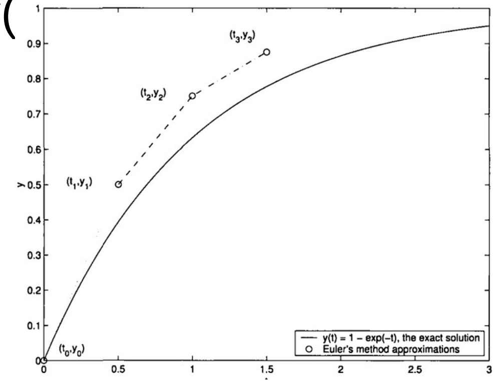
IV. Ordinary Differential equations

We are concerned here with the problem of solving differential equations, numerically. At first we concentrate on the ***initial value problem*** : Find a function y(

$$\frac{dy}{dt} = f(t, y(t)), \quad y(t_0) = y_0,$$



Geometric derivation of Euler's method.

Euler's method

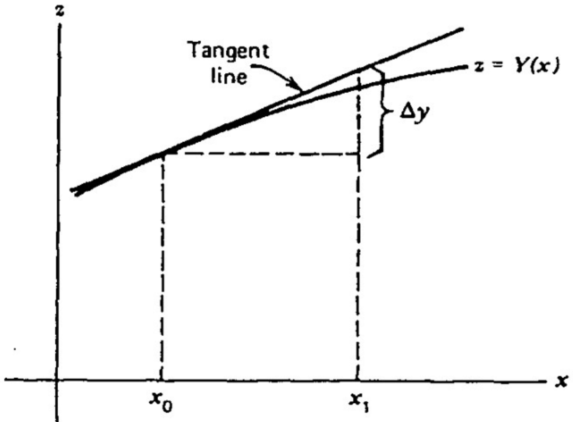
$$y' = f(t, y), \quad y(t_0) = y_0 \quad y(t_n + h) \approx y(t_n) + hf(t_n, y(t_n)).$$

heun's method

$$y_{n+1} = y_n + \frac{h}{2}\{f(x_n, y_n) + f[x_n + h, y_n + hf(x_n, y_n)]\}$$

Modified Euler's method

$$y_{n+1} = y_n + hf[x_n + \frac{h}{2}, y_n + \frac{h}{2}f(x_n, y_n)]$$



Geometric interpretation of Euler's method

Runge-Kutta method

$$y' = f(t, y), \quad y(t_0) = y_0$$

RK2

$$y_{n+1} = y_n + \frac{h}{2}\left[f(x_n, y_n) + f(x_{n+1}, y_n + hf(x_n, y_n))\right]$$

RK4

$$\begin{aligned}
k_1 &= hf(t_n, y_n), \\
k_2 &= hf\left(t_n + \frac{1}{2}h, y_n + \frac{1}{2}k_1\right), \\
k_3 &= hf\left(t_n + \frac{1}{2}h, y_n + \frac{1}{2}k_2\right), \\
k_4 &= hf(t_n + h, y_n + k_3), \\
y_{n+1} &= y_n + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4).
\end{aligned}$$

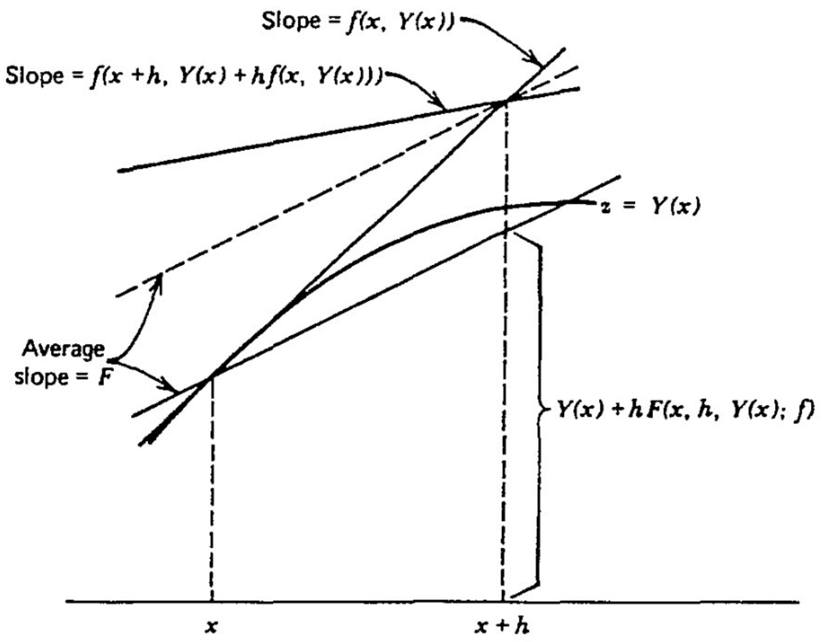


Illustration of Runge-Kutta method