Mathématiques 02 (série de TD N°01)

Exercice N°01

Montrer que F est une primitive de la fonction f dans les cas suivants:

1. $F(x) = x\arcsin(x) + \sqrt{1-x^2}$ et $f(x) = \arcsin(x)$.
2. $F(x) = x\arctan(x) - \dfrac{1}{2}\ln(x^2+1)$ et $f(x) = \arctan(x)$.

Exercice N°02

Calculer les intégrales suivantes

1. $\int \cos(x)\sin^3 x dx$.
2. $(\star)\int \dfrac{(\ln(x))^5}{x}dx$.
3. $\int \cos(x)\sin(x)dx$.
4. $\int \dfrac{1+\ln(x)}{1+x\ln(x)}dx$.
5. $(\star)\int \tan(x)dx$.
6. $(\star)\int \dfrac{\sin(x)}{\cos^2(x)}dx$.
7. $(\star)\int \dfrac{x}{1+x^2}dx$.
8. $\int \dfrac{\cos(x)}{1+\sin^2(x)}dx$.
9. $\int \dfrac{1}{3+x^2}dx$.
10. $(\star)\int \dfrac{e^x}{2+e^{2x}}dx$.
11. $\int \dfrac{1}{\sqrt{1+x^2}}dx$.
12. $\int (e^x)^{n+1}dx$.
13. $\int \dfrac{\cos(\log x)}{x}dx$.
14. $\int \dfrac{\frac{1}{x}}{\sqrt{1-(\ln(x))^2}}dx$.
15. $\int sh(e^x)e^x dx$.
16. $\int x^2e^x dx$.
17. $(\star)\int x^n\ln(x)dx$.
18. $\int_0^\pi x^2\sin(nx)dx$.
19. $(\star)\int (x-1)\cos(x)dx$.
20. $\int \dfrac{1}{\sin(x)}dx$.
21. $(\star)\int \dfrac{1}{\cos(x)}dx$.
22. $\int \sqrt{1+x^2}dx$.
23. $\int \dfrac{1}{x^2-1}dx$.
24. $\int \dfrac{1}{(x^2-3x+2)}dx$.
25. $\int \dfrac{1}{(x^2-2x+3)}dx$.
26. $\int \dfrac{1}{(x-1)^2(x+2)}dx$.
27. $(\star)\int \dfrac{2x}{(x^2+3)(x+2)}dx$.
28. $(\star)\int_2^3 \dfrac{x^2-1}{(x-1)^3(x^2+2)}dx$.
29. $\int \dfrac{x}{\sqrt{x^2+x+2}}dx$.

Exercice N°03

Soient $p, q \in \mathbb{N}^\star$ tel que $I = \int_{-\pi}^{\pi}\cos(px)\cos(qx)dx$; $J = \int_{-\pi}^{\pi}\sin(px)\sin(qx)dx$

1. Calculer $I+J$ et $I-J$
2. D'eduire I et J.

Primitives des fonctions usuelles

la fonction f	Les fonctions primitives F
$x \mapsto a$	$x \mapsto ax + c$
$x \mapsto x^n$	$x \mapsto \frac{1}{n+1}x^{n+1} + c$
$x \mapsto g'(x) \times g(x)^n$	$x \mapsto \frac{1}{n+1}g(x)^{n+1} + c$
$x \mapsto \frac{g(x)'}{g(x)}$	$x \mapsto \ln\lvert g(x)\rvert + c$
$x \mapsto \frac{g(x)'}{g(x)^n}$	$x \mapsto \frac{-1}{(n-1)g(x)^{n-1}} + c$
$x \mapsto e^x$	$x \mapsto e^x + c$
$x \mapsto e^{ax+b}$	$x \mapsto \frac{1}{a}e^{ax+b} + c$
$x \mapsto g'(x)e^{g(x)}$	$x \mapsto e^{g(x)} + c$
$x \mapsto \frac{g'(x)}{2\sqrt{g(x)}}$	$x \mapsto \sqrt{g(x)} + c$
$x \mapsto \frac{1}{1+x^2}$	$x \mapsto \arctan(x) + c$
$x \mapsto \frac{1}{a+x^2}, a > 0$	$x \mapsto \frac{1}{\sqrt{a}}\arctan\left(\frac{x}{\sqrt{a}}\right) + c$
$x \mapsto \frac{g'(x)}{1+g^2(x)}$	$x \mapsto \arctan(g(x)) + c$
$x \mapsto \cos(x)$	$x \mapsto \sin(x) + c$
$x \mapsto \cos(ax+b), a \neq 0$	$x \mapsto \frac{1}{a}\sin(ax+b) + c$
$x \mapsto \sin(x)$	$x \mapsto -\cos(x) + c$
$x \mapsto \sin(ax+b), a \neq 0$	$x \mapsto -\frac{1}{a}\cos(ax+b) + c$
$x \mapsto \frac{1}{\sqrt{1-x^2}}$	$x \mapsto \arcsin(x)$
$x \mapsto \frac{-1}{\sqrt{1-x^2}}$	$x \mapsto \arccos(x)$
$x \mapsto \frac{1}{cos^2(x)}$	$\tan(x)$

•Le changement de variable $t = tan\left(\frac{x}{2}\right)$ **.**

Si on pose $t = tan\left(\frac{x}{2}\right)$, on trouve $\boxed{dx = \frac{2}{1+t^2}dt}$, $\boxed{\sin(x) = \frac{2t}{1+t^2}}$, $\boxed{\tan(x) = \frac{2t}{1-t^2}}$, $\boxed{\cos(x) = \frac{1-t^2}{1+t^2}}$

• Quelques formules de trigonométrie:

1. $\cos(a+b) = \cos(a)\cos(b) - \sin(a)\sin(b)$.
2. $\cos(a-b) = \cos(a)\cos(b) + \sin(a)\sin(b)$.
3. $\sin(a+b) = \sin(a)\cos(b) + \sin(b)\cos(a)$.
4. $\sin(a-b) = \sin(a)\cos(b) - \sin(b)\cos(a)$.
5. $\sin(2a) = 2\sin(a)\cos(a)$.
6. $\cos(2a) = \cos^2(a) - \sin^2(a) = 2cos^2(a) - 1 = 1 - 2sin^2(a)$.
7. $\cos^2(a) = \frac{\cos(2a)+1}{2}$.
8. $\sin^2(a) = \frac{1-\cos(2a)}{2}$.