Department of Physics
Analytical Mechanics
Series 3

Exercise 1:
A particle of mass mmm is constrained to move along a cycloid represented by the following parametric equations:

$$x = a(\varphi - sin\varphi)$$

$$y = a(1 + cos\varphi)$$

influence of a gravitational field. Write the Lagrangian for the problem. Then write the expression for the Hamiltonian and the equations of motion. Use the change of variables $u = cos(\varphi/2)$ and write the equation of motion. Deduce that the particle oscillates with a period to be determined.

Exercise 2:
Show that when H does not explicitly depend on time t, it is constant and equal to the total energy of the system.

Exercise 3: The Hamiltonian of a system of dimensions is given by:

$$H(x, y, P_x, P_y) = \frac{1}{2m}(P_x^2 + P_y^2) + V(x, y)$$

What are the conditions on $V(x, y)$ for $\vec{L}$ (the angular momentum) to be conserved.

Exercise 4:
A particle of mass mmm moves in the xy-plane under the action of a central force that depends only on the distance of the particle from the origin. Write the Hamiltonian and the Hamilton equations.

Exercise 5:
Show the following relations:$\vec{L} = \vec{r} \times \vec{P}$. where $\vec{r}$ is the position vector, $\vec{P}$ is the momentum, and $\vec{L}$ is the angular momentum.

[Lx,Ly]=Lz, [Ly,Lz]=Lx, [Lz,Lx]=Ly,
[Px,Ly]=Pz, [Pz,Lx]=−Py, [Px,Lx]=0

Exercise 6:
Find the Hamiltonian of a particle of mass m in a uniform gravitational field g.
a) We want to perform a change of variables $(q, p) \to (Q, P)$ such that $P = E$, where E is the mechanical energy of the system. Using the Poisson brackets, find $Q(q, p, t)$ such that the change of variables is a canonical transformation.
b) Solve the time-dependent Hamilton-Jacobi equation and write the canonical transformation $F_2(q, P, t)$.
c) Find the new variables (Q,P) in terms of the old variables (q,p). Compare the result with that from part (a).