Mohamed Boudiaf University - M'sila — Faculty of Technology
Department of Civil Engineering — University year 2024-2025
Module : Probability-Statistics

Worksheet n°3

(COMBINATORIAL ANALYSIS)

Exercice n°1 :

In some countries, car number plates begin with a letter of the alphabet, followed by five digits. Calculate how many number plates are possible if :

a) The first digit following the letter cannot be 0.

b) The first letter cannot be O or I and the first digit cannot be 0 or 1.

Exercice n°2 :

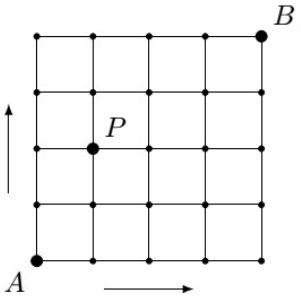
9 people are seated around a round table.

1. How many different ways can they sit ? (only the relative position of the nine people in relation to each other is taken into account). relative to each other)
2. Same question, but people A and B want to be near to each other.

Exercice n°3 :

Find the number of anagrams of the word MISSISSIPPI. Of these anagrams, how many begin and end with the letter S ?

Exercice n°4 : On this 4×4 grid, you can only move to the right or upwards.



a) How many paths are there from point A to point B ?

b) How many of these paths pass through point $P\,(1;2)$?

Exercice n°5 :

Let $C_n^p = \frac{n!}{p!(n-p)!}$,, where $n, p \in \mathbb{N}$ and $p \leq n$.

1) Show that : $a)\ C_n^0 = C_n^n \qquad b)\ C_n^p = C_n^{n-p} \qquad c)\ C_{n+1}^{p+1} = C_n^{p+1} + C_n^p$.

2) Using Newton's Binomial formula $(a+b)^n = \sum_{p=0}^{n} C_n^i a^{n-p} b^p$, calculate

$$A = \sum_{p=0}^{n} C_n^p \quad , \quad B = \sum_{p=0}^{n} C_n^p\,(-1)^p.$$

3) Show that $\forall n, p \in \mathbb{N}$ (with $p \leq n$) : $pC_n^p = nC_{n-1}^{p-1}$, and calculate : $F = 1C_n^1 + 2C_n^2 + ... + nC_n^n$.

4) What is the coefficient of x^6 in the development of $(x+2)^8$ and $(x^2-5)^7$?

Exercice n°6 :

An urn contains 12 balls numerated from 1 to 12. 3 balls are drawn simultaneously.

i) Determine the number of different draws.

ii) Same question if these three balls are drawn successively.

iii)* What if, after each draw, the ball is put back into the urn.

Exercice n°7 :

Consider the set $E = \{1; 2; 3; 4; 5; 6\}$. Using the 6 digits of this set, each taken only once, how many distinct numbers can be formed in each of the following cases :

a) Numbers of 6 digits ?

b) Numbers of 4 digits ?

c) Numbers with 4 digits starting with 3 ?

d) Numbers of 4 digits containing the digit 3 ?

e) Numbers of 4 digits containing the digits 3 and 6 ?

f) What is a 3-combinations of a set E if repetition is allowed ?