Université de Mohamed Boudiaf M'sila
Faculté des Sciences et Technologies
Module :Math 02

Année Universitaire 2019-2020
Licence ST LMD (1^{ere} année)

Solutions d'exercices (série de TD N°02)

Solution de l'Exercice 1 *Donner le type des équations différentielles suivantes (sans les résoudre)*

equation	*forme*	*type*
① $xy' = (x-1)\,y$	$xy' - (x-1)\,y = 0$	*(EDLH 01) équation déffirentielle lineaire homogène d'ordre 01*
② $(1+y^2)\,y' = x$	$(1+y^2)\,y' = x$	*(EDS) équation déffirentielle à variables separables*
③ $y'\sin x\cos x - 3y = -3y^{\frac{2}{3}}\sin^3 x$	$y' - \dfrac{3}{\sin x\cos x}y = 3\dfrac{\sin^2 x}{\cos x}y^{\frac{2}{3}}$	*(EDB) équation déffirentielle du Bernoulli,* $\alpha = \dfrac{2}{3}$
④ $y' = \dfrac{x-y}{x+y}$	$y' = \dfrac{1-\frac{y}{x}}{1+\frac{y}{x}} = F\left(\dfrac{y}{x}\right)$	*(EDH) équation déffirentielle homogène*
⑤ $y' - \dfrac{y}{1-x^2} = 1+x$	$y' - \dfrac{y}{1-x^2} - (1+x) = 0$	*(EDL 01) équation déffirentielle lineaire complete*

Solution de l'Exercice 2 *Résoudre les équations suivantes:*

① $$(1+e^x)\,yy' = e^x \iff yy' = \frac{e^x}{1+e^x}$$
$$\iff \int y\,dy = \int \frac{e^x}{1+e^x}dx$$
$$\iff \frac{1}{2}y^2 = \ln(1+e^x) + c,\ \ c \in \mathbb{R}$$
$$\iff y^2 = 2\ln(1+e^x) + 2c \iff \boxed{y = \pm\sqrt{2\ln(1+e^x)+c'}},\ c' = 2c$$

② $$\tan(x)\sin^2(y)\,dx + \cos^2(x)\cot(y)\,dy = 0 \iff \cos^2(x)\cot(y)\,dy = -\tan(x)\sin^2(y)\,dx$$
$$\iff \frac{cot(y)}{\sin^2(y)}dy = -\frac{tan(x)}{\cos^2(y)}dx,\quad \left[cot(x) = \frac{\cos(x)}{\sin(x)}\right]$$
$$\iff \int \frac{\cos(y)}{\sin^2(y)}dy = -\int \frac{\sin(x)}{\cos^2(x)}dx$$
$$\iff \frac{-1}{2\sin^2(y)} = \frac{-1}{2\cos^2(x)} + c$$
$$\iff \frac{1}{\sin^2(y)} = \frac{1}{\cos^2(x)} + c' = \frac{1+c'\cos^2(x)}{\cos^2(x)},\qquad c' = -2c$$
$$\iff \sin^2(y) = \frac{\cos^2(x)}{1+c'\cos^2(x)}$$
$$\iff \boxed{y = \arcsin\left(\pm\sqrt{\frac{\cos^2(x)}{-1+c'\cos^2(x)}}\right)}$$

③ $\frac{e^y}{e^y+1}y' = \frac{1}{x} \iff \int \frac{e^y}{(e^y+1)}dy = \int \frac{1}{x}dx$

$\iff \ln(e^y+1) = \ln|x| + c,\ c \in \mathbb{R}$

$\iff e^{\ln(e^y+1)} = e^c e^{ln|x|}$

$\iff e^y + 1 = \pm e^c x = kx,\ k = \pm e^c$

$\iff e^y = kx - 1$

$\iff \boxed{y = \ln(kx-1)},\quad avec\ x > \frac{1}{k}.$

④ $3e^x \tan(y)\,dx + \frac{1-e^x}{\cos^2(y)}dy = 0 \iff \frac{1-e^x}{cos^2(y)}dy = -3e^x\tan(y)dx$

$\iff \frac{1}{\tan(y)} \times \frac{1}{\cos^2(y)}dy = \frac{-3e^x}{1-e^x}dx$

$\iff \int \frac{1}{\tan(y)}\left(1+\tan^2(y)\right)dy = \int \frac{-3e^x}{1-e^x}dx, \left[\frac{1}{\cos^2(y)} = 1+\tan^2(y)\right]$

$\iff \int \left(\frac{1}{\tan(y)} + \tan(y)\right)dy = 3\ln|1-e^x| + c,\ c \in \mathbb{R}$

$\iff \int \left(\frac{\cos(y)}{\sin(y)} + \frac{\sin(y)}{\cos(y)}\right)dy = \ln|1-e^x|^3 + c,\ c \in \mathbb{R}$

$\iff \ln|\sin(y)| - \ln|\cos(y)| = \ln|1-e^x|^3 + c$

$\iff \ln\left|\frac{\sin(y)}{\cos(y)}\right| = 3\ln|1-e^x| + c$

$\iff \ln|\tan(y)| = \ln\left|(1-e^x)^3\right| + c$

$\iff e^{\ln|\tan(y)|} = e^c e^{\ln\left|(1-e^x)^3\right|}$

$\iff \tan(y)| = k\ln\left|(1-e^x)^3\right|,\ k \pm c$

$\iff \boxed{y = \arctan\left(k\ln\left|(1-e^x)^3\right|\right)}$

⑤ $y'\tan(x) = y \iff \frac{y'}{y} = \frac{1}{\tan(x)} \iff \int \frac{1}{y}dy = \int \frac{\cos(x)}{\sin(x)}dx \iff \ln|y| = \ln|\sin(x)| + c,\ c \in \mathbb{R}$

$\iff y = \pm e^c \sin(x)$

$\iff \boxed{y = k\sin(x)},\ k = \pm e^c$

⑥ $(x^2+1)\,y' = y^2 + 4 \iff \frac{y'}{y^2+4} = \frac{1}{x^2+1}$

$\iff \int \frac{1}{y^2+4}dy = \int \frac{1}{x^2+1}dx$

$\iff \frac{1}{2}\arctan\left(\frac{y}{2}\right) = \arctan(x) + c,\ c \in \mathbb{R}$

$\iff \arctan\left(\frac{y}{2}\right) = 2\arctan(x) + 2c \iff \frac{y}{2} = \tan(2\arctan(x) + k),\ k = 2c$

$\iff \boxed{y = 2\tan(2\arctan(x) + k)}$