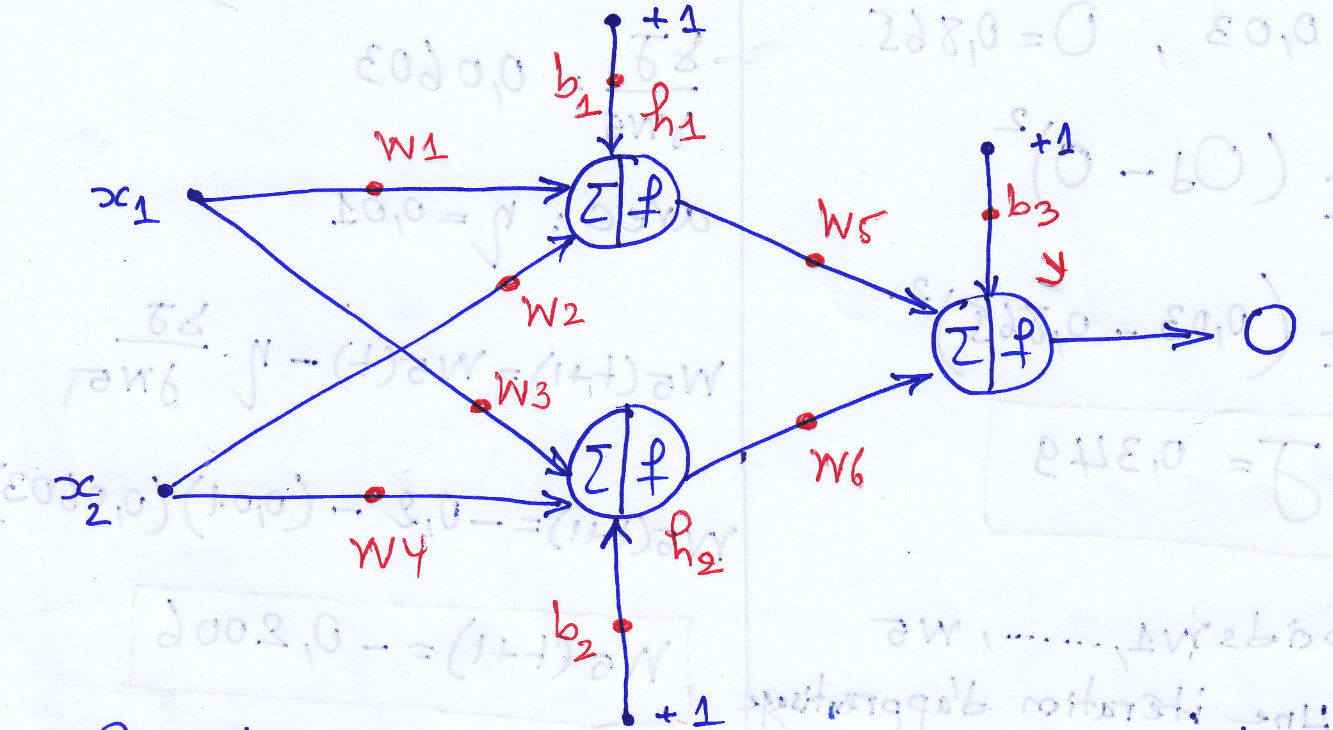
Exercice n=9

1- Calcule la sortie O :



Couche d'entré   Couche caché   couche de sortie

$$h_1 = x_1 \cdot w_1 + x_2 \cdot w_2 + b_1 = (0{,}1)(0{,}5) + (0{,}3)(0{,}1) + 0{,}4$$

$$\boxed{h_1 = 0{,}48}$$

$$O_{h_1} = \frac{1}{1 + e^{-h_1}} = 0{,}618 \Rightarrow \boxed{O_{h_1} = 0{,}618}$$

$$h_2 = x_1 \cdot w_3 + x_2 w_4 + b_2 = (0{,}1)(0{,}62) + (0{,}3)(0{,}2) - 0{,}1 \Rightarrow \boxed{h_2 = 0{,}022}$$

$$O_{h_2} = \frac{1}{1 + e^{-h_2}} \Rightarrow \boxed{O_{h_2} = 0{,}506}$$

$$y = w_5 \cdot O_{h_1} + w_6 \cdot O_{h_2} + b_3 = (-0{,}2)(0{,}618) + (0{,}506)(0{,}3) + 1{,}83$$

$$\boxed{y = 1{,}858} \qquad O = \frac{1}{1 + e^{-y}} \Rightarrow \boxed{O = 0{,}865}$$

2- Determination de l'erreur Quadratique :

$O_d = 0,03$, $O = 0,865$

$$J = \frac{1}{2}(O_d - O)^2$$

$$J = \frac{1}{2}(0,03 - 0,865)^2$$

$$\Rightarrow \boxed{J = 0,349}$$

3- les poids W1, ..., W5 aprés une itération d'apprentissage En utilisant la rétropropagation de l'erreur : avec ($\eta = 0,01$)

On commance par W5 :

$$W_5(t+1) = W_5(t) - \eta \frac{\partial J}{\partial W_5}$$

$$\frac{\partial J}{\partial W_5} = \frac{\partial J}{\partial O} \cdot \frac{\partial O}{\partial y} \cdot \frac{\partial y}{\partial W_5}$$

$$\frac{\partial J}{\partial O} = -(O_d - O) = O - O_d = 0,835$$

$$\frac{\partial O}{\partial y} = O \cdot (1 - O) = 0,1168$$

$$\frac{\partial y}{\partial W_5} = O_{h1} = 0,618$$

alors :

$$\frac{\partial J}{\partial W_5} = (0,835) \cdot (0,1168) \cdot (0,618)$$

$$\frac{\partial J}{\partial W_5} = 0,0603$$

avec : $\eta = 0,01$

$$W_5(t+1) = W_5(t) - \eta \frac{\partial J}{\partial W_5}$$

$$W_5(t+1) = -0,2 - (0,01)(0,0603)$$

$$\boxed{W_5(t+1) = -0,2006}$$

* - Adaptation de W6

$$W_6(t+1) = W_6(t) - \eta \frac{\partial J}{\partial W_6}$$

$$\frac{\partial J}{\partial W_6} = \frac{\partial J}{\partial O} \cdot \frac{\partial O}{\partial y} \cdot \frac{\partial y}{\partial W_6}$$

$$\frac{\partial y}{\partial W_6} = O_{h_2} = 0,506$$

$$\frac{\partial J}{\partial W_6} = (0,835)(0,1168)(0,506)$$

$$\frac{\partial J}{\partial W_6} = 0,049$$

$$W_6(t+1) = W_6(t) - \eta \frac{\partial J}{\partial W_6}$$

$$W_6(t+1) = 0,3 - (0,01)(0,049)$$

$$\boxed{W_6(t+1) = 0,2995}$$

* Adaptation de W1 :

$$W_1(t+1) = W_1(t) - \eta \frac{\partial J}{\partial W_1}$$

$$\frac{\partial J}{\partial W_1} = \boxed{\frac{\partial J}{\partial O} \cdot \frac{\partial O}{\partial y} \cdot \frac{\partial y}{\partial O_{h_1}} \cdot \frac{\partial O_{h_1}}{\partial h_1} \cdot \frac{\partial h_1}{\partial W_1}}$$

$$\frac{\partial J}{\partial O} = 0{,}835$$

$$\frac{\partial O}{\partial y} = 0{,}1168$$

$$\frac{\partial y}{\partial O_{h_1}} = W_5 = -0{,}2$$

$$\frac{\partial O_{h_1}}{\partial h_1} = O_{h_1}(1 - O_{h_1}) = 0{,}2361$$

$$\frac{\partial h_1}{\partial W_1} = x_1 = 0{,}1$$

alors :

$$\frac{\partial J}{\partial W_1} = (0{,}835)(0{,}1168)(-0{,}2)(0{,}2361) \cdot (0{,}1)$$

$$\boxed{\frac{\partial J}{\partial W_1} = -4{,}6053 \times 10^{-4}}$$

$$W_1(t+1) = 0{,}5 - (0{,}01)(-4{,}6053 \times 10^{-4})$$

$$\boxed{W_1(t+1) = 0{,}5000046053}$$

* Adaptation de W2

$$\frac{\partial J}{\partial W_2} = \frac{\partial J}{\partial O} \cdot \frac{\partial O}{\partial y} \cdot \frac{\partial y}{\partial O_{h_1}} \cdot \frac{\partial O_{h_1}}{\partial h_1} \cdot \frac{\partial h_1}{\partial W_2}$$

$$\frac{\partial J}{\partial O} = 0{,}835$$

$$\frac{\partial O}{\partial y} = 0{,}1168$$

$$\frac{\partial y}{\partial O_{h_1}} = W_5 = -0{,}2$$

$$\frac{\partial O_{h_1}}{\partial h_1} = 0{,}2361$$

$$\frac{\partial h_1}{\partial W_2} = x_2 = 0{,}3$$

alors

$$\frac{\partial J}{\partial W_2} = (0{,}835)(0{,}1168)(-0{,}2)(0{,}2361)(0{,}3)$$

$$\boxed{\frac{\partial J}{\partial W_2} = -0{,}0014}$$

$$W_2(t+1) = 0{,}1 - \left[(0{,}01)(-0{,}0014)\right]$$

$$\boxed{W_2(t+1) = 0{,}100014}$$

* Adaptation de W3

$$\frac{\partial J}{\partial W_3} = \frac{\partial J}{\partial O} \cdot \frac{\partial O}{\partial y} \cdot \frac{\partial y}{\partial O_{h_2}} \cdot \frac{\partial O_{h_2}}{\partial h_2} \cdot \frac{\partial h_2}{\partial W_3}$$

$$\frac{\partial J}{\partial O} = 0{,}835$$

$$\frac{\partial O}{\partial y} = 0{,}1168$$

$$\frac{\partial h_2}{\partial W_3} = x_1 = 0{,}1$$

$$\frac{\partial y}{\partial O_{h_2}} = W_6 = 0{,}3$$

$$\frac{\partial O_{h_2}}{\partial h_2} = O_{h_2}(1 - O_{h_2}) = 0{,}25$$

$$\frac{\partial J}{\partial w_3} = (0{,}835)(0{,}1168)(0{,}3)(0{,}25)(0{,}1)$$

$$\frac{\partial J}{\partial w_3} = 7{,}3146 \times 10^{-4}$$

$$W_3(t+1) = 0{,}62 - (0{,}01)(7{,}3146 \times 10^{-4})$$

$$\boxed{W_3(t+1) = 0{,}6199}$$

* Adaptation de W_4

$$\frac{\partial J}{\partial w_4} = \frac{\partial J}{\partial O} \cdot \frac{\partial O}{\partial y} \cdot \frac{\partial y}{\partial O_{h_2}} \cdot \frac{\partial O_{h_2}}{\partial h_2} \cdot \frac{\partial h_2}{\partial w_4}$$

$$\frac{\partial J}{\partial O} = 0{,}835$$

$$\frac{\partial O}{\partial y} = 0{,}1168$$

$$\frac{\partial y}{\partial O_{h_2}} = 0{,}3$$

$$\frac{\partial O_{h_2}}{\partial h_2} = O_{h_2}(1 - O_{h_2}) = 0{,}25$$

$$\frac{\partial h_2}{\partial w_4} = x_2 = 0{,}3$$

$$\frac{\partial J}{\partial w_4} = (0{,}835)(0{,}1168)(0{,}3)(0{,}25)(0{,}3)$$

$$\boxed{\frac{\partial J}{\partial w_4} = 0{,}0022}$$

$$W_4(t+1) = 0{,}2 - (0{,}01)(0{,}0022)$$

$$\boxed{W_4(t+1) = 0{,}199978}$$

Sous forme Matricel :

La sortie O du résaux s'écrit sous la forme

$$O = f\left[Z \cdot O_h + Z_0\right]$$

tq $O_h = f\left[\underbrace{W \cdot X + W_0}_{h}\right]$

avec : $\begin{cases} W\ (4\times 2) \quad X\ (2\times 1) \\ Z\ (1\times 2) \\ Z_0\ (1\times 1) \\ W_0\ (2\times 1) \end{cases}$

$$h = W \cdot X + W_0 = \begin{bmatrix} h_1 \\ h_2 \end{bmatrix}$$

$$O_h = \begin{pmatrix} f(h_1) \\ f(h_2) \end{pmatrix}$$

$$W = \begin{bmatrix} w_1 & w_2 \\ w_3 & w_4 \end{bmatrix} ; \quad X = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

$$W_0 = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix} ; \quad Z = \begin{bmatrix} w_5 & w_6 \end{bmatrix}$$

$$Z_0 = b_3$$

l'adaptation sous forme matriciel

$$\begin{cases} Z(t+1) = Z(t) - \eta \underbrace{(O - O_d) \cdot O \cdot (1 - O)}_{\delta_s} \cdot O_h^T \\ W(t+1) = W(t) - \eta \cdot \delta_s \cdot Z^T .* O_h .* (1 - O_h) \cdot x^T \end{cases}$$