1) Introduction

Through this chapter, we aim to advance from studying the motion a material point (kinematics) and the forces causing this motion (dynamics), to exploring energy changes and the work done by applying force. when a force affects an object, it performs work that results in the transfer or conversion of energy. thus, any applied force on a system has a visible effect 'the work done by the force) and hidden effect , which involves the transfer or conversion of energy.

2) Work of a force

Any action of a force that results in the displacement of an object is known as work of force. Work of force is a means of transferring or transforming energy from one form to another.

Work is a scalar quantity, and is symbolized by the symbol $\boldsymbol{W}$.

The physical dimension of work is: $[W] = M\,L^2\,T^{-2}$, *where:*

$\boldsymbol{M}$ *represents mass.*

$\boldsymbol{L}$ *represents distance or displacement.*

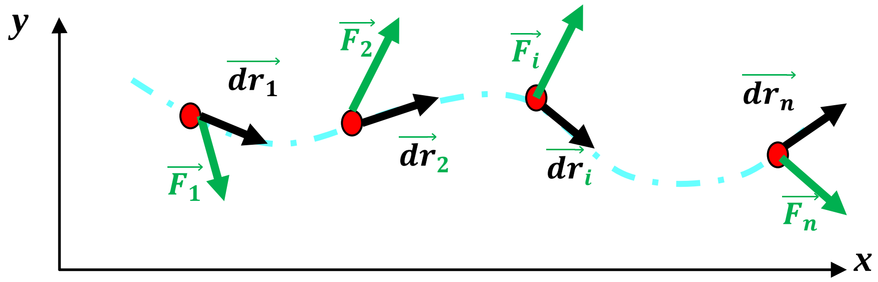
$\boldsymbol{T}$ *represents time.*

The unit of work in the International System of Units (SI) is the joule (J), and the joule can be expressed as follows:

$$1\ joule = 1\ kg \cdot m^2/s^2$$

2-1) Work Done by a Varying Force

consider object moving along a curved path and affected by a force may vary in magnitude or in direction or in both as shown the figure:



We divide the path of the body's motion into elementary displacements $\overrightarrow{dr}$ *(* $\overrightarrow{dr}$ *tangent to the path) where for each displacement, the force can be approximated to be constant in both magnitude and direction, and the elementary work* dW *done by the force* $\overrightarrow{F_i}$ *is written as follows:*

$$dW_i(\overrightarrow{F_i}) = \overrightarrow{F_i}\,.\,\overrightarrow{dr_i}$$

The total work done as the object moves from A to B is the sum of all the elementary works done along each elementary displacement:

$$W_{A\to B}\left(\vec{F}\right) = dW_1 + \cdots + dW_i + \cdots + dW_n = \overrightarrow{F_1}\,.\,\overrightarrow{dr_1} + \ldots + \overrightarrow{F_i}\,.\,\overrightarrow{dr_i} + \cdots + \overrightarrow{F_n}\,.\,\overrightarrow{dr_n}$$

By dividing the path into a large number "n" of elemental displacements, therefore to calculate the total work done by the force along the path, we calculate the integral dW_i

$$W_{A\to B}\left(\vec{F}\right) = \sum_{i=1}^{n} dW_i = \int_A^B dW_i = \int_A^B \vec{F}\,.\,\overrightarrow{dr}$$

To accomplish this integration, it is sufficient to write the components of both the force and displacement vectors, and this is according to the coordinate system adopted in the study. For example:

- ***Cartesian coordinates***

In the ***Cartesian coordinates*** *elementary displacement vector* $\overrightarrow{dr}$ *and the force vector* $\vec{F}$ *are written as :*

$$\begin{cases} \overrightarrow{dr} = dx\,\vec{\imath} + dy\,\vec{\jmath} + dz\,\vec{k} \\ \vec{F} = F_x\,\vec{\imath} + F_y\,\vec{\jmath} + F_z\,\vec{k} \end{cases}$$

$$\Rightarrow W_{A\to B}\left(\vec{F}\right) = \int_{x_A}^{x_B} F_x\,dx + \int_{y_A}^{y_B} F_y\,dy + \int_{z_A}^{z_B} F_z\,dz$$

- ***Cylindrical coordinate***

In the ***Cylindrical coordinates*** *elementary displacement vector* $\overrightarrow{dr}$ *and the force vector* $\vec{F}$ *are written as :*

$$\begin{cases} \overrightarrow{dr} = dr\,\overrightarrow{u_r} + r\,d\theta\,\overrightarrow{u_\theta} + dz\,\vec{k} \\ \vec{F} = F_r\,\overrightarrow{u_r} + F_\theta\,\overrightarrow{u_\theta} + F_z\,\vec{k} \end{cases}$$

$$\Rightarrow W_{A\to B}\left(\vec{F}\right) = \int_{r_A}^{r_B} F_r\, dr + \int_{\theta_A}^{\theta_B} rF_\theta\, d\theta + \int_{z_A}^{z_B} F_z\, dz$$

❖ ***Spherical coordinates***

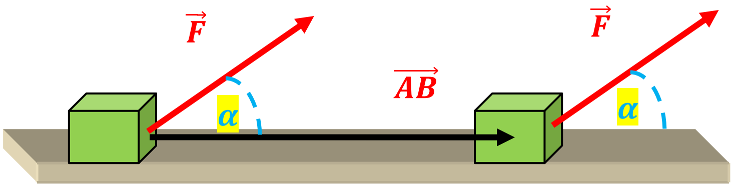
*In the **Spherical coordinates** elementary displacement vector* $\overrightarrow{dr}$ *and the force vector* $\vec{F}$ *are written as :*

$$\begin{cases} \overrightarrow{dr} = dr\, \overrightarrow{u_r} + r\, d\theta\, \overrightarrow{u_\theta} + r\sin\theta\, d\varphi\, \overrightarrow{u_\varphi} \\ \vec{F} = F_r\, \overrightarrow{u_r} + F_\theta\, \overrightarrow{u_\theta} + F_\varphi\, \overrightarrow{u_\varphi} \end{cases}$$

$$\Rightarrow W_{A\to B}\left(\vec{F}\right) = \int_{r_A}^{r_B} F_r\, dr + \int_{\theta_A}^{\theta_B} rF_\theta\, d\theta + \int_{\varphi_A}^{\varphi_B} r\sin\theta\, F_\varphi\, d\varphi$$

2-2) Work Done by a Constant Force

Consider an abject moved along a straight line and affected by a constant force $\vec{F}$ *(in both magnitude and direction that causes it to move from position A to position B, where* $\overrightarrow{AB}$ *represents the vector of its displacement, as shown in the following figure:*



Work done by a constant force can be defined as the scalar product between the force $\vec{F}$ *and its displacement* $\overrightarrow{AB}$*. the mathematical expression of the force work is given as:*

$$W_{A\to B}\left(\vec{F}\right) = \vec{F}\,.\overrightarrow{AB} = \left\|\vec{F}\right\|.\left\|\overrightarrow{AB}\right\| \cos\alpha = \begin{cases} \left\|\vec{F}\right\|.\left\|\overrightarrow{AB}\right\| & if\ \alpha = 0 \\ 0 & if\ \alpha = \dfrac{\pi}{2} \\ -\left\|\vec{F}\right\|.\left\|\overrightarrow{AB}\right\| & if\ \alpha = \pi \end{cases}$$

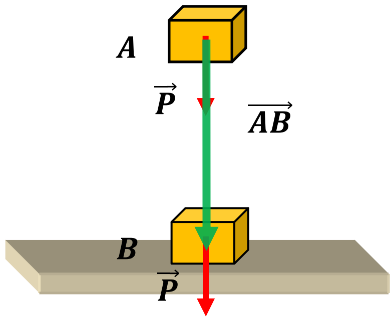
where α *is the angle between* $\vec{F}$ *and* $\overrightarrow{AB}$

A key point:* If the value of the work done by a force is positive, this means that the work is** *driving* **and the force is in the same direction as the motion (. If the work is negative, this indicates that the work is** *resistive, and the force is in the opposite direction to the motion.**

3) Examples of some force works

3-1) Work Done by a Weight

the work of the weight force $\vec{P}$ *of a block falls from point A to point B as shown in the figure.*

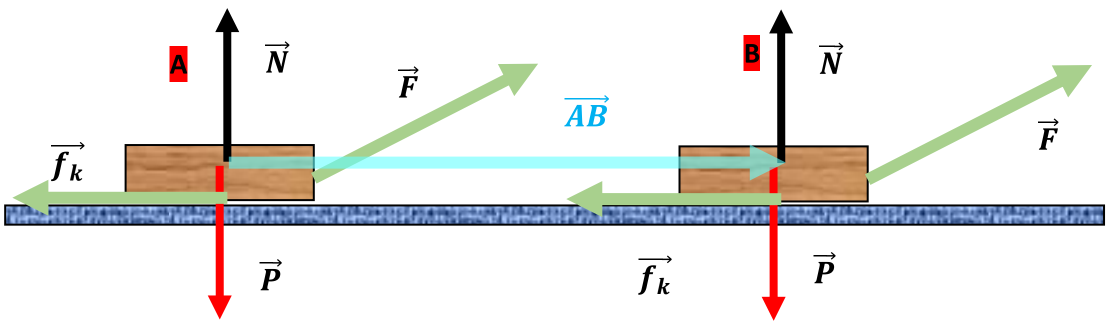


$$W_{A\to B}\left(\vec{P}\right) = \vec{P}\,.\overrightarrow{AB} = m\,g\,.\left\|\overrightarrow{AB}\right\|\,cos\,0$$

$$W_{A\to B}\left(\vec{P}\right) = \vec{P}\,.\overrightarrow{AB} = m\,g\,.\left\|\overrightarrow{AB}\right\|$$

3-2) Work Done by Friction

A block moves on a rough surface where it is subjected during its movement to the weight force $\vec{\boldsymbol{P}}$*, the normal force* $\vec{\boldsymbol{N}}$*, the force of tension* $\vec{\boldsymbol{T}}$*, and the force of kinetic friction* $\vec{\boldsymbol{f}}_{\boldsymbol{k}}$ *as shown in the figure.*

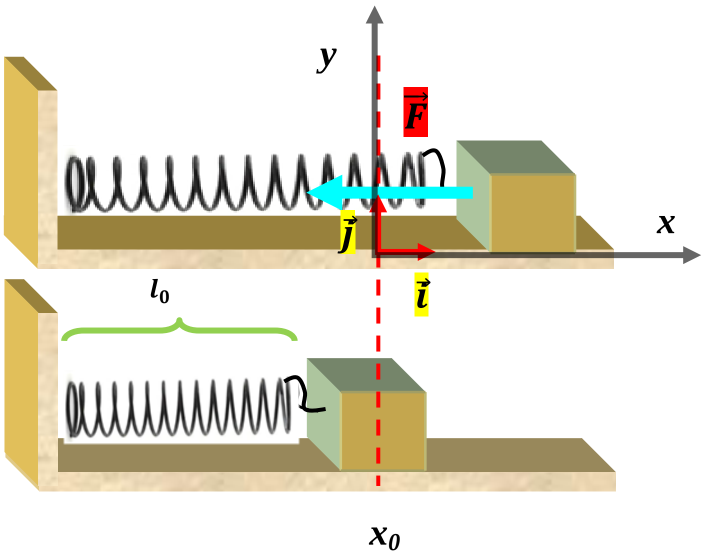


The work of the kinetic friction force during the movement of the mass from position A to position B is given by the relation:

$$W_{A\to B}\left(\vec{\boldsymbol{f}}_{\boldsymbol{k}}\right) = \vec{\boldsymbol{f}}_{\boldsymbol{k}}\,.\overrightarrow{AB} = \left\|\vec{\boldsymbol{f}}_{\boldsymbol{k}}\right\|.\left\|\overrightarrow{AB}\right\|\,cos\,\alpha = -\left\|\vec{\boldsymbol{f}}_{\boldsymbol{k}}\right\|.\left\|\overrightarrow{AB}\right\|$$

2-3) Work Done by a Spring Force

The following figure represents a block connected to a light spring of constant K fixed at the other end to a horizontal, frictionless surface.



The expression for the spring force, which is a non-constant force in both magnitude and direction, is written as follows:

$$\vec{F} = -\,k\,x\,\vec{\imath}$$

*where **x** represents the displacement from the equilibrium position, Thus, the expression for the work of the spring force when a displacement occurs from position **x** to the equilibrium position x_0 =0 is as follows is as follows:*

$$\begin{cases} \overrightarrow{dr} = dx\,\vec{\imath} + dy\,\vec{\jmath} + dz\,\vec{k} \\ \vec{F} = F_x\,\vec{\imath} = -\,k\,x\,\vec{\imath} \end{cases}$$

$$W_{x\to x_0}\left(\vec{F}\right) = \int_x^{x_0} F_x\,dx = \int_x^{x_0} -\,k\,x\,dx = -\frac{1}{2}\,k\,x^2\Big|_x^{x_0} = -\frac{1}{2}\,k\,x^2$$

3) Conservative and Nonconservative Forces

*We say that a force is **conservative** if the work it does when a body moves from one position to another is independent of the path followed but only on the initial and final positions.*

*We say that a force is **non-conservative** if the work it does from one place to another changes according to the path followed.*

*Conservative forces are also distinguished by the fact that they are derived from potential **U**, unlike non-conservative forces. We write:*

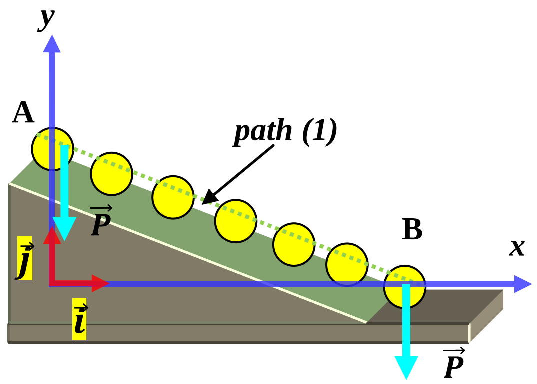
$$\text{conservative force} \Rightarrow \vec{F} = -\overrightarrow{grad}\, U$$

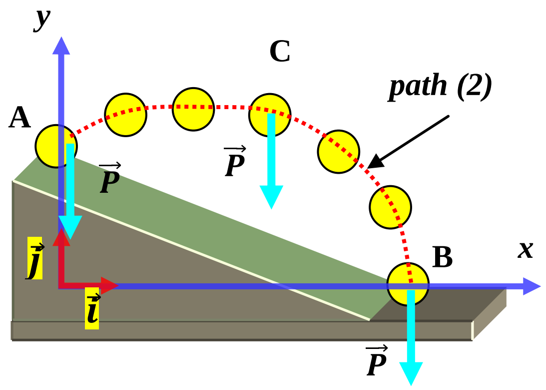
the rotational of conservative force equal to zero $\overrightarrow{rot} \wedge \vec{F} = \vec{\nabla} \wedge \vec{F} = curl\, \vec{F} = 0$

where the formula of $\vec{\nabla}$ *is given in the Cartesian, Cylindrical, and spherical coordinates systems as follow:*

$$\begin{cases} Cartesian: \quad \vec{\nabla} = \dfrac{\partial}{\partial x}\vec{\imath} + \dfrac{\partial}{\partial y}\vec{\jmath} + \dfrac{\partial}{\partial z}\vec{k} \\ Cylindrical: \quad \vec{\nabla} = \dfrac{\partial}{\partial r}\overrightarrow{u_r} + \dfrac{1}{r}\dfrac{\partial}{\partial \theta}\overrightarrow{u_\theta} + \dfrac{\partial}{\partial z}\vec{k} \\ Spherical: \quad \vec{\nabla} = \dfrac{\partial}{\partial r}\overrightarrow{u_r} + \dfrac{1}{r}\dfrac{\partial}{\partial \theta}\overrightarrow{u_\theta} + \dfrac{1}{r\sin\theta}\dfrac{\partial}{\partial \varphi}\overrightarrow{u_\varphi} \end{cases}$$

As an example of a conservative force, consider the following example, which involves the motion of a block from position A to position B via two different paths.





a) Calculating the work done by the weight force according to path (1)

$$\text{the postion } A \begin{cases} x_A = 0 \\ y_A \end{cases} \quad \text{the postion } B \begin{cases} x_B \\ y_B = 0 \end{cases} \quad \text{the weight force vector } \vec{P} = -mg\,\vec{\jmath}$$

the displacement vector $\overrightarrow{AB} = (x_B - x_A)\vec{\imath} + (y_B - y_A)\vec{\jmath} = (x_B)\vec{\imath} + (-y_A)\vec{\jmath}$

$$W_{A\to B}\left(\vec{P}\right) = \vec{P}\,.\overrightarrow{AB} = mgy_A$$

b) Calculating the work done by the weight force according to path (2)

$$\text{the postion } A \begin{cases} x_A = 0 \\ y_A \end{cases} \quad \text{the postion } C \begin{cases} x_C \\ y_C \end{cases} \quad \text{the postion } B \begin{cases} x_B \\ y_B = 0 \end{cases}$$

the displacement vector $\overrightarrow{AC} = (x_C - x_A)\vec{\imath} + (y_C - y_A)\vec{\jmath} = (x_C)\vec{\imath} + (y_C - y_A)\vec{\jmath}$

the displacement vector $\overrightarrow{CB} = (x_B - x_C)\vec{\imath} + (y_B - y_C)\vec{\jmath} = (x_B - x_C)\vec{\imath} + (-y_C)\vec{\jmath}$

$$\text{the weight force vector } \vec{P} = -mg\,\vec{\jmath}$$

$$W_{A\to C}\left(\vec{P}\right) = \vec{P}\,.\overrightarrow{CB} = (-mg\,\vec{\jmath})\,.\left((x_C)\vec{\imath} + (y_C - y_A)\vec{\jmath}\right) = -mgy_C + mg\,y_A$$

$$W_{C\to B}\left(\vec{P}\right) = \vec{P}\,.\overrightarrow{CB} = (-mg\,\vec{\jmath})\,.\left((x_B - x_C)\vec{\imath} + (-y_C)\vec{\jmath}\right) = +mgy_C$$

$$W_{A\to C}\left(\vec{P}\right) + W_{C\to B}\left(\vec{P}\right) = -mgy_C + mg\,y_A + mgy_C = mg\,y_A = W_{A\to B}\left(\vec{P}\right)$$

4) Power of a force

The Power is defined the time rate of doing work, it expresses by the derivative of work with respect to time, and is written as follows:

$$P = \frac{dW}{dt} = \frac{\vec{F}\,.\overrightarrow{dr}}{dt} = \vec{F}\,.\vec{v}$$

The SI unit of power is joules per second (J/s) and is called the watt (W).

1 W = 1 J/s = 1 kg.m^2/s^3

5) Work-Energy Theorem

The work done by this net force on a moving block from an initial position $\vec{r_i}$ *to a final position* $\vec{r_f}$ *can be calculated using the following steps :*

- Using the fundamental principle of motion:

$$\vec{F} = \frac{d\vec{P}}{dt} \Rightarrow d\vec{P} = \vec{F}\,dt$$

The effect of the force $\vec{F}$ *on the block results in a change in the linear momentum* $\vec{P}$ *by* $d\vec{P}$*.*

$$\vec{P} = m\,\vec{v}$$

$$d\vec{P} = m\,d\vec{v} + \vec{v}\,dm$$

If the mass is constant, the velocity of the moving block changes:

$$d\vec{P} = m\,d\vec{v}$$

$$\begin{cases} d\vec{P} = \vec{F}\,dt \\ d\vec{P} = m\,d\vec{v} \end{cases} \Rightarrow \vec{F}\,dt = m\,d\vec{v}$$

We multiply both sides of this equation by $\vec{v}$, we find:

$$\vec{F}.\vec{v}\,dt = m\vec{v}\,.d\vec{v}$$

given that $\vec{v} = \frac{d\vec{r}}{dt}$

$$\vec{F}.\frac{d\vec{r}}{dt}\,dt = m\vec{v}\,.d\vec{v}$$

$$\vec{v}\,.d\vec{v} = \|\vec{v}\|\,.\|d\vec{v}\|\cos\theta$$

where θ confined between $\vec{v}$ *and* $d\vec{v}$ and, $\|\vec{v}\| = v$ *and* $\|d\vec{v}\| = dv$

$$\vec{F}\,.d\vec{r} = mv\,.dv \quad \Rightarrow \int_{\vec{r_i}}^{\vec{r_f}} \vec{F}\,.d\vec{r} = \int_{v_i}^{v_f} mv\,.dv$$

$$W_{\vec{r_i}\to\vec{r_f}}\left(\vec{F}\right) = \frac{1}{2}m\,v^2\Big|_{v_i}^{v_f}$$

The quantity $\frac{1}{2}m\,v^2$ *expresses the kinetic energy of an object of mass* m *moving with a velocity of* v*. According to the expression obtained* $\left(W_{\vec{r_i}\to\vec{r_f}}\left(\vec{F}_{external}\right) = \Delta E_K\right)$ *the change in kinetic energy between two moments is equal to the net work done by the* ***external*** *forces acting on the body.*

This statement relates to the ***work-energy theorem****, which states that the net work done on an object is equal to the change in its kinetic energy.*

5-1) Translational Kinetic energy

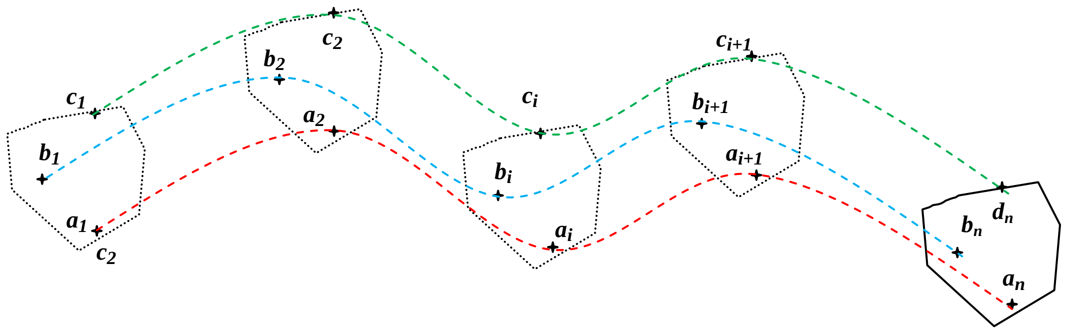
A block is said to be in translational motion if all points of the body, during their motion:

1. *Move along identical paths*

2. *Travel the same distance in the same period of time.*

the following figure *represents the paths of three points on a block moving in translational motion, where these points follow identical paths. Also, all points travel the same distance in the same amount of time.*

$$within\ a\ period\ of\ time\ \Delta t = t_{i+1} - t_1 \ \Rightarrow \|\overrightarrow{a_i a_{a+1}}\| = \left\|\overrightarrow{b_i b_{a+1}}\right\| = \|\overrightarrow{c_i c_{a+1}}\|$$



The expression for the kinetic energy of a body moving in a translational motion is written as follows:

$$E_K = \sum_{i=1}^{n} \frac{1}{2} m_i \, {v_i}^2$$

$$where \;\; v_1 = v_2 = \cdots . v_i \ldots = v \quad , and \quad m = \sum_{i=1}^{N} m_i$$

$$\boxed{E_K = \frac{1}{2} m \, v^2}$$

5-2) Rotational Kinetic Energy

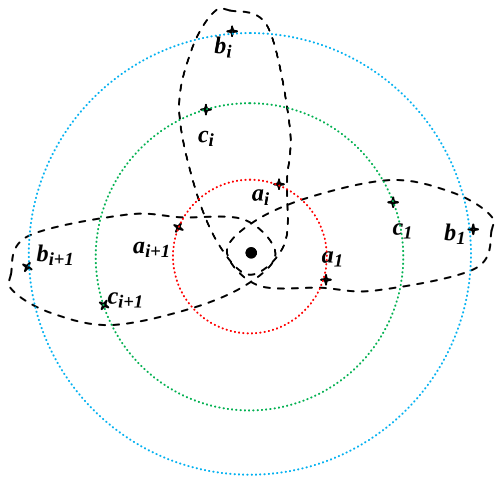
A block is said to be in rotational motion if all points of the block during their motion:

1. Move along circular paths

2. All circular paths have the same center of rotation.

3. The distance traveled increases over the same time interval as one moves away from the axis of rotation.

The following figure represents the paths of three points on a block moving in rotational motion, where all points trace circular paths with the same center, and all points cover different distances that increase as we move farther from the center of rotation.



$$within\ a\ period\ of\ time\ \Delta t = t_{i+1} - t_1 \Rightarrow \|\overrightarrow{a_i a_{a+1}}\| < \|\overrightarrow{b_i b_{a+1}}\| < \|\overrightarrow{c_i c_{a+1}}\|$$

The expression for the kinetic energy of a body moving in a translational motion is written as follows:

$$E_K = \sum_{i=1}^{n} \frac{1}{2} m_i\, {v_i}^2$$

$where\ \ v_i = r_i\, \dot{\theta}\ \ (\dot{\theta}\ is\ the\ angulat\ velocity)$

$$E_K = \sum_{i=1}^{n} \frac{1}{2} m_i\, {r_i}^2\, \dot{\theta}^2 = \frac{1}{2}\, \dot{\theta}^2 \sum_{i=1}^{n} m_i\, {r_i}^2 = \frac{1}{2}\, \dot{\theta}^2 I\ \ where\ I = \sum_{i=1}^{n} m_i\, {r_i}^2\ \ is\ moment\ of\ inertia$$

6) Potential energy

A body is said to have potential energy if its state or position enables it to perform work (or movement) under the influence of conservative forces, such as gravity or elastic force, without the need for additional forces to generate this work. For example: the state of an elevated body allows it to move downward under the influence of its conservative gravitational force. Similarly, the state of a compressed spring allows it to move and return to its natural state under the influence of elastic force.

The work of the conservative forces can be expressed from a state function called potential energy. Thus, the change in potential energy is written as follows:

$$\Delta E_P = E_{P_f} - E_{P_i} = -W_{i \to f}\left(\vec{F}_{conservative}\right)$$

the negative sign arises because the work done is opposite to the change in potential energy. That is, if the object moves from a state of low potential energy to a state of high potential energy, the work must be negative.

For an elementary change in potential energy, we write:

$$dE_P = -\,dW(\vec{F}) = -\vec{F}\,.\,\overrightarrow{dr} \;\Rightarrow E_P = -\int \vec{F}\,.\,\overrightarrow{dr}$$

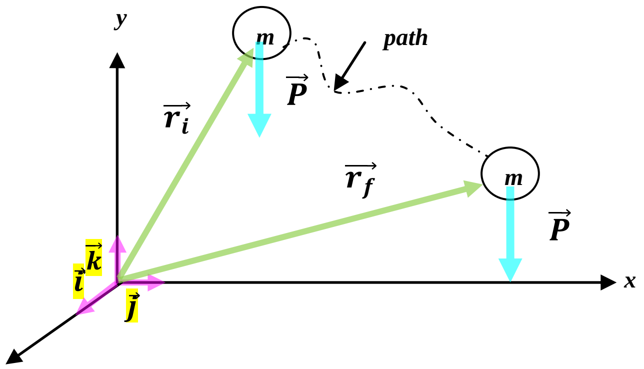
$$\vec{F} = -\overrightarrow{grad}E_P = -\vec{\nabla}E_P$$

$\vec{\nabla}$ *is given in the Cartesian, Cylindrical, and spherical coordinates systems as follow:*

$$\begin{cases} Cartesian: \quad \vec{\nabla} = \dfrac{\partial}{\partial x}\vec{i} + \dfrac{\partial}{\partial y}\vec{j} + \dfrac{\partial}{\partial z}\vec{k} \\ Cylindrical: \quad \vec{\nabla} = \dfrac{\partial}{\partial r}\overrightarrow{u_r} + \dfrac{1}{r}\dfrac{\partial}{\partial \theta}\overrightarrow{u_\theta} + \dfrac{\partial}{\partial z}\vec{k} \\ Spherical: \quad \vec{\nabla} = \dfrac{\partial}{\partial r}\overrightarrow{u_r} + \dfrac{1}{r}\dfrac{\partial}{\partial \theta}\overrightarrow{u_\theta} + \dfrac{1}{r\sin\theta}\dfrac{\partial}{\partial \varphi}\overrightarrow{u_\varphi} \end{cases}$$

6-1) Gravitational Potential Energy

To calculate the gravitational potential energy of an object, we write:



$$\vec{F} = -\vec{\nabla}E_P = -\left(\frac{\partial E_P}{\partial x}\vec{i} + \frac{\partial E_P}{\partial y}\vec{j} + \frac{\partial E_P}{\partial z}\vec{k}\right) \;\; and \;\; \vec{P} = -mg\,\vec{k}$$

$$-\left(\frac{\partial E_P}{\partial x}\vec{i} + \frac{\partial E_P}{\partial y}\vec{j} + \frac{\partial E_P}{\partial z}\vec{k}\right) = -mg\,\vec{k}$$

$$\frac{\partial E_P}{\partial z} = mg \;\Rightarrow \partial E_P = mg\,\partial z \;\Rightarrow E_P = \int mg\,\partial z = mgz + Constant$$

or

$$W_{\vec{r_i}\to\vec{r_f}}(\vec{P}) = \int_{\vec{r_i}}^{\vec{r_f}} dW_i = \int_{\vec{r_i}}^{\vec{r_f}} \vec{P}\,.\,\vec{dr} \quad \textbf{where} \begin{cases} \vec{dr} = dx\,\vec{\imath} + dy\,\vec{\jmath} + dz\,\vec{k} \\ \vec{P} = -mg\,\vec{k} \end{cases}$$

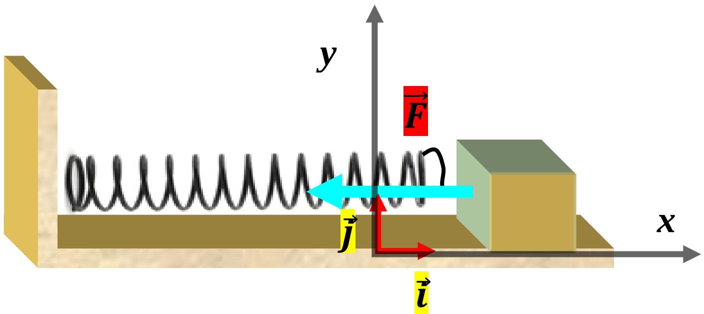
$$\vec{r_f} = x_i\,\vec{\imath} + y_i\,\vec{\jmath} + z_i\,\vec{k} \quad \textbf{and} \quad \vec{r_f} = x_f\,\vec{\imath} + y_f\,\vec{\jmath} + z_f\,\vec{k}$$

$$\Rightarrow W_{\vec{r_i}\to\vec{r_f}}(\vec{P}) = \int_{z_i}^{z_f} -mg\,dz = -mg(z_f - z_i)$$

$$\Delta E_P = E_{P_f} - E_{P_i} = -W_{\vec{r_i}\to\vec{r_f}}(\vec{P}) = mg(z_f - z_i)$$

6-2) The Elastic Potential Energy

To calculate the spring potential energy of an object, we write:



$$\vec{F} = -\vec{\nabla}E_P = -\left(\frac{\partial E_P}{\partial x}\vec{\imath} + \frac{\partial E_P}{\partial y}\vec{\jmath} + \frac{\partial E_P}{\partial z}\vec{k}\right) \quad and \quad \vec{F} = -kx\,\vec{\imath}$$

$$-\left(\frac{\partial E_P}{\partial x}\vec{\imath} + \frac{\partial E_P}{\partial y}\vec{\jmath} + \frac{\partial E_P}{\partial z}\vec{k}\right) = -kx\,\vec{\imath}$$

$$\frac{\partial E_P}{\partial x} = k\,x \Rightarrow \partial E_P = k\,x\,\partial x \Rightarrow E_P = \int k\,x\,\partial x = \frac{1}{2}\,k\,x^2 + Constant$$

7) Mechanical energy

The total mechanical energy of an object $\boldsymbol{E_T}$ is defined as the sum of all of the kinetic energies $\boldsymbol{E_K}$ of the objects within the system plus all of the potential energies $\boldsymbol{E_P}$.

$$\boldsymbol{E_T = E_K + E_P}$$

According to the kinetic energy theory, which states that the change in kinetic energy between two initial and final positions is the result of all the work of the external conservative and non-conservative forces acting on the system.

$$W_{\vec{r_i}\to\vec{r_f}}(\vec{F}_{extern\,al}) = \Delta E_K = W_{\vec{r_i}\to\vec{r_f}}(\vec{F}_{CON}) + W_{\vec{r_i}\to\vec{r_f}}(\vec{F}_{NCON})$$

$$E_K(\vec{r_f}) - E_K(\vec{r_i}) = W_{\vec{r_i}\to\vec{r_f}}(\vec{F}_{CON}) + W_{\vec{r_i}\to\vec{r_f}}(\vec{F}_{NCON})$$

$$W_{\vec{r_i}\to\vec{r_f}}(\vec{F}_{CON}) = -\Delta E_P = -(E_P(\vec{r_f}) - E_P(\vec{r_i}))$$

$$E_K(\vec{r_f}) - E_K(\vec{r_i}) + \left(E_P(\vec{r_f}) - E_P(\vec{r_i})\right) = W_{\vec{r_i}\to\vec{r_f}}(\vec{F}_{NCON})$$

$$\left(E_K(\vec{r_f}) + E_P(\vec{r_f})\right) - (E_K(\vec{r_i}) + E_P(\vec{r_i})) = W_{\vec{r_i}\to\vec{r_f}}(\vec{F}_{NCON})$$

According to the definition of mechanical energy, we can write

$$\begin{cases} E_K(\vec{r_i}) + E_P(\vec{r_i}) = E_T(\vec{r_i}) \\ E_K(\vec{r_f}) + E_P(\vec{r_f}) = E_T(\vec{r_f}) \end{cases} \Rightarrow \Delta E_T = E_T(\vec{r_f}) - E_T(\vec{r_i}) = W_{\vec{r_i}\to\vec{r_f}}(\vec{F}_{NCON})$$

Therefore, the change in the mechanical energy of the system is equal to the total work of the non-conservative forces.