Exercice n°1 :

1) :

$$G(p) = \frac{K}{p(p+3)(p+4)}$$

$$G(p) = \frac{K/12}{p\left(\frac{p}{3}+1\right)\left(\frac{p}{4}+1\right)}$$

$$G(j\omega) = \frac{K/12}{j\omega\left(j\frac{\omega}{3}+1\right)\left(j\frac{\omega}{4}+1\right)}$$

$$|G(j\omega)|_{db} = 20\log\frac{K}{12} - 20\log\omega - 20\log\sqrt{\left(\frac{\omega}{3}\right)^2+1} - 20\log\sqrt{\left(\frac{\omega}{4}\right)^2+1}$$

$$\mathrm{Arg}\,G(j\omega) = 0 - \frac{\pi}{2} - \mathrm{Arctg}\frac{\omega}{3} - \mathrm{Arctg}\frac{\omega}{4}$$

$$|F_1|_{db} = 20\log\frac{K}{12} - 20\log\omega \qquad \mathrm{Arg}(F_1) = -\frac{\pi}{2}$$

$$|F_2|_{db} = -20\log\sqrt{\left(\frac{\omega}{3}\right)^2+1} \qquad \mathrm{Arg}(F_2) = -\mathrm{Arctg}\left(\frac{\omega}{3}\right)$$

$$|F_3|_{db} = -20\log\sqrt{\left(\frac{\omega}{4}\right)^2+1} \qquad \mathrm{Arg}(F_3) = -\mathrm{Arctg}\left(\frac{\omega}{4}\right).$$

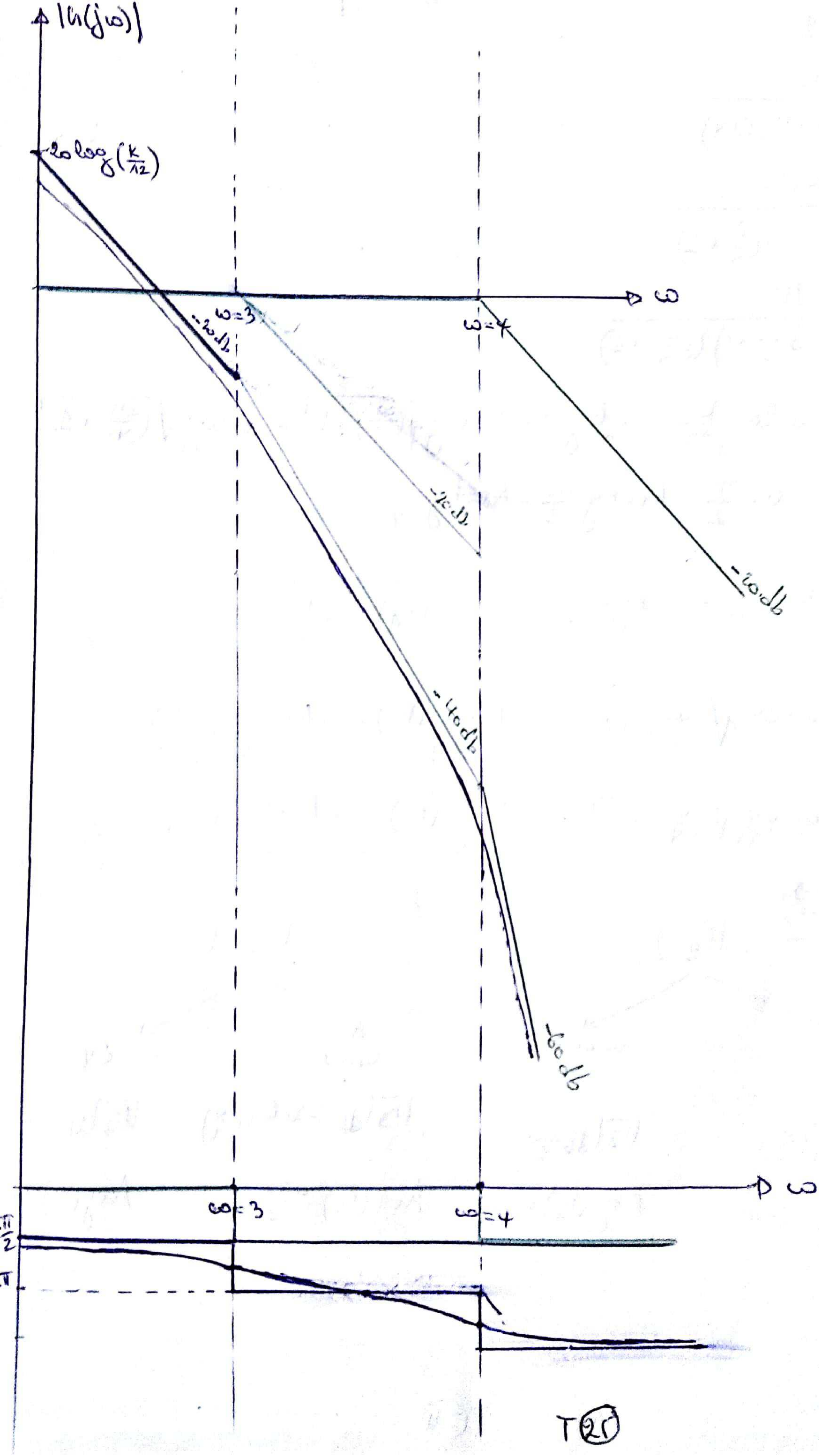
Les Asymptotes :

$|F_2|_{db}$

- $\omega \gg 3$: $|F_2|_{db} = -20\log\left(\frac{\omega}{3}\right)$; $\mathrm{Arg}(F_2) = -\frac{\pi}{2}$
- $\omega \ll 3$: $|F_2|_{db} = 0$; $\mathrm{Arg}(F_2) = 0$

$|F_3|_{db}$

- $\omega \gg 4$: $|F_3|_{db} = -20\log\left(\frac{\omega}{4}\right)$; $\mathrm{Arg}(F_3) = -\frac{\pi}{2}$
- $\omega \ll 4$: $|F_3|_{db} = 0$; $\mathrm{Arg}(F_3) = 0$



T(21)

Diagramme de Nyquist : (Exercice 02)

1) $$G_1(p) = \frac{1+p}{p^2(1+2p)(1+3p)}$$

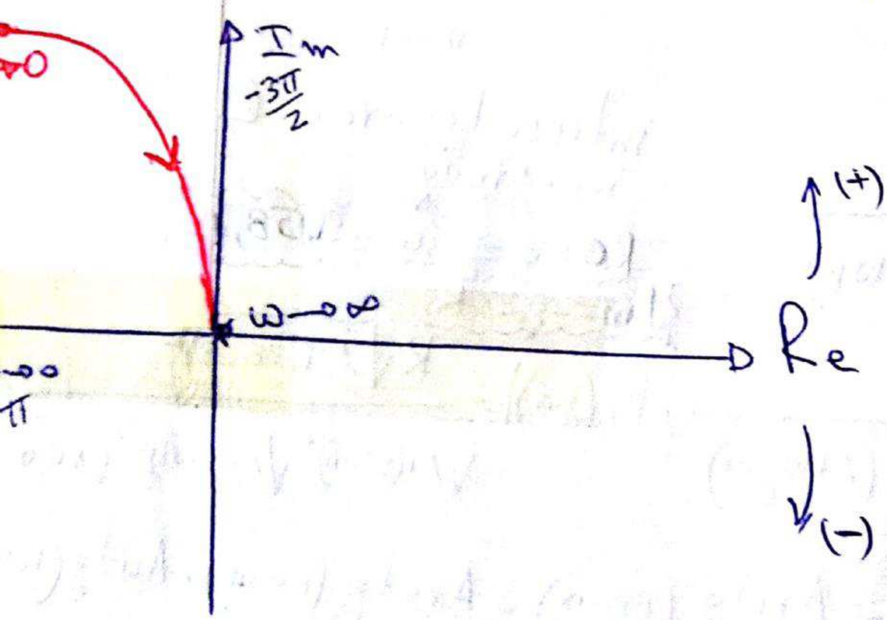
$$G_1(j\omega) = \frac{1+j\omega}{-\omega^2(1+2j\omega)(1+3j\omega)}$$

$$|G_1(j\omega)| = \frac{\sqrt{1+\omega^2}}{\omega^2\left(\sqrt{1+(2\omega)^2}\right)\left(\sqrt{1+9\omega^2}\right)} = \frac{\sqrt{1+\omega^2}}{\omega^2\sqrt{1+4\omega^2}\sqrt{1+9\omega^2}}$$

$$\mathrm{Arg}(G_1(j\omega)) = \mathrm{Arctg}\,\omega - \left[\frac{\pi}{2}+\frac{\pi}{2}+\mathrm{Arctg}(2\omega)+\mathrm{Arctg}\,3\omega\right]$$

$$\omega \to 0 \left\{ |G_1(j\omega)| = \infty \quad \mathrm{Arg}(G_1(j\omega)) = -\pi \right.$$

$$\omega \to \infty \left\{ |G_1(j\omega)| = 0 \quad \mathrm{Arg}(G_1(j\omega)) = -\frac{3\pi}{2} \right.$$



$$G_1(p) = \overbrace{\frac{-(1-\omega^2)}{\omega^2(1+4\omega^2)(1+9\omega^2)}}^{Re} + j\underbrace{\frac{(4+6\omega^2)}{\omega(1+4\omega^2)(1+9\omega^2)}}_{Im}$$

$$\omega \to 0 \left\{ \begin{array}{l} Re = -\infty \\ Im = +\infty \end{array} \right.$$

$$\omega \to \infty \left\{ \begin{array}{l} Re = 0 \\ Im = 0 \end{array} \right.$$

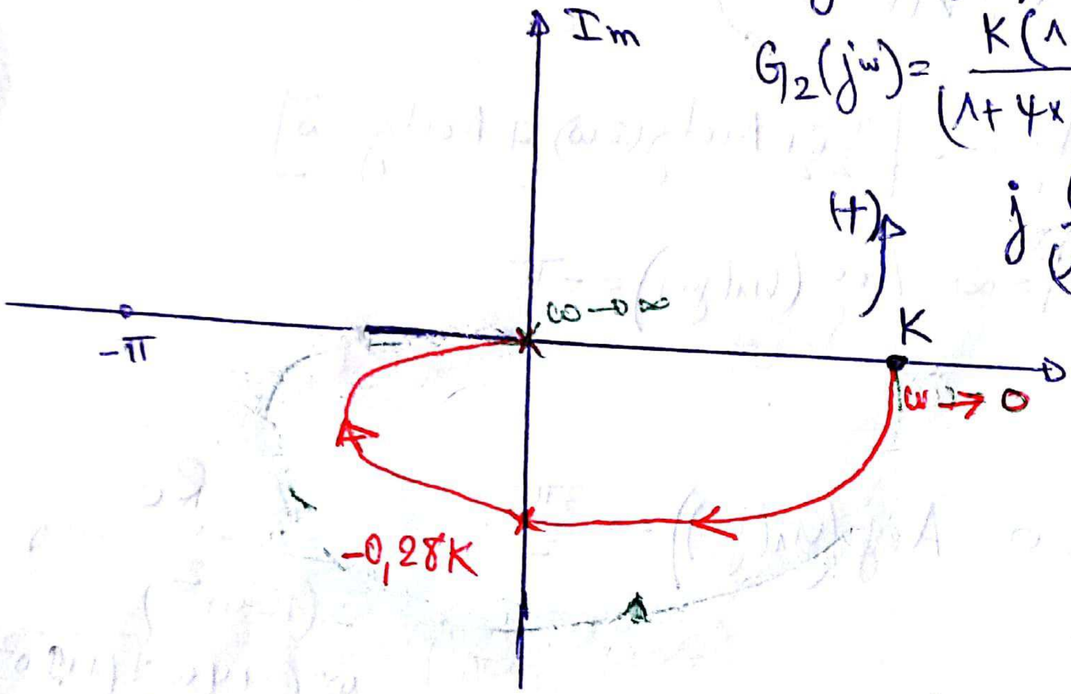
2) $$G_2(p) = \frac{K}{(1+0{,}02p)(1+0{,}002p)}, \quad G_2(j\omega) = \frac{K}{(1+0{,}02j\omega)(1+0{,}002j\omega)}$$

$$|G_2(j\omega)| = \frac{K}{\sqrt{1+(0{,}02\omega)^2}\sqrt{1+(0{,}002\omega)^2}}$$

$$\text{Arg}(G_2(j\omega)) = -\text{Arctg}\,0{,}02\omega - \text{Arctg}\,0{,}002\omega$$

$\omega = 0 \Rightarrow |G_2(j\omega)| = K \quad \text{Arg}(G_2(j\omega)) = 0$

$\omega \to \infty \quad |G_2(j\omega)| = 0 \quad \text{Arg}(G_2(j\omega)) = -\pi$



$$G_2(j\omega) = \frac{K(1-4\times10^{-5}\omega^2)}{(1+4\times10^{-4}\omega^2)(1+4\times10^{-6}\omega^2)} + j\frac{(-0{,}022\omega)K}{(1+4\times10^{-4}\omega^2)(1+4\times10^{-6}\omega^2)}$$

$\omega \to 0^+ \begin{cases} Re = K \\ Im = 0 \end{cases}$

$\omega \to \infty \begin{cases} Re = 0 \\ Im = 0 \end{cases}$

intersection avec l'axe imaginaire

$Re = 0 \Rightarrow \boxed{\omega = 158{,}11} \Rightarrow \boxed{Im = -0{,}28K}$

(3) $$G_3(p) = \frac{K(1-20p)}{(1+20p)(1+60p)(1+10p)}$$

$$G_3(j\omega) = \frac{K(1-20j\omega)}{(1+20j\omega)(1+60j\omega)(1+10j\omega)}, \quad |G_3(j\omega)| = \frac{K\cancel{\sqrt{1+(20\omega)^2}}}{\sqrt{1+(60\omega)^2}\cancel{\sqrt{1+(20\omega)^2}}\sqrt{1+(10\omega)^2}}$$

$$\text{Arg}(G_3(j\omega)) = \text{Arctg}(-20\omega) - \text{Arctg}(20\omega) - \text{Arctg}(60\omega) - \text{Arctg}(10\omega)$$

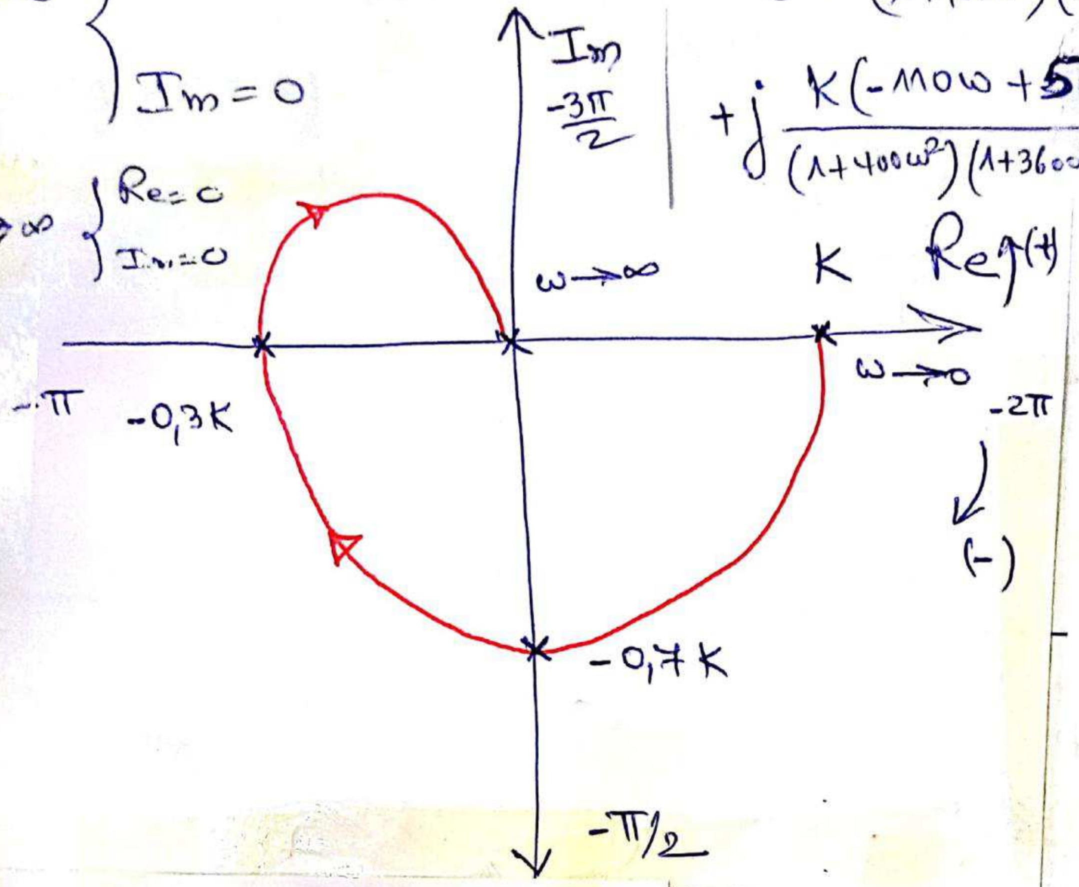
$\omega \to 0 \begin{cases} |G_3(j\omega)| = K \\ \text{Arg}(G_3(j\omega)) = 0 \end{cases}$

$\omega \to \infty \begin{cases} |G_3(j\omega)| = 0 \\ \text{Arg}(G_3(j\omega)) = -2\pi \end{cases}$

$$G_3(j\omega) = \frac{K(1-3800\omega^2 + 24\times10^4\omega^4)}{(1+400\omega^2)(1+3600\omega^2)(1+100\omega^2)} + j\,\frac{K(-110\omega + 52\times10^3\omega^3)}{(1+400\omega^2)(1+3600\omega^2)(1+100\omega^2)}$$

$\omega \to 0 \begin{cases} Re = K \\ Im = 0 \end{cases}$

$\omega \to \infty \begin{cases} Re = 0 \\ Im = 0 \end{cases}$



→ le pt d'insersection avec l'axe reele

$Im = 0 \Rightarrow \boxed{\omega = 0{,}046} \Rightarrow \boxed{Re = -0{,}30\,K}$

→ le pt d'intersection avec l'ax Imaginaire

$Re = 0 \Rightarrow \boxed{\omega = 0{,}0164} \Rightarrow \boxed{Im = -0{,}7\,K}$

Exercice n° 2 =

$$G_1(p) = \frac{K}{p(1+T_1p)(1+T_2p)} = \frac{K}{T_1T_2p^3 + (T_1+T_2)p^2 + p}$$

-a- Critère de Routh - Hurwitz :

$$F.T.B.F = \frac{K}{(T_1T_2)p^3 + (T_1+T_2)p^2 + p + K}$$

CNNS = $K > 0$

C.N.S

$$\begin{array}{lll} p^3 & T_1T_2 & 1 \\ p^2 & T_1+T_2 & K \\ p^1 & \frac{(T_1+T_2) - KT_1T_2}{T_1+T_2} & \\ p^0 & K & \end{array}$$

syst stable $\Rightarrow (T_1+T_2) > KT_1T_2$

$$\Rightarrow KT_1T_2 < (T_1+T_2)$$

$$\Rightarrow \boxed{K < \frac{T_1+T_2}{T_1T_2}}$$

Le diagramme de Bode = possant $\tau_1 = \frac{1}{T_1}$; $\tau_2 = \frac{1}{T_2}$

avec $\tau_1 < \tau_2$

$$G_1(p) = \frac{K}{p\left(1+\frac{p}{\tau_1}\right)\left(1+\frac{p}{\tau_2}\right)}$$

$$G_1(j\omega) = \frac{K}{j\omega\left(1+j\frac{\omega}{\tau_1}\right)\left(1+j\frac{\omega}{\tau_2}\right)}, \quad |G_1(j\omega)|_{db} = \underbrace{20\log(K) - 20\log\omega}_{1} \; \underbrace{-20\log\sqrt{1+\left(\frac{\omega}{\tau_1}\right)^2}}_{2} \; \underbrace{-20\log\sqrt{1+\left(\frac{\omega}{\tau_2}\right)^2}}_{3}$$

$$\mathrm{Arg}\left[G_1(j\omega)\right] = \underbrace{-\frac{\pi}{2}}_{1} \underbrace{-\,\mathrm{Arctg}\left(\frac{\omega}{z_1}\right)}_{2} \underbrace{-\,\mathrm{Arctg}\left(\frac{\omega}{z_2}\right)}_{3}$$

$|F_2|_{db}$

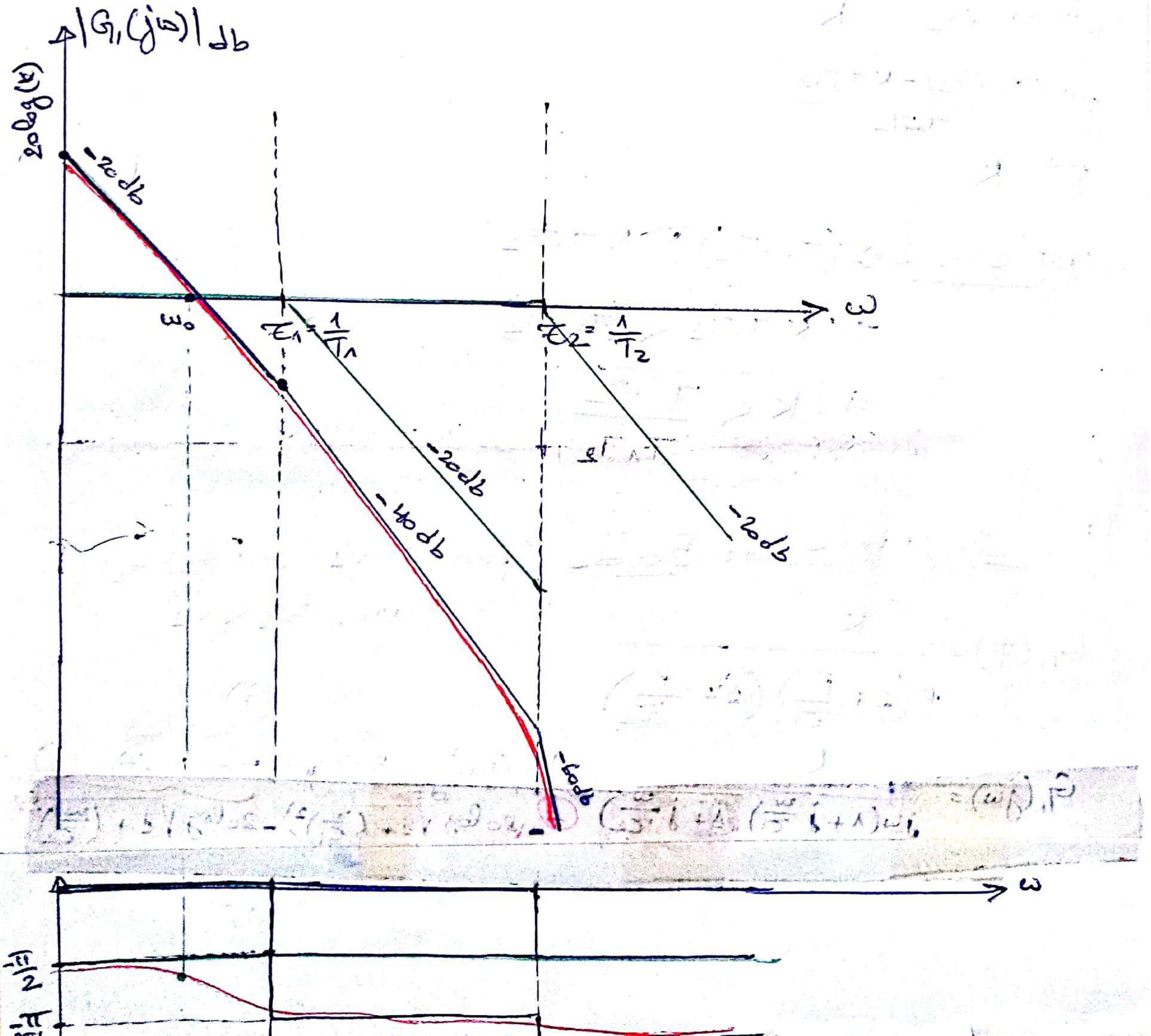
$\omega \gg z_1$: $|F_2|_{db} = -20\log\left(\frac{\omega}{z_1}\right)$, $\mathrm{Arg}(F_2) = -\frac{\pi}{2}$

$\omega \ll z_1$: $|F_2|_{db} = 0$, $\mathrm{Arg}(F_2) = 0$

$|F_3|_{db}$

$\omega \gg z_2$: $|F_3|_{db} = -20\log\left(\frac{\omega}{z_2}\right)$, $\mathrm{Arg}(F_3) = -\frac{\pi}{2}$

$\omega \ll z_2$: $|F_3|_{db} = 0$, $\mathrm{Arg}(F_3) = 0$



Le systéme est stable selon le diagramme de Bode

K << Klimite → syst stable

K >> Klimite → syst instable

$K = K_{limite}$ $(\omega = \omega_0)$ on a : $\begin{cases} |G_1 j(\omega)| = 1 \\ \angle G_1(j\omega) = -\pi \end{cases}$

$$Arg[G_1(j\omega_0)] = -\frac{\pi}{2} - Arctg\left(\frac{\omega_0}{\tau_1}\right) - Arctg\left(\frac{\omega_0}{\tau_2}\right) = -\pi$$

$$+Arctg\left(\frac{\omega_0}{\tau_1}\right) + Arctg\left(\frac{\omega_0}{\tau_2}\right) = -\pi + \frac{\pi}{2} = +\frac{\pi}{2}$$
$$= Arctg(\infty)$$

$$Arctg(a) + Arctg(b) = arctg\left(\frac{a+b}{1-ab}\right)$$

$$arctg\left(\frac{\frac{\omega_0}{\tau_1} + \frac{\omega_0}{\tau_2}}{1 - \frac{\omega_0}{\tau_1} \cdot \frac{\omega_0}{\tau_2}}\right) = Arctg(\infty)$$

$$\Rightarrow 1 - \frac{\omega_0^2}{\tau_1 \tau_2} = 0 \Rightarrow \omega_0^2 = \tau_1 \tau_2 = \frac{1}{T_1 T_2}$$

$$\Rightarrow \omega_0 = \frac{1}{\sqrt{T_1 T_2}}$$

$$\frac{K_{lim}}{\omega_0\sqrt{1+\frac{\omega_0^2}{\tau_1^2}}\sqrt{1+\frac{\omega_0^2}{\tau_2^2}}}=1$$

On ramplaçant $\omega_0=\frac{1}{\sqrt{T_1T_2}}$ on trouve =

$$\frac{K_{lim}\sqrt{T_1T_2}}{\frac{T_1+T_2}{\sqrt{T_1T_2}}}=1 \Rightarrow \boxed{K_{lim}=\frac{T_1+T_2}{T_1T_2}}$$

$K<\frac{T_1+T_2}{T_1T_2}$ syst stable

$K>\frac{T_1+T_2}{T_1T_2}$ syst instable.

$K=\frac{T_1+T_2}{T_1T_2}$ limite de stabilité

$$\Delta\Phi=\pi+\mathrm{Arg}\left[T(j\omega_0)\right]$$

$$\Delta\Phi=\pi-\frac{\pi}{2}-\mathrm{Arctg}\left(\frac{T_1}{\sqrt{T_1T_2}}\right)-\mathrm{Arctg}\left(\frac{T_2}{\sqrt{T_1T_2}}\right)$$

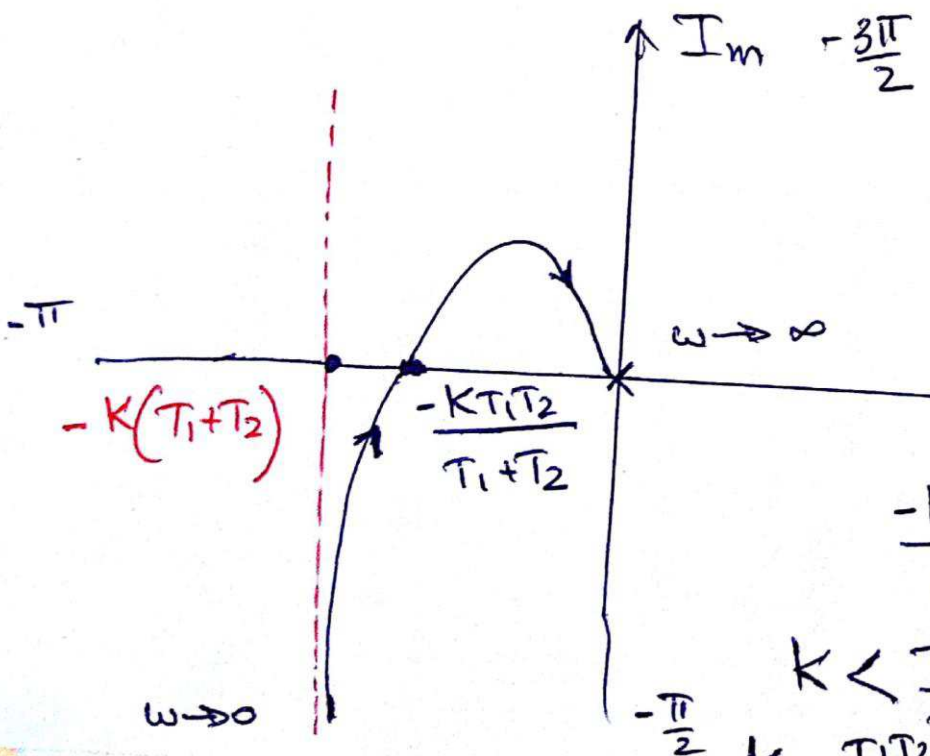
$$\boxed{\Delta\Phi=\frac{\pi}{2}-\mathrm{Arctg}\left(\frac{T_1}{\sqrt{T_1T_2}}\right)-\mathrm{Arctg}\left(\frac{T_2}{\sqrt{T_1T_2}}\right)}$$

→ Stabilité par Nyquist :

$$G_1(j\omega) = \underbrace{\frac{-K(T_1+T_2)}{\left[1+(\omega T_1)^2\right]\left[1+(\omega T_2)^2\right]}}_{Re} + j\,\underbrace{\frac{K(\omega^2 T_1 T_2 - 1)}{\omega\left[1+(\omega T_1)^2\right]\left[1+(\omega T_2)^2\right]}}_{Im}$$

$$\omega \to 0 \quad \begin{cases} \lim\limits_{\omega\to 0} Re = -K(T_1+T_2) \\ \lim\limits_{\omega\to 0} Im = -\infty \end{cases} \quad , \ Arg\left[G_1(j\omega)\right] = \frac{-\pi}{2}$$

$$\omega \to \infty \quad \begin{cases} \lim\limits_{\omega\to \infty} Re = 0 \\ \lim\limits_{\omega\to \infty} Im = 0 \end{cases} \quad Arg\left[G_1(j\omega)\right] = \frac{-3\pi}{2}$$



$Im = 0 \Rightarrow \omega = \frac{1}{\sqrt{T_1T_2}}$

$\Rightarrow Re = \frac{-K(T_1T_2)}{(T_1+T_2)}$

$\frac{-KT_1T_2}{T_1+T_2} > -1$ → syst stable

$K < \frac{T_1T_2}{T_1+T_2}$ syst stable

$K = \frac{T_1T_2}{T_1+T_2}$ limite de stabilité