the direction

1.2-Types of vectors

1.2.1- Free vector

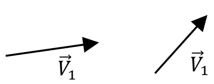
It is a vector where the application point can be transferred to any point in space.

1.2.2- Sliding vector

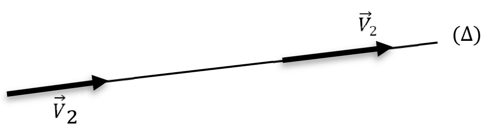
It is a vector where the application point can move along its line of action

1.2.3- Bound vector

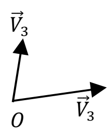
It is a vector where the point of application is fixed and defined by the coordinates of its origin



Free vector



Sliding vector



Bound vector

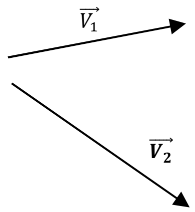
2- Operation on vectors

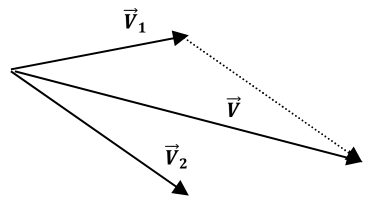
2.1- Sum of vectors (resultant):

Relative to an orthonormal$(\vec{\imath},\vec{\jmath},\vec{k})$ basis, the sum of two vectors is a vector, where the components are added two to two respectively

$$\vec{V}_1 = x_1\vec{\imath} + y_1\vec{\jmath} + z_1\vec{k} \qquad \textbf{and} \qquad \vec{V}_2 = x_2\vec{\imath} + y_2\vec{\jmath} + z_2\vec{k}$$

$$\Rightarrow \quad \vec{V} = \vec{V}_1 + \vec{V}_2 = (x_1 + x_2)\vec{\imath} + (y_1 + y_2)\vec{\jmath} + (z_1 + z_2)\vec{k}$$



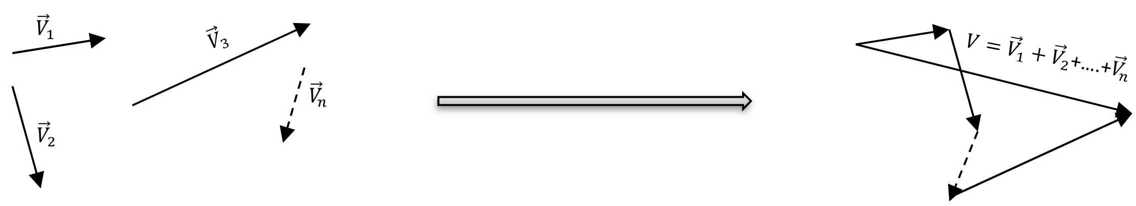


Note:

For multiple vectors, the sum of the respective components added together represents the components of the resultant vector.

$$\vec{V} = \vec{V}_1 + \vec{V}_2 + \cdots + \vec{V}_n = (x_1 + x_2 + \cdots + x_n)\vec{\imath} + (y_1 + y_2 + \cdots + y_n)\vec{\jmath} + (z_1 + z_2 + \cdots + z_n)\vec{k}$$

$$\vec{V} = x\vec{\imath} + y\vec{\jmath} + z\vec{k}$$



$$\begin{cases} x = \sum_{l=1}^{n} x_l \\ y = \sum_{l=1}^{n} y_l \\ z = \sum_{l=1}^{n} z_l \end{cases}$$

2.2- Product of vectors:

a- Scalar product and projection:

The scalar product of two vectors $\vec{V}_1$ and $\vec{V}_2$, is a scalar denoted $\vec{V}_1 \circ \vec{V}_2$, which is equal to the sum of the products of the corresponding components taken pairwise.

$$\vec{V}_1 = x_1\vec{\imath} + y_1\vec{\jmath} + z_1\vec{k} \quad \text{and} \quad \vec{V}_2 = x_2\vec{\imath} + y_2\vec{\jmath} + z_2\vec{k}$$

$$\Longrightarrow \qquad V = \vec{V}_1 \circ \vec{V}_2 = (x_1.x_2) + (y_1.y_2) + (z_1.z_2)$$

Note:

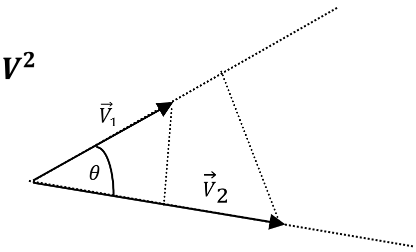
- For the unit vectors of the orthonormal basis, we have:

$$\begin{cases} \vec{\imath} \circ \vec{\imath} = \vec{\jmath} \circ \vec{\jmath} = \vec{k} \circ \vec{k} = 1 \\ \vec{\imath} \circ \vec{\jmath} = \vec{\imath} \circ \vec{k} = \vec{\jmath} \circ \vec{k} = 0 \end{cases}$$

- The square of the modulus of the vector is:

$$\vec{V} \circ \vec{V} = (x.x) + (y.y) + (z.z) = x^2 + y^2 + z^2 = V^2$$

$$\Longrightarrow |\vec{V}| = V = \sqrt{x^2 + y^2 + z^2}$$



- The scalar product can also be defined as follows:

$$\vec{V}_1 \circ \vec{V}_2 = |\vec{V}_1|.|\vec{V}_2| \cos(\vec{V}_1, \vec{V}_2) = |\vec{V}_1|.|\vec{V}_2| \cos(\theta)$$

- The square of the modulus of a vector can be given by:

$$\vec{V}_1 \circ \vec{V}_1 = |\vec{V}_1|.|\vec{V}_2| \cos(\vec{V}_1, \vec{V}_1) = V_1^2$$

<u>Properties</u>:

- The scalar product is commutative

$$\vec{V}_1 \circ \vec{V}_2 = \vec{V}_2 \circ \vec{V}_1$$

- The scalar product is distributive with respect to addition

$$\vec{V}_1 \circ (\vec{V}_2 + \vec{V}_3) = \vec{V}_1 \circ \vec{V}_2 + \vec{V}_1 \circ \vec{V}_3$$

- The scalar product geometrically represents the projection of one vector onto the direction of another

$$\begin{cases} \vec{V} \circ \vec{i} = (x\vec{i} + y\vec{j} + z\vec{k}) \circ \vec{i} = x \\ \vec{V} \circ \vec{j} = (x\vec{i} + y\vec{j} + z\vec{k}) \circ \vec{j} = y \\ \vec{V} \circ \vec{k} = (x\vec{i} + y\vec{j} + z\vec{k}) \circ \vec{k} = z \end{cases}$$

- The scalar product is zero if:

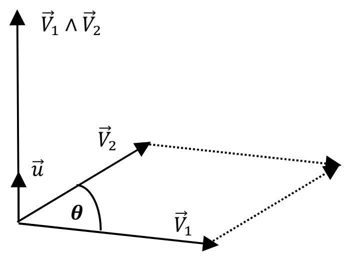
$$|\vec{V}_1| = 0, |\vec{V}_2| = 0 \text{ or } \vec{V}_1 \perp \vec{V}_2$$

b- Vector product and oriented surface:

The cross product of two vectors, $\vec{V}_1$ and $\vec{V}_2$, is a vector denoted $\vec{V}_1 \wedge \vec{V}_2$ and given by:

$$\vec{V}_1 \wedge \vec{V}_2 = \begin{vmatrix} \vec{i} & \vec{j} & k \\ x_1 & y_1 & z_1 \\ x_2 & y_2 & z_2 \end{vmatrix} = (y_1.z_2 - y_2.z_1)\vec{i} - (x_1.z_2 - x_2.z_1)\vec{j} + (x_1.y_2 - x_2.y_1)\vec{k}$$

Also defined as follows:



$$\vec{V}_1 \wedge \vec{V}_2 = |\vec{V}_1|.|\vec{V}_2| \sin(\vec{V}_1, \vec{V}_2)\,\vec{u} = |\vec{V}_1|.|\vec{V}_2| \sin(\theta)\,\vec{u}$$

$\vec{u}$: is a unit vector

$\vec{u} \perp (\vec{V}_1 et \vec{V}_2)$

Properties:

- The vector product is noncommutative (anticommutative)

$$\vec{V}_1 \wedge \vec{V}_2 = -\vec{V}_2 \wedge \vec{V}_1$$

- The vector product is distributive with respect to the addition

$$\vec{V}_1 \wedge (\vec{V}_2 + \vec{V}_3) = \vec{V}_1 \wedge \vec{V}_2 + \vec{V}_1 \wedge \vec{V}_3$$

- The resulting vector of the cross product is always perpendicular to the operand vectors.
- The vector product obeys the rule of circular permutation

$$\begin{cases} \vec{i} \wedge \vec{j} = \vec{k} \\ \vec{j} \wedge \vec{k} = \vec{i} \\ \vec{k} \wedge \vec{i} = \vec{j} \end{cases} \qquad \text{and } \vec{i} \wedge \vec{i} = \vec{j} \wedge \vec{j} = \vec{k} \wedge \vec{k} = \mathbf{0}$$

- The vector product is zero if:

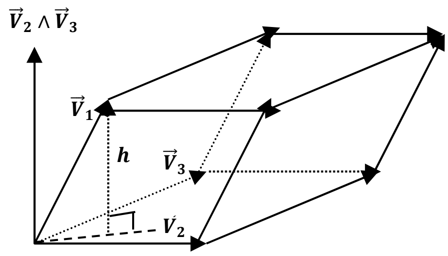
$$\left|\vec{V}_1\right| = 0\ ,\left|\vec{V}_2\right| = 0 \text{ or } \vec{V}_1 \parallel \vec{V}_2$$

- The cross product geometrically represents the area of the oriented surface formed by operand vectors.

c- Triple product:

❖ <u>The scalar triple product</u>

The scalar triple product, is a scalar defined as:



$$\vec{V}_1 \circ \left(\vec{V}_2 \wedge \vec{V}_3\right) = W$$

<u>Properties</u>:

- The scalar triple product is invariant by cyclic permutation

$$\vec{V}_1 \circ \left(\vec{V}_2 \wedge \vec{V}_3\right) = \vec{V}_3 \circ \left(\vec{V}_1 \wedge \vec{V}_2\right) = \vec{V}_2 \circ \left(\vec{V}_3 \wedge \vec{V}_1\right)$$

- The scalar triple product is zero if:

$\left|\vec{V}_1\right| = 0\ \left|\vec{V}_2\right| = 0\left|\vec{V}_3\right| = 0,$ or $\vec{V}_1, \vec{V}_2$ and $\vec{V}_3$ are coplanar

- Geometrically, the scalar triple product represents the volume formed by the operand vectors.

❖ <u>The vector triple product</u>

The vector triple product is a vector defined by the following relation:

$$\vec{V}_1 \wedge \left(\vec{V}_2 \wedge \vec{V}_3\right) = (\vec{V}_1 \circ \vec{V}_3)\vec{V}_2 - (\vec{V}_1 \circ \vec{V}_2)\vec{V}_3 = \alpha\vec{V}_2 + \beta\vec{V}_3 = \vec{W}$$

<u>Remark:</u>

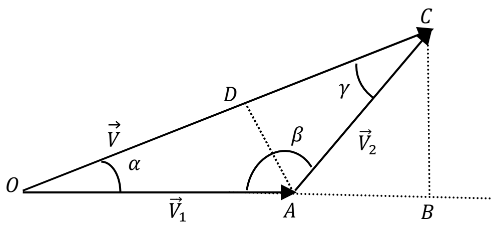
The multiplication of a vector by a scalar is a vector (it is a homothety)

$$\lambda\vec{V} = \vec{W}$$

3- Rule of sines

$$\vec{V} = \vec{V}_1 + \vec{V}_2$$

$$|\vec{V}| = \sqrt{(\vec{V}_1 + \vec{V}_2) \circ (\vec{V}_1 + \vec{V}_2)} = \sqrt{|V_1| + |V_2| + 2|V_1| \circ |V_2|}$$



- The triangles ABC and OBC give:

$$\begin{cases} \sin(\alpha) = \dfrac{BC}{OC} \\ \sin(\pi - \beta) = \dfrac{BC}{AC} \end{cases} \Rightarrow \quad OC.\sin(\alpha) = AC.\sin(\beta) \quad \Rightarrow \frac{|\vec{V}|}{\sin(\beta)} = \frac{|\vec{V}_2|}{\sin(\alpha)}$$

- The triangles OAD and give:ACD

$$\begin{cases} \sin(\alpha) = \dfrac{AD}{OA} \\ \sin(\gamma) = \dfrac{AD}{AC} \end{cases} \Rightarrow \quad OA.\sin(\alpha) = AC.\sin(\gamma) \quad \Rightarrow \frac{|\vec{V}_1|}{\sin(\gamma)} = \frac{|\vec{V}_2|}{\sin(\alpha)}$$

$$\Rightarrow \frac{|\vec{V}|}{\sin(\beta)} = \frac{|\vec{V}_2|}{\sin(\alpha)} = \frac{|\vec{V}_1|}{\sin(\gamma)}$$

4- Derived from a vector

In a Cartesian orthonormal basis, the vector is expressed $\vec{a}$ by:

$$\vec{a} = x\vec{\imath} + y\vec{\jmath} + z\vec{k}$$

If it is variable, its derivative comes down to differentiating these components.

$$\frac{d\vec{a}}{dt} = \frac{dx}{dt}\vec{\imath} + \frac{dy}{dt}\vec{\jmath} + \frac{dz}{dt}\vec{k}$$

- The derivative of the sum of the vectors is equal to the sum of the derivatives of these vectors

$$\frac{d(\vec{a}+\vec{b})}{dt}=\frac{d\vec{a}}{dt}+\frac{d\vec{b}}{dt}$$

- The derivative of the product of the vectors is equal to

$$\frac{d(\vec{a}\circ\vec{b})}{dt}=\vec{b}\circ\frac{d\vec{a}}{dt}+\vec{a}\circ\frac{d\vec{b}}{dt} \qquad \textit{for the scalar product}$$

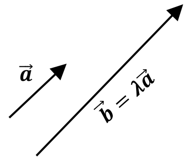
$$\frac{d(\vec{a}\wedge\vec{b})}{dt}=\vec{a}\wedge\frac{d\vec{b}}{dt}+\frac{d\vec{a}}{dt}\ \wedge\ \vec{b} \qquad \textit{for the cross product}$$

I - *Coordinate systems*

1- Introduction

- Two vectors are linearly dependent if one vector can be expressed in terms of the other.

$\vec{b} = \lambda\vec{a}$ where"$\boldsymbol{\lambda}$" is a real



vectors linearly dependant

- Two vectors are linearly independent if any of the vectors cannot be expressed in terms of the other.

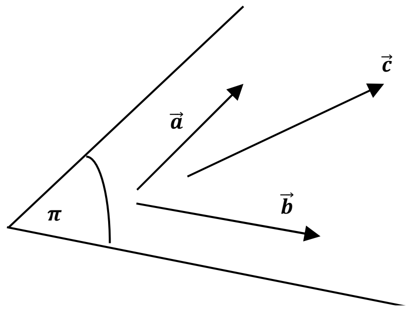
Remarks:

- In a plane, a vector can be expressed as a linear combination of two linearly independent vectors.

$$\vec{c} = \alpha\vec{a} + \beta\vec{b}$$

- The case can be generalized to three dimensions and more

$$\vec{v} = \alpha\vec{a} + \beta\vec{b} + \gamma\vec{c} + \cdots$$



vectors lineairly independant

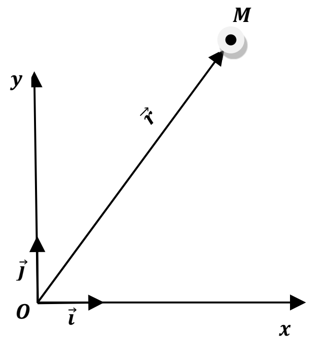
- The three vectors $\vec{a}, \vec{b}, \vec{c}$ form a basis if they are linearly independent.
    - If they are pairwise orthogonal, they form an orthogonal basis.
    - If they are normalized, the basis is called orthonormal.

2- Representation in the plan

2.1- Cartesian (Rectangular)coordinates $[(x,y) \rightarrow (\vec{i},\vec{j})]$

In the plane we choose an orthonormal basis $(\vec{i},\vec{j})$ where the coordinates of the point "M" are (x,y)

Location of "M" :



The point M position is given by the vector $\overrightarrow{OM}$ such that:

$$\overrightarrow{OM} = \vec{r} = x\vec{i} + y\vec{j}$$

The module is:

$$|\overrightarrow{OM}| = |\vec{r}| = \sqrt{x^2 + y^2}$$

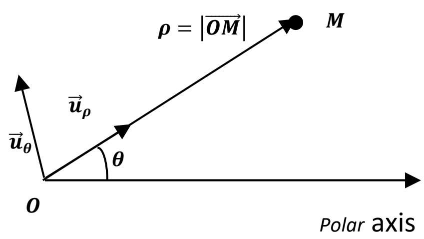
2.2- Polar coordinates$[(\rho,\theta) \rightarrow (\vec{u}_\rho, \vec{u}_\theta)]$

If we choose a local base $(\vec{u}_\rho, \vec{u}_\theta)$. "$O$" taken arbitrarily as the pole. The unit vector $\vec{u}_\rho$ is oriented along the vector $\overrightarrow{OM}$. The direction passing through the pole "O" is the polar axis, taken as a reference to define the angle (coordinate) "θ". The other coordinate "ρ" is the magnitude of the vector $\overrightarrow{OM}$.

$$\overrightarrow{OM} = \rho\vec{u}_\rho$$

The module is:

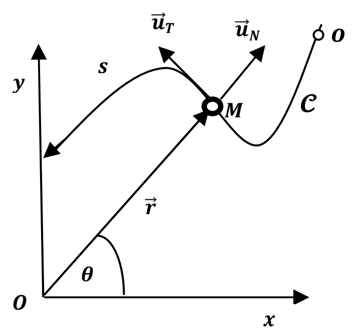
$$|\overrightarrow{OM}| = \rho$$



2.3- Intrinsic coordinates $[(\vec{u}_N, \vec{u}_T)]$

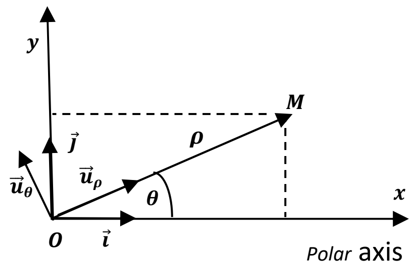
We cannot represent the point in the intrinsic coordinate system unless we know the curve "C" of the trajectory, which is taken as the axis. Equipped with an origin, the distance $\widehat{oM}$ is denoted as "s".

$$\widehat{oM} = s \quad \text{and} \quad \overrightarrow{OM} = \vec{r}$$



2.4- Relationship between the coordinates of the different systems

- In Cartesian coordinates: $\overrightarrow{OM} = \vec{r} = x\vec{\imath} + y\vec{\jmath}$
- In Polar coordinates: $\overrightarrow{OM} = \rho\vec{u}_\rho$
- If we make a choice such that the polar axis is superimposed with the $\overrightarrow{ox}$ axis



We will have:

$$\begin{cases} \vec{u}_\rho = cos(\theta)\,\vec{\imath} + sin(\theta)\vec{\jmath} \\ \vec{u}_\theta = -sin(\theta)\,\vec{\imath} + cos(\theta)\vec{\jmath} \end{cases}$$

Then: $$\overrightarrow{OM} = \vec{r} = x\vec{\imath} + y\vec{\jmath} = \rho\vec{u}_\rho = \rho\, cos(\theta)\,\vec{\imath} + \rho\, sin(\theta)\,\vec{\jmath}$$

By comparison we will get:

$$\begin{cases} x = \rho\cos\theta \\ y = \rho\sin\theta \end{cases} \Leftrightarrow \begin{cases} \rho = \sqrt{x^2 + y^2} \\ \theta = arctg(y/x) \end{cases}$$

Note:

Polar coordinates and intrinsic coordinates should not be merge (confused).

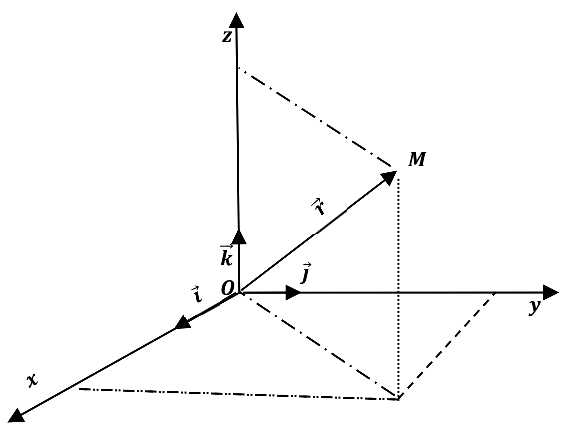
3- Representation in space

3.1- Cartesian (Rectangular)coordinates $[(x, y, z) \rightarrow (\vec{\imath}, \vec{\jmath}, \vec{k})]$

In space, the location of the point "M" is expressed by the (x, y, z) coordinates in an orthonormal basis $(\vec{\imath}, \vec{\jmath}, \vec{k})$. in such a way that:

$$\overrightarrow{OM} = \vec{r} = x\vec{\imath} + y\vec{\jmath} + z\vec{k}$$

$\overrightarrow{OM}$: is the position vector of the point M



The module is:

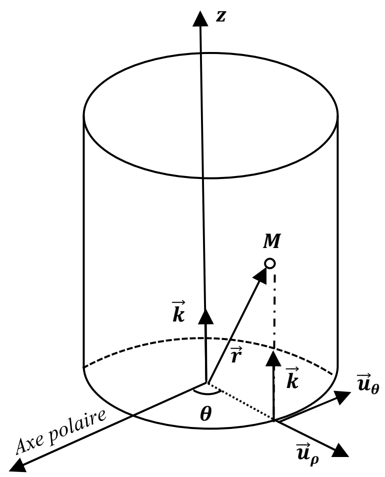
$$|\overrightarrow{OM}| = |\vec{r}| = \sqrt{x^2 + y^2 + z^2}$$

$\boldsymbol{x}$: is the projection of $\overrightarrow{\boldsymbol{OM}}$ on the direction $\vec{\boldsymbol{\imath}}$

$\boldsymbol{y}$: is the projection of $\overrightarrow{\boldsymbol{OM}}$ on the direction $\vec{\boldsymbol{\jmath}}$

$\boldsymbol{z}$: is the projection of $\overrightarrow{\boldsymbol{OM}}$ on the direction $\vec{\boldsymbol{k}}$

3.2- Coordinates cylindrical $[(\rho,\theta,z) \rightarrow (\vec{u}_\rho,\vec{u}_\theta,\vec{k})]$



To locate a point "$\boldsymbol{M}$" in space, instead of using a Cartesian system, other systems can be used. Among these, the cylindrical system. In this system, we imagine that point "$\boldsymbol{M}$" is on the surface of a cylinder with axis $\overrightarrow{\boldsymbol{OZ}}$, radius $\boldsymbol{\rho}$, and "some" base.

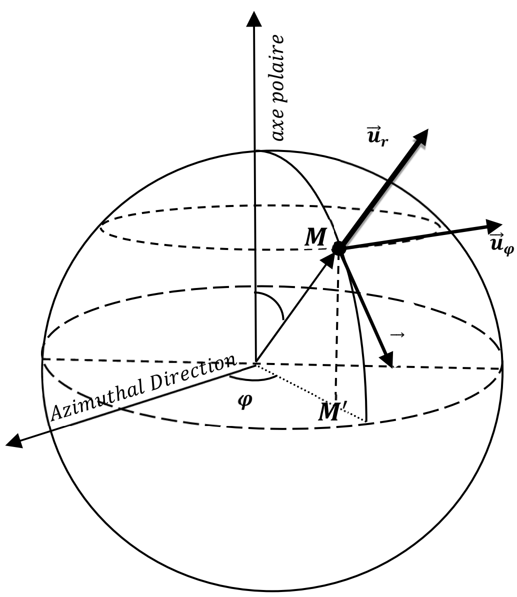
The projection of $\overrightarrow{\boldsymbol{OM}}$, on the base of the cylinder is located by $(\boldsymbol{\rho},\boldsymbol{\theta})$.

So $\quad \overrightarrow{\boldsymbol{OM}} = \vec{\boldsymbol{r}} = \boldsymbol{\rho}\vec{\boldsymbol{u}}_{\boldsymbol{\rho}} + \boldsymbol{z}\vec{\boldsymbol{k}}$

And $\quad |\overrightarrow{\boldsymbol{OM}}| = |\vec{\boldsymbol{r}}| = \sqrt{\boldsymbol{\rho}^2 + \boldsymbol{z}^2}$

3.3- Spherical coordinates $[(r,\theta,\varphi) \rightarrow (\vec{u}_r,\vec{u}_\theta,\vec{u}_\varphi)]$

Another system allows us to locate a point "$\boldsymbol{M}$" in space. In this system, it is imagined that point "$\boldsymbol{M}$" is on the surface of a sphere with radius "$\boldsymbol{r}$" and center "$\boldsymbol{O}$" . This center is taken as the origin, and called pole. It is located in the equatorial plane.



In spherical coordinates, a point "$\boldsymbol{M}$" is characterized by the linear variable "$\boldsymbol{r}$", and the angular variables "$\boldsymbol{\varphi},\boldsymbol{\theta}$".

- "$\boldsymbol{\theta}$" polar angle: Angle between the polar axis taken arbitrarily and the direction $\overrightarrow{\boldsymbol{OM}}$.
  "$\boldsymbol{O}$" is the center of this sphere.
- The projection of "$\boldsymbol{M}$" on the Equatorial plane is "$\boldsymbol{M'}$ ". It is located by the azimuthal angle "$\boldsymbol{\varphi}$" with respect to an arbitrary direction axis (azimuthal direction) in that plane.

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$$\overrightarrow{OM} = \vec{r} = |\vec{r}|\vec{u}_r$$

- $\vec{u}_r$: radial unit vector (in the direction of the radius $\overrightarrow{OM}$)
- $\vec{u}_\theta$: unit vector tangent to the great circle (all circles of radius $\overrightarrow{OM}$).
- $\vec{u}_\varphi$: unit vector tangent to parallels (circles parallel to the equator).

3.4- Relationship between the coordinates of the different systems

3.4- 1 Relationship between Cartesian coordinates and cylindrical coordinates

- In Cartesian coordinates: $\overrightarrow{OM} = \vec{r} = x\vec{i} + y\vec{j} + z\vec{k}$
- In cylindrical coordinates: $\overrightarrow{OM} = \rho\vec{u}_\rho + z\vec{k}$

With $\vec{u}_\rho = \cos\theta\,\vec{i} + \sin\theta\,\vec{j}$

$$\overrightarrow{OM} = \vec{r} = x\vec{i} + y\vec{j} + z\vec{k} = \rho\cos\theta\,\vec{i} + \rho\sin\theta\,\vec{j} + z\vec{k}$$

$$\begin{cases} x = \rho\cos\theta \\ y = \rho\sin\theta \\ z = z \end{cases} \Leftrightarrow \begin{cases} \rho = \sqrt{x^2 + y^2} \\ \theta = arctg(y/x) \end{cases}$$

3.4- 2 Relationship between Cartesian and spherical coordinates

- In Cartesian coordinates: $\overrightarrow{OM} = \vec{r} = x\vec{i} + y\vec{j} + z\vec{k}$
- In spherical coordinates: $\overrightarrow{OM} = |\vec{r}|\vec{u}_r = r\vec{u}_r$

With $\vec{u}_r = \sin\theta\cos\varphi\,\vec{i} + \sin\theta\sin\varphi\,\vec{j} + \cos\theta\,\vec{k}$

So:

$$\begin{cases} x = r\sin\theta\cos\varphi \\ y = r\sin\theta\sin\varphi \\ z = r\cos\theta \end{cases} \Leftrightarrow \begin{cases} r = \sqrt{x^2 + y^2 + z^2} \\ \varphi = arctg(y/x) \\ \theta = arcos\left(\frac{z}{\sqrt{x^2+y^2+z^2}}\right) \end{cases}$$