

Worksheet n°3

( COMBINATORIAL ANALYSIS)

**Exercise n°1 :**

In some countries, car number plates begin with a letter of the alphabet, followed by five digits. Calculate how many number plates are possible if :

- a) The first digit following the letter cannot be 0.
- b) The first letter cannot be O or I and the first digit cannot be 0 or 1.

**Exercise n°2 :**

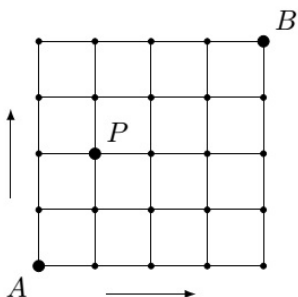
9 people are seated around a round table.

- 1. How many different ways can they sit ? (only the relative position of the nine people in relation to each other is taken into account). relative to each other)
- 2. Same question, but people A and B want to be near to each other.

**Exercise n°3 :**

Find the number of anagrams of the word MISSISSIPPI. Of these anagrams, how many begin and end with the letter S ?

**Exercise n°4 :** On this  $4 \times 4$  grid, you can only move to the right or upwards.



- a) How many paths are there from point A to point B ?
- b) How many of these paths pass through point P (1; 2) ?

**Exercise n°5 :**

Let  $\binom{n}{r} = \frac{n!}{r!(n-r)!}$ , where  $n, r \in \mathbb{N}$  and  $r \leq n$ .

1) Show that : a)  $\binom{n}{n} = \binom{n}{0}$  , b)  $\binom{n}{r} = \binom{n}{n-r}$  (symmetry formula)

c)  $\binom{n+1}{r+1} = \binom{n}{r} + \binom{n}{r+1}$

2) Using Newton's Binomial formula  $(a+b)^n = \sum_{r=0}^n \binom{n}{r} a^{n-r} b^r$  , calculate

$$A = \sum_{r=0}^n \binom{n}{r} \quad , \quad B = \sum_{r=0}^n \binom{n}{r} (-1)^r.$$

**3)** What is the coefficient of  $x^6$  in the development of  $(x + 2)^8$  and  $(x^2 - 5)^7$ ?

**Exercice n°6 :**

An urn contains 12 balls numerated from 1 to 12.

3 balls are drawn simultaneously.

- i) Determine the number of different draws.
- ii) Same question if these three balls are drawn successively.
- iii)\* What if, after each draw, the ball is put back into the urn.

**Exercice n°7 :**

Consider the set  $E = \{1; 2; 3; 4; 5; 6\}$ . Using the 6 digits of this set, each taken only once, how many distinct numbers can be formed in each of the following cases :

- a) Numbers of 6 digits?
- b) Numbers of 4 digits?
- c) Numbers with 4 digits starting with 3?
- d) Numbers of 4 digits containing the digit 3?
- e) Numbers of 4 digits containing the digits 3 and 6?