Exercise (Chap 1): Dimensional analysis

Exercise

In order to find the equations giving the position of a freely falling body, the students have giving different equations:

Eq. 1:
$$x = v_0 t + \frac{1}{2} g t^2$$

Eq. 2:
$$x = v_0 t^2 + \frac{1}{2} g t$$

Choose the correct equation by using dimensional analysis.

Answer

If we analyze the first equation dimensionally, we have:

$$[L] = \frac{[L]}{[T]} + \frac{[L]}{[T]^2} + \frac{[L]}{[T]^2} + \frac{[T]^2}{[T]^2}$$
$$[L] = [L] + [L]$$
$$[L] = [L]$$

Eq. 1 is correct with respect to dimensional analysis.

If we analyze the first equation dimensionally, we have:

$$[L] = \frac{[L]}{[T]} [T]^{2} + \frac{[L]}{[T]^{2}} [T]$$
$$[L] = \frac{[L]}{[T]} \frac{[T]}{[T]} [T] + \frac{[L]}{[T][T]} \frac{[T]}{[T]}$$
$$[L] = [L][T] + \frac{[L]}{[T]}$$
$$[L] = [L][T] + \frac{[L]}{[T]}$$

Eq. 2 is meaningless and incorrect with respect to dimensional analysis.