First-year Master of Theoretical Physics

**Quantum Field Theory**

The college year 2020-2021

Chapter 1

# Canonical quantization of the real scalar field

**التكميم القانوني للحقول السلمية الحقيقية**

## A Review of special relativity

مراجعة للنسبية الخاصة

The coordinates of an object or event in 4-dimensional space-time, which known by Minkowski space, form a *contravariant* four-vector whose four components have upper indices :

إحداثيات كائن أو حدث في الفضاء الزماني-المكاني رباعي الأبعاد ، المعروف بفضاء مينكوفسكي ، تشكل شعاعا مخالف للتغير رباعي حيث تكون أدلته علوية:

$$x^{\mu}\left(x^{0}\equiv ct, x^{1}\equiv x, x^{2}\equiv y, x^{3}\equiv z\right) \tag{1}$$

Here $c$ represents the speed of light in a vacuum. The *covariant* four-vector whose four covariant have lower indices :

هنا $c$ تمثل سرعة الضوء في الفراغ. المتجه الرباعي تكون أدلته سفلية يسمى شعاع رباعي موافق التغير:

$$x_{\mu}\left(x_{0}=x^{0}, x_{1}=-x^{1}\equiv -x, x_{2}=-x^{2}\equiv -y, x_{3}=-x^{3}\equiv -z\right) \tag{2}$$

In general, this term can be generalized to all quadratic vectors in the Minkowski four-dimensional space :

و بشكل عام يمكن تعميم هذا الاصطلاح على كل الاشعة الرباعية في فضاء مينكوفسكي رباعي البعد:

$$\begin{aligned} &a^{\mu}\left(a^{0}, a^{1}, a^{2}, a^{3}\right) \\ &a_{\mu}\left(a_{0}=a^{0}, a_{1}=-a^{1}, a_{2}=a^{2}, a_{3}=-a^{3}\right) \end{aligned} \tag{3}$$

A *contravariant* four-vector (upper indices) are related to covariant four vector (lower indices) with the metric tensor $g^{\mu\nu}$ as follows :

ترتبط الاشعة الرباعية المخالفة التغير(الأدلة العلوية) بالأشعة الرباعية الموافقة التغير(الأدلة السفلية) عن طريق مترية الفضاء كمايلي:

$$a^{\mu} = \sum_{\nu=0}^{3} g^{\mu\nu} a_{\nu} \equiv g^{\mu\nu} a_{\nu} \qquad (4)$$

As the index repeated twice, once upper and lower, is considered a sum index without mentioning the sum symbol (the Einstien summation convention).

حيث أن الدليل المتكرر مرتين مرة في الاعلى و أخرى في الاسفل يعتبر دليل جمع بدون ذكر رمز الجمع (أصطلاح إنشتين للجمع)

the metric tensor $g^{\mu\nu}$ in 4-dimensional space-time, which known by Minkowski space as follows :

$$g^{\mu\nu} \equiv g_{\mu\nu} = \begin{pmatrix} +1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} \equiv diag(+,-,-,-) \qquad (5)$$

The metric tensor $g^{\mu\nu}$ satisfies the following orthogonality condition :

علاقة التعامد التالية: $g^{\mu\nu}$ تحقق مترية الفضاء

$$g^{\mu\nu} g_{\nu\alpha} = \delta^{\mu}_{\alpha} = \begin{cases} 1 & \text{for} \quad \mu = \alpha \\ 0 & \text{for} \quad \mu \neq \alpha \end{cases}$$

Now, the scalar product in Minkowski space is defined as follows :

الان الجداء السلمي في فضاء مينكوفسكي يعرف كمايلي:

$$\begin{aligned} AB \equiv A^{\mu} B_{\mu} = A_{\mu} B^{\mu} = g_{\mu\nu} A^{\mu} B^{\nu} = g^{\mu\nu} A_{\mu} B \nu \\ = A^0 B^0 - A_x B_x - A_y B_y - A_z B_z \end{aligned} \qquad (6)$$

Lorentz transformations are linear transformations on the components of four-vectors which leave invariant scalar product :

تحويلات لورنتز هي علاقات خطية بين الاشعة الرباعية المقاسة في جمل عطالية مختلفة في فضاء مينكوفسكي و التي تحفظ قيمة الجداء السلمي:

$$A'^{\mu} = \Lambda^{\mu}{}_{\nu} A^{\nu}$$
$$x'^{\mu} = \Lambda^{\mu}{}_{\nu} x^{\nu} \qquad (7)$$

Where $\Lambda^{\mu}{}_{\nu}$ is given by :

$$\Lambda^{\mu}{}_{\nu} = \frac{\partial x'^{\nu}}{\partial x^{\nu}} \qquad (8)$$

When the inertial reference R' is moved parallel to (Ox) axis of the reference (R) we have the special case :

عندما تتحرك الجملة العطالية R' وفق المحور (Ox) للجملة العطالية (R) لدينا الحالة الخاصة المعروفة:

$$\begin{cases} A'^{0} = \gamma\left(A^{0} - \beta A_{x}\right) \\ A'_{x} = \gamma\left(A_{x} - \beta A^{0}\right) \\ A'_{y} = A_{y} \\ A'_{z} = A_{z} \end{cases} \Rightarrow \begin{cases} A^{0} = \gamma\left(A'^{0} + \beta A'_{x}\right) \\ A_{x} = \gamma\left(A'_{x} + \beta A'^{0}\right) \\ A_{y} = A'_{y} \\ A'_{z} = A_{z} \end{cases} \qquad (9)$$

Here $\gamma = \frac{1}{\sqrt{1-\beta^{2}}}$ and $\beta = \frac{V_{O'/0}}{c}$ . The interval $ds^{2}$ between two events in Minkowski space is invariant under all Lorentz transformations :

تعرف الفترة الزمكانية التي تفصل بين حدثين في فضاء مينكوفسكي كمايلي:

$$\begin{aligned} ds^{2} &= g_{\mu\nu} dx^{\mu} dx^{\nu} \\ &= c^{2}(dt)^{2} - (dx)^{2} - (dy)^{2} - (dz)^{2} \end{aligned} \qquad (10)$$

It is classified as follows:

$$\begin{aligned} ds^{2} &= g_{\mu\nu} dx^{\mu} dx^{\nu} \succ 0 \quad \text{Time - like} \\ &= g_{\mu\nu} dx^{\mu} dx^{\nu} \prec 0 \quad \text{Space - like} \\ &= g_{\mu\nu} dx^{\mu} dx^{\nu} = 0 \quad \text{Light - like} \end{aligned} \qquad (11)$$

Differential operators $\partial_\mu$ and $\partial^\mu$, in Minkowski space, are defined by :

المؤثرات التفاضلية $\partial_\mu$ و $\partial^\mu$ في فضاء مينكوفسكي تعرف كمايلي:

$$\begin{aligned}\partial_\mu &\equiv \frac{\partial}{\partial x^\mu} = \left(\frac{\partial}{\partial x^0} = \frac{1}{c}\frac{\partial}{\partial t}, \frac{\partial}{\partial x^1} = \frac{\partial}{\partial x}, \frac{\partial}{\partial x^2} = \frac{\partial}{\partial y}, \frac{\partial}{\partial x^3} = \frac{\partial}{\partial z}\right) \\ &= \left(\frac{1}{c}\frac{\partial}{\partial t}, \frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}\right) \\ &= \left(\frac{1}{c}\frac{\partial}{\partial t}, \vec{\nabla}\right)\end{aligned} \tag{12}$$

And

$$\begin{aligned}\partial^\mu &\equiv \frac{\partial}{\partial x_\mu} = \left(\frac{\partial}{\partial x_0} = \frac{1}{c}\frac{\partial}{\partial t}, \frac{\partial}{\partial x_1} = -\frac{\partial}{\partial x}, \frac{\partial}{\partial x_2} = -\frac{\partial}{\partial y}, \frac{\partial}{\partial x_3} = -\frac{\partial}{\partial z}\right) \\ &= \left(\frac{1}{c}\frac{\partial}{\partial t}, -\frac{\partial}{\partial x}, -\frac{\partial}{\partial y}, -\frac{\partial}{\partial z}\right) \\ &= \left(\frac{1}{c}\frac{\partial}{\partial t}, -\vec{\nabla}\right)\end{aligned} \tag{13}$$

And the d'Almbertian :

و الدالومبارسيان:

$$\begin{aligned}\partial_\mu \partial^\mu &\equiv \partial^\mu \partial_\mu \\ &= \left(\frac{1}{c}\frac{\partial}{\partial t}, \vec{\nabla}\right)\left(\frac{1}{c}\frac{\partial}{\partial t}, -\vec{\nabla}\right) \\ &= \frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \frac{\partial^2}{\partial x^2} - \frac{\partial^2}{\partial y^2} - \frac{\partial^2}{\partial z^2} \\ &= \frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \Delta \equiv \partial^2\end{aligned} \tag{14}$$

The conserved four-momentum is given by :

الشعاع الرباعي للحركة المحفوظ يعرف كمايلي:

$$\begin{aligned} P^{\mu} &\equiv \left(\frac{E}{c}, \vec{p}\right) \\ P_{\mu} &\equiv \left(\frac{E}{c}, -\vec{p}\right) \\ P^{\mu}P_{\mu} &= \left(\frac{E}{c}, \vec{p}\right)\left(\frac{E}{c}, -\vec{p}\right) \\ &= \left(\frac{E}{c}\right)^2 - \vec{p}^{\,2} = \mathrm{m}_0^2\mathrm{c}^2 \end{aligned} \tag{15}$$

**A review of relativistic quantum mechanics :**

The Shrodinger equation in quantum mechanics is the operator equation corresponding to the nonrelativistic expression for the energy :

معادلة شرودينغر في ميكانيك الكم هي معادلة توافق الطاقة اللانسبية:

$$E = \frac{\vec{p}^{\,2}}{2m} \tag{16}$$

In the coordinates representations, by means of the operator substitution prescriptions :

في فضاء الاحداثيات و باستعمال الطرق الوصفية للتكميم:

$$\begin{cases} E \to i\hbar \dfrac{\partial}{\partial t} \\ \vec{p} \to \dfrac{\hbar}{i}\vec{\nabla} \end{cases} \tag{17}$$

Action on the wave function one finds for a free particle :

بالتاثير على الدالة الموجية نجد بالنسبة لجسيم حر:

$$-\frac{\hbar^2}{2m}\vec{\nabla}^2\Psi(\vec{r},t) = i\hbar\frac{\partial\Psi(\vec{r},t)}{\partial t} \tag{18}$$

The equations (17) and (18) are not covariant. Thus, the covariant relativistic equation can be obtained by starting with the covariant equation for the conserved four-momentum :

المعادلتين (17) و (18) غير صامدتين (المقصود تتغير اثناء الانتقال من جملة عطالية لاخرى), المعادلة النسبية الصامدة يمكن الحصول عليها من المعادلة الصامدة للشعاع الرباعي للحركة:

$$P^{\mu}P_{\mu} = \left(\frac{E}{c}\right)^2 - \vec{p}^{\,2} = \mathrm{m}_0^2\mathrm{c}^2$$
$$\Rightarrow \mathrm{E}^2 = \vec{p}^{\,2}\mathrm{c}^2 + m_0^2 c^4 \quad (19)$$

Substitution of operators gives the Klein-Gordon equation for real or complex function :

يعطي تعويض المؤثرات معادلة كلاين-جوردون من أجل الدوال الحقيقية أو المركبة:

$$-\hbar^2\vec{\nabla}^2\varphi(\vec{r},t) + m_0^2 c^4 \varphi(\vec{r},t) = -\hbar^2 \frac{\partial^2\varphi(\vec{r},t)}{\partial t^2} \quad (20)$$

We can rewrite the Klein-Gordon equation in a manifestly covariant form as :

يمكننا إعادة كتابة معادلة كلاين-جوردون بصيغة رباعية على النحو التالي:

$$\left(\partial^2 + \frac{m_0^2 c^2}{\hbar^2}\right)\varphi(\vec{r},t) = 0 \quad (21)$$

The plane waves are present solutions to the Klein-Gordon equation :

الأمواج الكستوية تعتبر حلولا لمعادلة كلين-غوردن من الشكل:

$$\varphi(\vec{r},t) = \exp\left(i\vec{k}\vec{r} - \omega t\right) \quad (22)$$

Allows us to relations between $\vec{k}, \omega$ and $m_0^2$ as follows :

مما يسمح بإيجاد العلاقة بين المقادير الفيزيائية كمايلي:

$$\hbar^2\omega^2 = \hbar^2 c^2 \vec{k}^{\,2} + m_0^2 c^4$$
$$\Rightarrow \hbar\omega = \pm\sqrt{\hbar^2 c^2 \vec{k}^{\,2} + m_0^2 c^4} \quad (23)$$

Thus, we obtain two modes of energy, the positive mode, and the negative mode of energy solutions. If $k^{\mu}=\left(\frac{\omega}{c},\vec{k}\right)$ present a 4-vector, allows us the covariant form of the solutions of the Klein-Gordon equation :

و هكذا نتحصل على نمطين للطاقة' النمط الموجب و النمط السالب, أذا كان $k^{\mu}=\left(\frac{\omega}{c},\vec{k}\right)$ يمثل الشعاع الرباعي الموجي هذا يسمح بإعادة صياغة حلول معادلة كلين-غوردن:

$$\varphi(\vec{r},t)=\exp\left(ik_{\mu}x^{\mu}\right)=\exp\left(\frac{i}{\hbar}p_{\mu}x^{\mu}\right) \tag{24}$$

The general solutions of the Klein-Gordon equation can be rewritten as a superposition of plane wave solutions :

الحلول العامة لمعادلة كلين-قوردن هي تركيب خطي للامواج المستوية:

$$\varphi(\vec{r},t)=\int\frac{d^{3}p}{2E_{p}(2\prod)^{3}}\left[a(\vec{p})\exp\left(-\frac{i}{\hbar}p_{\mu}x^{\mu}\right)+b^{*}(\vec{p})\exp\left(\frac{i}{\hbar}p_{\mu}x^{\mu}\right)\right] \tag{25}$$

The coefficients $a(\vec{p})$ and $b^{*}(\vec{p})$are the amplitudes of two independent solutions. For real scalar field $a(\vec{p})$ equal $b(\vec{p})$.

المعاملان $a(\vec{p})$ و $b^{*}(\vec{p})$ يمثلان السعات لحلين مستقلين, من أجل الحقول السلمية يتساوى المعاملان,

To generate the continuity equation, we consider the following combinations :

لايجاد معادلة الاستمراية نعتبر التركيب التالي:

$$\Psi^{*}(\text{Shrodinger - equation})-\Psi(\text{Shrodinger - equation})^{*}$$

We obtain the continuity equation :

و نتحصل على معادلة الاستمرارية:

$$\frac{\partial\rho}{\partial t}+div\vec{j}=0 \tag{26}$$

Where

$$\rho = \Psi^* \Psi = |\Psi|^2$$
$$\vec{j} = -\frac{i\hbar}{2m_0}\left(\Psi^* \vec{\nabla}\Psi - \Psi\vec{\nabla}\Psi^*\right) \tag{27}$$

For the Klein-Gordon equation, the continuity equation, we have :

بالنسبة لمعادلة كلين-قوردن معادلة الاستمرارية بجدها بنفس الطريقة:

$$\rho = \frac{i\hbar}{2m_0 c^2}\left(\varphi^* \frac{\partial \varphi}{\partial t} - \varphi \frac{\partial \varphi^*}{\partial t}\right)$$
$$\vec{j} = -\frac{i\hbar}{2m_0}\left(\varphi^* \vec{\nabla}\varphi - \varphi\vec{\nabla}\varphi^*\right) \tag{28}$$

In the above equation $\rho$, is no longer a positive define.

في المعادلة أعلاه $\rho$ ليست دوما موجبة,

To obtain the Klein-Gordon equation in an external electromagnetic field, we make the two replacements :

$$\begin{cases} E \to E - qV \\ \vec{p} \to \vec{p} - q\dfrac{\vec{A}}{c} \end{cases} \Rightarrow p^\mu \to p^\mu - \frac{q}{c}A^\mu \tag{31}$$

Where $A^\mu = \left(\frac{V}{c}, \vec{A}\right)$ is the 4-quadrivector potential in the Minkowski space, thus the Eq. (19) becomes as follows :

حيث أن $A^\mu = \left(\frac{V}{c}, \vec{A}\right)$ يمثل الشعاع الرباعي للكمون في فضاء مينكوفسكي' و منه تصبح المعادلة (19) كمايلي:

$$(E - qV)^2 = \left(\vec{p} - q\frac{\vec{A}}{c}\right)^2 c^2 + m_0^2 c^4 \tag{32}$$

Now, we combined two equations (17) and (32) to obtain :

الان نركب بين المعادلتين (17) و (18) لنجد:

$$\left(i\hbar\frac{\partial}{\partial t} - qV\right)^2 \varphi(x) = \left[\left(-i\hbar\vec{\nabla} - q\frac{\vec{A}}{c}\right)^2 c^2 + m_0^2 c^4\right]\varphi(x) \tag{33}$$

It is well known that the Euler-Lagrange equation in classical mechanics is given by :

من المعلوم أن معادلة إيلر-لاغرانج في الميكانيك الكلاسيكي تعطى بالعبارة التالية:

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_i}\right)-\frac{\partial L}{\partial q_i}=0 \qquad (34)$$

Where $L\left(q_i,\dot{q}_i\right)=T\left(\dot{q}_i\right)-V(q_i)$ is the Lagrangian function, $T\left(\dot{q}_i\right)$ is the kinetic energy, and $V(q_i)$ is the potential interaction. The canonical momentum $p_i$ defined as :

حيث أن $L\left(q_i,\dot{q}_i\right)=T\left(\dot{q}_i\right)-V(q_i)$ هو دالة لاغرانج’ $T\left(\dot{q}_i\right)$ هي الطاقة الحركية و $V(q_i)$ كمون التفاعل,

العزم القانوني $p_i$ يعرف كمايلي:

$$p_i=\frac{\partial L}{\partial \dot{q}_i} \qquad (35)$$

The Hamiltonian function defined as a function of $q_i$ and $p_i$ as follows :

تعرف دالة الهاميلتونيان بدلالة $q_i$ و $p_i$ كمايلي:

$$H(q_i,p_i)=p_i\,\dot{q}_i-L\left(q_i,\dot{q}_i\right) \qquad (36)$$

The Hamiltonian’s equations of motion can be obtained from Eq ; (36) as follows :

معادلات الحركة لهاملتون تستنتج من المعادلة (36):

$$\begin{cases}\dot{p}_i=-\dfrac{\partial H}{\partial q_i}\\ \dot{q}_i=\dfrac{\partial H}{\partial p_i}\end{cases} \qquad (37)$$

The Euler-Lagrange equations (19) are not covariant. Let us replace the generalized coordinate $q_i$ with a field $\varphi$. A covariant form of the action would involve a Lagrangian density via :

معادلات إيلر-لاغرانج رقم (19) غير صامدة, لنعوض الاحداثية المعممة $q_i$ بالحقل $\varphi$ , الصيغة الصامدة للفعل تشمل كثافة اللاغرنجيان:

$$\begin{aligned} S &= \int L dt \\ &= \int \Im d^3 x dt \\ &= \int \Im d^4 x dt \end{aligned} \tag{38}$$

Where $d^4x = d^3xdt$ is an element of volume and $L = \int \Im d^3 x$. The term $\left(-\frac{\partial L}{\partial q_i}\right)$ in Eq . (34) replaced by $\left(-\frac{\partial \Im}{\partial \varphi}\right)$ and the time derivative $\frac{d}{dt}$ should be replaced with the covariant derivative $\partial_\mu$, allows us to gets the covariate Euler-Lagrange equation :

حيث أن $d^4x = d^3xdt$ يمثل عنصر الحجم و $L = \int \Im d^3 x$, الحد $\left(-\frac{\partial L}{\partial q_i}\right)$ في المعادلة (34) يعوض ب: $\left(-\frac{\partial \Im}{\partial \varphi}\right)$ و مؤثر التفاضل الزمني $\frac{d}{dt}$ يجب تعويضه بالمشتق الصامد $\partial_\mu$ مما يسمح بإيجاد معادلة إيلر-لاغرانج الصامدة:

$$\partial_\mu \left( \frac{\partial \Im}{\partial(\partial_\mu \varphi)} \right) - \frac{\partial \Im}{\partial \varphi} = 0 \tag{39}$$

The covariant momentum density $\Pi^\mu$ is defined by the following relation :

كثافة العزم الحركي $\Pi^\mu$ تعرف من خلال المعادلة:

$$\Pi^\mu = \frac{\partial \Im}{\partial(\partial_\mu \varphi)} \tag{40}$$

The canonical momentum $\Pi$ is defined by :

العزم الحركي القانوني $\Pi$ يعرف كمايلي:

$$\Pi \equiv \Pi^0 = \frac{\partial \Im}{\partial(\partial_0 \varphi)} \tag{41}$$

The energy-momentum tensor $T_{\mu\nu}$ is given by :

تنسور الطاقة-حركة $T_{\mu\nu}$ يعرف كمايلي:

$$T_{\mu\nu} = \prod_{\mu} \partial_{\nu}\varphi - g_{\mu\nu}\Im \qquad (42)$$

The Hamiltonian operator $H$ is given by :

مؤثر الهاميلتونيان $H$ يعطى بالعبارة:

$$H = \int \aleph d^3x \qquad (43)$$

The Hamiltonian density $\aleph$ is defined by :

كثافة الهاميلتونيان $\aleph$ تعرف كمايلي:

$$\aleph \equiv T_{00} = \prod \partial_0 \varphi - \Im \qquad (44)$$

**Homework** :

The massive Klein-Gordon Lagrangian density is :

$$\Im(\varphi, \partial_{\mu}\varphi) = \frac{1}{2}\left(\partial_{\mu}\varphi\partial^{\mu}\varphi - m^2\varphi^2\right)$$

A 1-Derive expression for the covariant momentum density and the covariate momentum

A 2-Derive the equation of motion

A 3-Derive expression for the energy-momentum tensor and the Hamiltonian density.

Let $\varphi \to \varphi + \delta\varphi$ be a variation of the fields, so the variation of Lagrangian density can be written as follows :

$$\begin{aligned}\delta\Im(\varphi, \partial_{\mu}\varphi) &= \frac{\partial\Im}{\partial(\partial_{\mu}\varphi)}\delta(\partial_{\mu}\varphi) + \frac{\partial\Im}{\partial\varphi}\delta\varphi \\ &= \left\{\frac{\partial\Im}{\partial\varphi} - \partial_{\mu}\left(\frac{\partial\Im}{\partial(\partial_{\mu}\varphi)}\right)\right\}\delta\varphi + \partial_{\mu}\left(\frac{\partial\Im}{\partial(\partial_{\mu}\varphi)}\delta\varphi\right)\end{aligned} \qquad (45)$$

The first term vanishes by the Euler-Lagrange equation, the density Lagrangian is invariant under the variation of the fields, then we require that the second term $\partial_\mu\left(\frac{\partial\Im}{\partial(\partial_\mu\varphi)}\delta\varphi\right)$ equal zero when $\frac{\partial\Im}{\partial(\partial_\mu\varphi)}\delta\varphi$ is conserved :

$$\partial_\mu j^\mu = 0$$
$$j^\mu = \frac{\partial\Im}{\partial(\partial_\mu\varphi)}\delta\varphi \tag{46}$$

The corresponding conserved charge is given by :

$$Q = \int d^3x j^0 \tag{47}$$

**Canonical quantization of the real scalar field**

**التكميم القانوني للحقل السلمي الحقيقي**

We have seen in the first section of chapter one that the canonical momenta $p_i$ and the Hamiltonian $H(q_i, p_i)$ were given by :

راينا في القسم الأول من الفصل الأول ان العزم القانوني $p_i$ و الهاميلتونيان $H(q_i, p_i)$ تم اعطائهم كمايلي:

$$p_i = \frac{\partial L\left(q_i, \dot{q}_i\right)}{\partial \dot{q}_i}$$
$$H(q_i, p_i) \equiv p_i\dot{q}_i - L\left(q_i, \dot{q}_i\right) = \sum_i p_i\dot{q}_i - L\left(q_i, \dot{q}_i\right) \tag{48}$$

Here all the physical quantities $q_i = q_i^+$, $p_i = p_i^+$ and $H(q_i, p_i) = H(q_i, p_i)^+$ are Hermitian operators on a Helbert space. It is well known that the operators $q_i = q_i^+$ and $p_i = p_i^+$ satisfy the following Canonical Commutation Relations (***CCRs***) are postulated :

كل القيم الفيزيائية $p_i = p_i^+$ , $q_i = q_i^+$ و $H(q_i, p_i) = H(q_i, p_i)^+$ هي ارميتيةفي فضاء هلبرت ,من المعلوم أن المؤثرات $q_i = q_i^+$ و $p_i = p_i^+$ تحقق علاقات التبادل القانونية المسلم به:

$$[q_i, p_i] = [q_i(t), p_i(t)] = q_i p_i - p_i q_i = q_i(t)p_i(t) - p_i(t)q_i(t) = i\hbar\delta_{ij}$$
$$[q_i, q_i] = [q_i(t), q_i(t)] = 0 \qquad (49)$$
$$[p_i, p_i] = [p_i(t), p_i(t)] = 0$$

It is important to notice that the ***CCRs*** are satisfied in both Shrodinger and Heisenberg pictures (SP and HP). We have in SP :

من المهم أن نشير إلى أن علاقات التبادل القانونية تتحقق في كل من صور شرودينغر و هايزنبرغ, لدينا في صورة هايزنبرغ:

$$i\hbar\frac{\partial}{\partial t}|\Psi(t)\rangle_s = H(q_i, p_i)|\Psi(t)\rangle_s$$
$$|\Psi(t)\rangle_s = \exp\left(-\frac{i}{\hbar}H(t-t_0)\right)|\Psi(t_0)\rangle_s \qquad (50)$$

Where $\Psi(\vec{r},t) = \langle\vec{r}|\Psi(t)\rangle_s$ and $t_0$ is the initial time. The connection between both SP and HP is given by :

حيث $\Psi(\vec{r},t) = \langle\vec{r}|\Psi(t)\rangle_s$ و $t_0$ هو الزمن الابتداىئ, العلاقة بين صورتي شرودينغر و هايزنبرغ تعطى كمايلي:

$$\begin{cases} |\Psi\rangle_H = \exp\left(\frac{i}{\hbar}H(t-t_0)\right)|\Psi(t)\rangle_s = |\Psi(t_0)\rangle_s \\ A_H(t) = \exp\left(\frac{i}{\hbar}H(t-t_0)\right)A_s \exp\left(-\frac{i}{\hbar}H(t-t_0)\right) \end{cases} \qquad (51)$$

And the equivalent physical form is given by :

و بشكل مكافئ لدينا أيضا:

$$\begin{cases} |\Psi(t)\rangle_s = \exp\left(-\frac{i}{\hbar}H(t-t_0)\right)|\Psi\rangle_H \\ A_s = \exp\left(-\frac{i}{\hbar}H(t-t_0)\right)A_H(t)\exp\left(\frac{i}{\hbar}H(t-t_0)\right) \end{cases} \qquad (52)$$

The expectations values are conserved quantities :

القيمة المتوسطة تبقى محفوظة:

$${}_s\langle\Psi(t)|A_S|\Psi(t)\rangle_s = {}_H\langle\Psi|A_H(t)|\Psi\rangle_H \qquad (53)$$

In the special case :

في الحالة الخاصة:

$$\begin{cases} q_i(t)=\exp\left(\frac{i}{\hbar}H(t-t_0)\right)q_i\exp\left(-\frac{i}{\hbar}H(t-t_0)\right) \\ p_i(t)=\exp\left(\frac{i}{\hbar}H(t-t_0)\right)p_i\exp\left(-\frac{i}{\hbar}H(t-t_0)\right) \end{cases}$$
$$\Rightarrow \qquad (54)$$
$$\begin{cases} q_i=\exp\left(-\frac{i}{\hbar}H(t-t_0)\right)q_i(t)\exp\left(\frac{i}{\hbar}H(t-t_0)\right) \\ p_i(t)=\exp\left(-\frac{i}{\hbar}H(t-t_0)\right)p_i(t)\exp\left(\frac{i}{\hbar}H(t-t_0)\right) \end{cases}$$

While

$$\begin{cases} H(t)=\exp\left(\frac{i}{\hbar}H(t-t_0)\right)H\exp\left(-\frac{i}{\hbar}H(t-t_0)\right) \\ \text{and} \\ H=\exp\left(-\frac{i}{\hbar}H(t-t_0)\right)H(t)\exp\left(\frac{i}{\hbar}H(t-t_0)\right) \end{cases} \Rightarrow H(t)=H \qquad (55)$$

**Homework :**

Show that, by differentiating Eq. (51) in an HP the operator $A_H(t)$ obeys the Heisenberg equation of motion :

بين أن بتفاضل المعادلة (51) في تمثيل هايزنبرغ فإن المؤثر $A_H(t)$ يخضع لمعادلة هايزنبرغ الحركية:

$$\frac{dA_H(t)}{dt}=\frac{i}{\hbar}[H,A_H(t)] \qquad (56)$$

And

$$\begin{cases} \frac{dq(t)}{dt}=\frac{p(t)}{m} \\ \frac{dp(t)}{dt}=-\frac{\partial V}{\partial t} \end{cases} \qquad (57)$$

Where $m$ and $V$ are the mass and interaction potential, respectively.

حيث $m$ و $V$ تمثلان الكتلة و كمون التفاعل,

**The second quantization :**

**التكميم الثاني:**

The second quantization procedure of scalar real field satisfied by considering both the field $\phi(t,\vec{r})$ and its conjugate $\Pi(t,\vec{r})$ as Hermitian operators ($\phi(t,\vec{r})=\phi^{+}(t,\vec{r})$ and $\Pi(t,\vec{r})=\Pi^{+}(t,\vec{r})$ ). To satisfies this object, we made the following translations from the Canonical Commutation Relations (***CCRs***) :

تحدد طريق التكميم الثاني للحقل السلمي الحقيقي باعتبار كل من الحقل $\phi(t,\vec{r})$ و عزمه القانوني $\Pi(t,\vec{r})$ مؤثرات ارميتية $\phi(t,\vec{r})=\phi^{+}(t,\vec{r})$ و $\Pi(t,\vec{r})=\Pi^{+}(t,\vec{r})$ , لانجاز ذلك ننطلق من علاقات التابدل القانونية في ميكانيك الكم:

$$\begin{aligned} q_i &\to \phi(t,\vec{r}) \\ p_i &\to \Pi(t,\vec{r}) \\ \delta_{ij} &\to \delta(\vec{r}-\vec{r}\,') \end{aligned} \qquad (58)$$

Here $\delta(\vec{r}-\vec{r}\,')$ is a 3-dimensional Dirac delta function. The ***CCRs*** is the first quantization, translated to the second quantization by considering the three-simultaneously transformations on the ***CCRs***, in Eq. (49) :

هنا $\delta(\vec{r}-\vec{r}\,')$ تمثل دلتا-ديرات ثلاثية البعد, علاقات التبادل القانونية تعتبر أساس للانتقال للتكميم الثاني بتطبيق قواعد الانتقال عليها المحددة في المعادلة (58):

$$\begin{aligned} [q_i,p_i]=i\hbar\delta_{ij} &\to [\phi(t,\vec{r}),\Pi(t,\vec{r}\,')]=i\hbar\delta(\vec{r}-\vec{r}\,') \\ [q_i,q_i]=0 &\to [\phi(t,\vec{r}),\phi(t,\vec{r}\,')]=0 \\ [p_i,p_i]=0 &\to [\Pi(t,\vec{r}),\Pi(t,\vec{r}\,')]=0 \end{aligned} \qquad (59)$$

The new above algebra is known by Equal Time Commutation Relation (***ETCCRs***). We have seen that the Hamiltonian operator $H$, in the first part of chapter one, is given by :

الجبر الجديد يعرف علاقات التبادل القانونية في نفس الزمن, راينا أن مؤثر الهاميلتونيان $H$ في القسم الأول من الفصل الأول يكتب كمايلي:

$$H=\int \aleph d^3x \qquad \text{(Part 1: 43)}$$

The Hamiltonian density $\aleph$ is defined by :

كثافة الهاميلتونيان $\aleph$ تعرف كمايلي:

$$\aleph \equiv T_{00}=\prod \partial_0\varphi-\Im \qquad \text{(Part 1:44)}$$

And $\Im(\varphi,\partial_{\mu}\varphi)=\frac{1}{2}\left(\partial_{\mu}\varphi\partial^{\mu}\varphi-m^{2}\varphi^{2}\right)$ is the Lagrangian density for the scalar real field. Allows us to get the Hamiltonian density $\aleph$ for the scalar real field as follows :

و $\Im(\varphi,\partial_{\mu}\varphi)=\frac{1}{2}\left(\partial_{\mu}\varphi\partial^{\mu}\varphi-m^{2}\varphi^{2}\right)$ هو كثافة اللاغرنجيان للحقل السلمي الحقيقي, يسمح بإيجاد كثافة الهاميلتونيان $\aleph$ للحقل السلمي الحقيقي:

$$\aleph=\frac{1}{2}\left[\Pi^{2}+\left(\vec{\nabla}\phi\right)^{2}+m^{2}\phi^{2}\right] \tag{60}$$

Now, applying the Heisenberg equation to obtain :

نطبق الان معادلة هايزنبرغ الحركية لنجد:

$$\begin{aligned}\frac{d\phi(t,\vec{r})}{dt}&=\frac{i}{\hbar}\left[H,\phi(t,\vec{r})\right]\\&=\frac{i}{\hbar}\int d\vec{r}\,'\left[\aleph(t,\vec{r}\,'),\phi(t,\vec{r})\right]\end{aligned} \tag{61}$$

Where $d\vec{r}\,'=dx'dy'dz'$. Using *ETCCRs* (Eq . (59)) and the expression of the Hamiltonian density for the scalar real field, the commutator $[\aleph(t,\vec{r}\,'),\phi(t,\vec{r})]$ will be taken the following result :

حيث أن $d\vec{r}\,'=dx'dy'dz'$, نستعمل *ETCCRs* (Eq . (59)) و كثافة الهاميلتونيان للحقل السلمي الحقيقي. المبدل $[\aleph(t,\vec{r}\,'),\phi(t,\vec{r})]$ سيصبح كمايليك

$$[\aleph(t,\vec{r}\,'),\phi(t,\vec{r})]=-i\Pi(t,\vec{r}\,')\delta(\vec{r}-\vec{r}\,') \tag{62}$$

Allows us to gets :

يسمح بإيجاد:

$$\frac{d\phi(t,\vec{r})}{dt}=\Pi(t,\vec{r}) \tag{63}$$

For the conjugate momenta $\Pi(t,\vec{r})$, applying the Heisenberg equation to obtain :

من أجل العزم القانوني $\Pi(t,\vec{r})$ نطبق معادلة هايزنبرغ الحركية:

$$\begin{aligned}\frac{d\Pi(t,\vec{r})}{dt}&=\frac{i}{\hbar}\left[H,\Pi(t,\vec{r})\right]\\&=\frac{i}{\hbar}\int d\vec{r}\,'\left[\aleph(t,\vec{r}\,'),\Pi(t,\vec{r})\right]\end{aligned} \tag{64}$$

Using ***ETCCRs*** (Eq . (59)) and the expression of the Hamiltonian density for the scalar real field, the commutator $[\aleph(t,\vec{r}\,'),\Pi(t,\vec{r})]$ will be taken the following result :

نستعمل ((Eq . (59)) ***ETCCRs*** و كثافة الهاميلتونيان للحقل السلمي الحقيقي لنجد المبدل $[\aleph(t,\vec{r}'),\Pi(t,\vec{r})]$ بالنتيجة التالية:

$$[\aleph(t,\vec{r}'),\Pi(t,\vec{r})] = i\vec{\nabla}'\phi(t,\vec{r}')\vec{\nabla}'\delta(\vec{r}-\vec{r}') + im^2\phi(t,\vec{r}')\delta(\vec{r}-\vec{r}') \qquad (65)$$

Allows us to gets :

يسمح بإيجاد:

$$\frac{d\Pi(t,\vec{r})}{dt} = \vec{\nabla}^2\phi(t,\vec{r}) - m^2\phi(t,\vec{r}) \qquad (66)$$

Combined between Eqs. (63) and (66) to show that the operator of the scalar real field $\varphi(\vec{r},t)$ is satisfied with the Klein-Gordon equation (in the system $c=\hbar=1$) :

نركب بين المعادلتين Eqs. (63) and (66) لنبين ان مؤثر الحقل السلمي $\varphi(\vec{r},t)$ تحقق معادلة كلين-غوردن (في نظام الوحدات $c=\hbar=1$):

$$\left(\partial^2 + m_0^2\right)\varphi(\vec{r},t) = 0 \qquad (67)$$

Here $\varphi(\vec{r},t)$ is an ***operator*** and ***not a function***. Because the operator of scalar real field $\varphi(\vec{r},t)$ satisfies the Klein-Gordon equation, it is possible to expand it of a propagating plane wave as follows :

هنا $\varphi(\vec{r},t)$ تعتبر مؤثر و ليس تابع, لان مؤثر الحقل السلمي $\varphi(\vec{r},t)$ تحقق معادلة كلين-غوردن فبالامكان نشرها وفق أمواج مستوية على الشكل التالي:

$$\varphi(\vec{r},t) = \int \frac{d^3p}{2E_p(2\Pi)^3}\left[a(\vec{p})\exp\left(-\frac{i}{\hbar}p_\mu x^\mu\right) + a^+(\vec{p})\exp\left(\frac{i}{\hbar}p_\mu x^\mu\right)\right] \qquad (68)$$

We have replaced in Eq. (Ch_1: Part 1, 25) both the numbers $a(\vec{p})$ and $a^*(\vec{p})$with the operators $a(\vec{p})$ and $a^+(\vec{p})$, respectively. The energy $E_p \equiv P^0 = \sqrt{\vec{p}^2 + m^2}$ in the natural system $c=\hbar=1$. We can write Eq. (68) as a sum of two moded, the first one is the positive mode while the second is the negative frequency mode as follows :

قمنا بتعويض الاعداد $a(\vec{p})$ و $a^*(\vec{p})$ ((Eq. (Ch_1: Part 1, 25)) على التوالي بالمؤثرات $a(\vec{p})$ و $a^+(\vec{p})$ , يمكن كتابة المعادلة رقم (68) كمجموع لنمطين الأول موجب و الثاني سالب التواتر على الشكل التالي:

$$\varphi^{+}(\vec{r},t)=\int\frac{d^{3}p}{2E_{p}(2\Pi)^{3}}a(\vec{p})\exp\left(-\frac{i}{\hbar}p_{\mu}x^{\mu}\right)$$
$$\varphi^{(-)}(\vec{r},t)=\int\frac{d^{3}p}{2E_{p}(2\Pi)^{3}}a^{+}(\vec{p})\exp\left(\frac{i}{\hbar}p_{\mu}x^{\mu}\right) \qquad (69)$$

The conjugate field $\Pi(t,\vec{r})$ is :

العزم القانوني $\Pi(t,\vec{r})$ بالشكل:

$$\Pi(\vec{r},t)=\int\frac{d^{3}p}{2E_{p}(2\Pi)^{3}}(-iE_{p})\left[a(\vec{p})\exp\left(-\frac{i}{\hbar}p_{\mu}x^{\mu}\right)-a^{+}(\vec{p})\exp\left(\frac{i}{\hbar}p_{\mu}x^{\mu}\right)\right] \qquad (70)$$

**Homework :**

Show that the operators $a(\vec{p})$ and $a^{+}(\vec{p})$ can be written as follows :

بين أن المؤثرين $a(\vec{p})$ و $a^{+}(\vec{p})$ بالإمكان كتابتهما على النحو:

$$a(\vec{p})=\int d\vec{r}\left[2E_{p}(2\Pi)^{3}\right]^{1/2}f_{p}^{*}(t,\vec{r})i\overset{\leftrightarrow}{\partial}_{0}\phi(t,\vec{r})$$
$$a^{+}(\vec{p}')=\int d\vec{r}'\left[2E_{p'}(2\Pi)^{3}\right]^{1/2}\phi(t',\vec{r}')i\overset{\leftrightarrow}{\partial}_{0}f_{p'}(t',\vec{r}') \qquad (71)$$

Where

$$\begin{cases} f_{p}^{*}(t,\vec{r})=\dfrac{1}{\left[2E_{p}(2\Pi)^{3}\right]^{1/2}}\exp\left(ip_{\mu}x^{\mu}\right) \\ \overset{\leftrightarrow}{\partial}_{0}=\overset{\rightarrow}{\partial}_{0}-\overset{\leftarrow}{\partial}_{0} \\ \int d\vec{r}\,f_{p}^{*}(t,\vec{r})i\overset{\leftrightarrow}{\partial}_{0}f_{p'}(t',\vec{r}')=\delta^{(3)}(\vec{p}-\vec{p}') \end{cases} \qquad (72)$$

Hence, their commutators is given by :

و عليه تكون مبادلاتهم كمايلي:

$$\left[a(\vec{p}),a^{+}(\vec{p}')\right]=-\int d\vec{r}\int d\vec{r}'(2\Pi)^{3}\left[4E_{p}E_{p'}\right]^{1/2}\left[f_{p}^{*}(t,\vec{r})i\overset{\leftrightarrow}{\partial}_{0}\phi(t,\vec{r}),\phi(t',\vec{r}')i\overset{\leftrightarrow}{\partial}_{0}f_{p'}(t',\vec{r}')\right]$$
$$=(2\Pi)^{3}\int d\vec{r}\int d\vec{r}'\left[4E_{p}E_{p'}\right]^{1/2}f_{p}^{*}(t,\vec{r})i\overset{\leftrightarrow}{\partial}_{0}f_{p'}(t',\vec{r}')\left[\phi(t,\vec{r}),\Pi(t',\vec{r}')\right] \qquad (73)$$
$$=(2\Pi)^{3}2E_{p}\delta^{(3)}(\vec{p}-\vec{p}')$$

Similarly, for the two commutators $[a(\vec{p}),a(\vec{p}')]$ and $\left[a^{+}(\vec{p}),a^{+}(\vec{p}')\right]$, one can obtain the following results :

بالمثل نجد المبدلان $[a(\vec{p}),a(\vec{p}')]$ و $\left[a^{+}(\vec{p}),a^{+}(\vec{p}')\right]$ على الشكل:

$$\begin{aligned}&[a(\vec{p}),a(\vec{p}')]=0\\&\left[a^{+}(\vec{p}),a^{+}(\vec{p}')\right]=0\end{aligned}\qquad(74)$$

We observe a complete similarity between the algebra of Eqs. (73) and (74) with the creation and annihilation operators of the quantum mechanical Harmonic oscillator. To profound this complete similarity further, we recall the operator's relations :

نلاحظ تشابه كامل بين الجبر في المعادلات (73) و (74) مع مؤثرات التكوين و الإلغاء في الهزاز التوافقي الكمي, لتعميق فهم التشابه نذكر بعلاقات المؤثرات:

$$\begin{aligned}&N(\vec{p})=a^{+}(\vec{p})a(\vec{p}')\\&N(\vec{p})|n(\vec{p})\rangle=n(\vec{p})|n(\vec{p})\rangle\\&[N(\vec{p}),N'(\vec{p})]=0\\&\left[N(\vec{p}),a^{+}(\vec{p})\right]=a^{+}(\vec{p})\\&[N(\vec{p}),a(\vec{p})]=-a(\vec{p})\end{aligned}\qquad(75)$$

Which allows us to get :

التي تسمح بإيجاد:

$$\begin{aligned}&N(\vec{p})a^{+}(\vec{p})|n(\vec{p})\rangle=(n(\vec{p})+1)a^{+}(\vec{p})|n(\vec{p})\rangle\\&N(\vec{p})a(\vec{p})|n(\vec{p})\rangle=(n(\vec{p})-1)a(\vec{p})|n(\vec{p})\rangle\end{aligned}\qquad(76)$$

This means that, if the operator $a(\vec{p})$ acting on the state $|n(\vec{p})\rangle$ reduces its eigenvalue by one, while $a^{+}(\vec{p})$ raises its eigenvalue by one. Besides, the number operator of occupation $N(\vec{p})$ is non-negative, because :

هذا يعني أن المؤثر $a(\vec{p})$ عندما يؤثر على الحالة $|n(\vec{p})\rangle$ فانه يخفض من قيمها الذاتية بواحدة في حين أن المؤثر $a^{+}(\vec{p})$ يضيف للقيم الذاتية واحدة, ضف لذلك أن مؤثر الاشغال $N(\vec{p})$ لن يكون سالبا لان:

$$\begin{aligned}0\leq\left(a(\vec{p})|n(\vec{p})\rangle\right)^{+}\left(a(\vec{p})|n(\vec{p})\rangle\right)&=\langle n(\vec{p})|a^{+}(\vec{p})a(\vec{p})|n(\vec{p})\rangle\\&=\langle n(\vec{p})|a^{+}(\vec{p})a(\vec{p}')n(\vec{p})|n(\vec{p})\rangle\\&=\langle n(\vec{p})|N(\vec{p})|n(\vec{p})\rangle\\&=\langle n(\vec{p})|n(\vec{p})|n(\vec{p})\rangle\\&=n(\vec{p})\langle n(\vec{p})||n(\vec{p})\rangle\\&=n(\vec{p})\end{aligned}\qquad(77)$$

The ground state $|0\rangle$ is defined by

الحالة الأساسية $|0\rangle$ تعرف كمايلي:

$$a(\vec{p})|0\rangle = 0 \qquad (78)$$

Let us now write the Hamiltonian operator $H$ as a function of the creation and annihilation operators $a(\vec{p})$ and $a^{+}(\vec{p})$:

لنعبر عن الهاميلتونينان $H$ بدلالة مؤثرات الالغاء و التكوين $a(\vec{p})$ و $a^{+}(\vec{p})$:

$$\begin{aligned} H &= \int \aleph d^3x \\ &= \int d\vec{r}\,\frac{1}{2}\left[\Pi(t,\vec{r})^2 + \left(\vec{\nabla}\phi(t,\vec{r})\right)^2 + m^2\phi(t,\vec{r})^2\right] \end{aligned} \qquad (79\text{-}1)$$

Using the explicit forms of the field operator and its conjugate expressed in Eqs. (68) and (70) to gets the following form of the Hamiltonian operator $H$ :

باستعمال العلاقات الصريحة لمؤثر الحقل السلمي و عزمه القانوني في المعادلتين (68) و (70) لايجاد الشكل التالي لمؤثر الهاميلتونيان $H$:

$$H = H_1 + H_2 + H_3 \qquad (79\text{-}2)$$

Where the three terms $H_1$ , $H_2$ and $H_3$ are given by :

حيث أن الحدود الثلاثة $H_1$ , $H_2$ و $H_3$ تعطى كمايلي:

$$\begin{aligned} H_1 &= \frac{1}{2}\int \frac{d^3p}{2E_p(2\Pi)^3}\int \frac{d^3p'}{2E_{p'}(2\Pi)^3}(-iE_p)(-iE_{p'})\int d\vec{r}\left[a(\vec{p})\exp\left(-\frac{i}{\hbar}p_\mu x^\mu\right) - a^{+}(\vec{p})\exp\left(\frac{i}{\hbar}p_\mu x^\mu\right)\right] \\ &\quad \left[a(\vec{p}')\exp\left(-\frac{i}{\hbar}p'_\mu x^\mu\right) - a^{+}(\vec{p}')\exp\left(\frac{i}{\hbar}p'_\mu x^\mu\right)\right] \\ H_2 &= \frac{1}{2}\int \frac{d^3p}{2E_p(2\Pi)^3}\int \frac{d^3p'}{2E_{p'}(2\Pi)^3}(-iE_p)(-iE_{p'})\int d\vec{r}\left[(i\vec{p})a(\vec{p})\exp\left(-\frac{i}{\hbar}p_\mu x^\mu\right) + (-i\vec{p})a^{+}(\vec{p})\exp\left(\frac{i}{\hbar}p_\mu x^\mu\right)\right] \\ &\quad \left[a(\vec{p}')(i\vec{p}')\exp\left(-\frac{i}{\hbar}p'_\mu x^\mu\right) + (-i\vec{p}')a^{+}(\vec{p}')\exp\left(\frac{i}{\hbar}p'_\mu x^\mu\right)\right] \\ H_3 &= \frac{m^2}{2}\int \frac{d^3p}{2E_p(2\Pi)^3}\int \frac{d^3p'}{2E_{p'}(2\Pi)^3}(-iE_p)(-iE_{p'})\int d\vec{r}\left[a(\vec{p})\exp\left(-\frac{i}{\hbar}p_\mu x^\mu\right) + a^{+}(\vec{p})\exp\left(\frac{i}{\hbar}p_\mu x^\mu\right)\right] \\ &\quad \left[a(\vec{p}')\exp\left(-\frac{i}{\hbar}p'_\mu x^\mu\right) + a^{+}(\vec{p}')\exp\left(\frac{i}{\hbar}p'_\mu x^\mu\right)\right] \end{aligned} \qquad (80)$$

Using the following special integrations :

باستعمال التكاملات الخاصة:

$$\int d\vec{r}\,\exp\left(\pm i\left(p_\mu + p'_\mu\right)x^\mu\right) = (2\Pi)^3\,\delta(\vec{p}+\vec{p}')\exp\left(\pm 2iE_p t\right)$$
$$\int d\vec{r}\,\exp\left(\pm i\left(p_\mu - p'_\mu\right)x^\mu\right) = (2\Pi)^3\,\delta(\vec{p}-\vec{p}') \qquad (81)$$

We get the following results :

نجد النتائج التالية:

$$H_1 = \frac{1}{4}\int \frac{d^3p}{2E_p(2\Pi)^3}\left[a(\vec{p})a^+(\vec{p}) + a^+(\vec{p})a(\vec{p})\right]$$
$$H_2 = \frac{1}{4}\int \frac{d^3p}{2E_p(2\Pi)^3}\frac{\vec{p}^2}{E_p}\left[a(\vec{p})a^+(\vec{p}) + a^+(\vec{p})a(\vec{p})\right] \qquad (82)$$
$$H_3 = \frac{1}{4}\int \frac{d^3p}{2E_p(2\Pi)^3}\frac{m^2}{E_p}\left[a(\vec{p})a^+(\vec{p}) + a^+(\vec{p})a(\vec{p})\right]$$

Which allows us to obtain the Hamiltonian operator $H$ as a function of the creation and annihilation operators $a(\vec{p})$ and $a^+(\vec{p})$ as follows :

مما يسمح بإيجاد مؤثر الهاميلتونيان $H$, بدلالة مؤثرات الالغاء و التكوين $a(\vec{p})$ و $a^+(\vec{p})$:

$$H = \frac{1}{2}\int \frac{d^3p}{2E_p(2\Pi)^3}E_p\left[a(\vec{p})a^+(\vec{p}) + a^+(\vec{p})a(\vec{p})\right] \qquad (83)$$

It is useful to calculate the commutators $\left[H, a^+(\vec{p})\right]$ and $\left[H, a(\vec{p})\right]$, using the Eq. (83) to obtain :

من المفيد حساب المبدلين $\left[H, a^+(\vec{p})\right]$ و $\left[H, a(\vec{p})\right]$, باستعمال المعادلة (83) نجد:

$$\left[H, a^+(\vec{p})\right] =$$
$$= \left[\frac{1}{2}\int \frac{d^3p}{2E_{p'}(2\Pi)^3}E_{p'}\left[a(\vec{p}')a^+(\vec{p}') + a^+(\vec{p}')a(\vec{p}')\right], a^+(\vec{p})\right]$$
$$= \frac{1}{2}\int \frac{d^3p}{2E_{p'}(2\Pi)^3}E_{p'}\left\{a^+(\vec{p})a(\vec{p}')a^+(\vec{p}')a^+(\vec{p}) + a^+(\vec{p}')a(\vec{p}')a^+(\vec{p}) - a^+(\vec{p})a(\vec{p}')a^+(\vec{p}') - a^+(\vec{p})a^+(\vec{p}')a(\vec{p}')\right\}$$
$$= \frac{1}{2}\int \frac{d^3p}{2E_{p'}(2\Pi)^3}E_{p'}\left\{a^+(\vec{p}')\left[a(\vec{p}'), a^+(\vec{p})\right] - \left[a(\vec{p}'), a^+(\vec{p})\right]a^+(\vec{p}')\right\} \qquad (84)$$
$$\frac{1}{2}\int \frac{d^3p}{2E_{p'}(2\Pi)^3}E_{p'}\left\{a^+(\vec{p}')(2\Pi)^3 2E_p\delta^{(3)}(\vec{p}-\vec{p}') + (2\Pi)^3 2E_p\delta^{(3)}(\vec{p}-\vec{p}')a^+(\vec{p}')\right\}$$
$$= +E_p a^+(\vec{p})$$

Similarly, for the $[H, a(\vec{p})]$, we get :

بشكل مماثل بالنسبة للمبدل $[H, a(\vec{p})]$ نجد:

$$[H, a(\vec{p})] = -E_p a(\vec{p}) \tag{85}$$

Allowing the following results :

يسمح بإيجاد النتائج:

$$\begin{aligned} Ha^+(\vec{p})|E\rangle &= \left(a^+(\vec{p})H + E_p a^+(\vec{p})\right)|E\rangle = \left(E + E_p\right)a^+(\vec{p})|E\rangle \\ Ha(\vec{p})|E\rangle &= \left(a(\vec{p})H - E_p a^+(\vec{p})\right)|E\rangle = \left(E - E_p\right)a(\vec{p})|E\rangle \end{aligned} \tag{86}$$

Thus, the operators $a(\vec{p})$ and $a^+(\vec{p})$ are the annihilation and creation operators. We can write the Hamiltonian operator as a function of the occupation number $N(\vec{p}) = a^+(\vec{p})a(\vec{p})$ by using the commutator which presents by Eq. (73) as follows :

و منه يكون المؤثران $a(\vec{p})$ و $a^+(\vec{p})$ للالغاء و التكوين, نستطيع كتابة الهاميلتونيان بدلالة مؤثر الاشغال $N(\vec{p}) = a^+(\vec{p})a(\vec{p})$ باستعمال المبدلات الوضحة في المعادلة (74):

$$H = \int \frac{d^3p}{2E_p(2\Pi)^3} E_p \left[ N(\vec{p}) + \frac{1}{2}\delta_{pp} \right] \tag{87}$$

Where $\delta_{pp'}$ is just denotes the quantity: $(2\Pi)^3 E_p \delta(\vec{p} - \vec{p}')$. Now, if we calculate the expectation values of energy $\langle 0|H|0\rangle$ we obtain :

حيث أن $\delta_{pp'}$ فقط ترمز للكمية $(2\Pi)^3 E_p \delta(\vec{p} - \vec{p}')$, أذا حسبنا القيمة المتوقعة للطاقة $\langle 0|H|0\rangle$ نجد:

$$\begin{aligned} \langle 0|H|0\rangle &= \int \frac{d^3p}{2E_p(2\Pi)^3} E_p \left[ \langle 0|N(\vec{p})|0\rangle N(\vec{p}) + \frac{1}{2}\delta_{pp}\langle 0\|0\rangle \right] \\ &= \int \frac{d^3p}{2E_p(2\Pi)^3} E_p \frac{1}{2}\delta_{pp} \to \infty \end{aligned} \tag{88}$$

We have used $N(\vec{p})|0\rangle = 0$ and $\langle 0\|0\rangle = 1$. Thus, the canonical quantization leads to a quantum theory *non-renormalizable*. Notice, that the eigenvalues of the operator $N(\vec{p})$ are non-negative, the eigenvalues $n(\vec{p}) = 0,1,2,....$ and known as occupation numbers. Furthermore, if we have multi-particle states with occupation numbers $n_1(\vec{p}_1), n_2(\vec{p}_2)...$ the global occupation numbers $n = n_1(\vec{p}_1) + n_2(\vec{p}_2) + ...$ in the state $|n(\vec{p})\rangle = |n_1(\vec{p}_1), n_2(\vec{p}_2),....\rangle$.

استعملنا $N(\vec{p})|0\rangle = 0$ و $\langle 0\|0\rangle = 1$, و منه التكميم القانوني يؤدي لنظرية غير-منظمة (متباعدة), تجدر الإشارة الى ان القيم الذاتية للمؤثر $N(\vec{p})$ ليست سالبة فهي $n(\vec{p}) = 0,1,2,....$ و تسمي بقيم الاشغال, زيادة على ذلك إذا كانت لدينا جملة متنوعة من الجسيمات $n_1(\vec{p}_1), n_2(\vec{p}_2)...$ العدد الإجمالي هو المجموع $n = n_1(\vec{p}_1) + n_2(\vec{p}_2) + ...$ في الحالة $|n(\vec{p})\rangle = |n_1(\vec{p}_1), n_2(\vec{p}_2),....\rangle$,

We have seen that the field operator $\varphi(\vec{r},t)$ composed of two moded, the first one is the positive mode $\varphi^{+}(\vec{r},t)$ while the second is the negative frequency mode $\varphi^{-}(\vec{r},t)$ as follows :

رأينا أن مؤثر الحقل $\varphi(\vec{r},t)$ مكون من نمطين الأول موجب $\varphi^{+}(\vec{r},t)$ بينما الثاني سالب التواتر على الشكل التالي:

$$
\begin{aligned}
\varphi^{+}(\vec{r},t) &= \int \frac{d^3p}{2E_p(2\Pi)^3} a(\vec{p}) \exp\left(-\frac{i}{\hbar} p_\mu x^\mu\right) \equiv \int \frac{d^3p}{2E_p(2\Pi)^3} a(\vec{p}) \exp(-ipx) \\
\varphi^{(-)}(\vec{r},t) &= \int \frac{d^3p}{2E_p(2\Pi)^3} a^{+}(\vec{p}) \exp\left(\frac{i}{\hbar} p_\mu x^\mu\right) \equiv \int \frac{d^3p}{2E_p(2\Pi)^3} a^{+}(\vec{p}) \exp(ipx)
\end{aligned} \tag{89}
$$

**The normal product :**

It is easy to get the product of two operators $\varphi(x \equiv \vec{r},t)$ and $\varphi(y \equiv \vec{r}',t')$ :

من السهل إيجاد الجداء العادي للمؤثرين $\varphi(x \equiv \vec{r},t)$ و $\varphi(y \equiv \vec{r}',t')$:

$$
\varphi(x)\varphi(y) = \varphi^{+}(x)\varphi^{+}(y) + \varphi^{+}(x)\varphi^{-}(y) + \varphi^{-}(x)\varphi^{+}(y) + \varphi^{-}(x)\varphi^{-}(y) \tag{90}
$$

Now, if we calculate the expectation values $\langle 0|\varphi^{+}(x)\varphi^{+}(y)|0\rangle$, $\langle 0|\varphi^{+}(x)\varphi^{-}(y)|0\rangle$ , $\langle 0|\varphi^{-}(x)\varphi^{+}(y)|0\rangle$ and $\langle 0|\varphi^{-}(x)\varphi^{-}(y)|0\rangle$, we obtain :

الان إذا حسبنا القيم المتوقعة $\langle 0|\varphi^{-}(x)\varphi^{+}(y)|0\rangle$ , $\langle 0|\varphi^{+}(x)\varphi^{-}(y)|0\rangle$, $\langle 0|\varphi^{+}(x)\varphi^{+}(y)|0\rangle$ و $\langle 0|\varphi^{-}(x)\varphi^{-}(y)|0\rangle$ نجد:

$$
\begin{aligned}
\langle 0|\varphi^{+}(x)\varphi^{+}(y)|0\rangle &= 0 \\
\langle 0|\varphi^{+}(x)\varphi^{-}(y)|0\rangle &\neq 0 \\
\langle 0|\varphi^{-}(x)\varphi^{+}(y)|0\rangle &= 0 \\
\langle 0|\varphi^{-}(x)\varphi^{-}(y)|0\rangle &= 0
\end{aligned} \tag{91}
$$

Thus, the expectation value of the second term of Eq. (90) is not zero, to this physical reason it is useful to introduce a new product known as the ***normal product*** instead of ordinary product, the normal product noted by $:\varphi(x)\varphi(y):$ or $N[\varphi(x)\varphi(y)]$ :

إذن القيمة المتوقعة للحد الثاني من المعادلة (90) ليست معدومة, لهذا السبب الفيزيائ من المفيد ادخال جداء جديد يعرف بالجداء الطبيعي بدلا من الجداء العادي يرمزله بالرمز: $:\varphi(x)\varphi(y):$ أو $N[\varphi(x)\varphi(y)]$ :

$$:\varphi(x)\varphi(y):=\varphi^{+}(x)\varphi^{+}(y)+\varphi^{-}(y)\varphi^{+}(x)+\varphi^{-}(x)\varphi^{+}(y)+\varphi^{-}(x)\varphi^{-}(y) \quad (92)$$

Thus, we obtain immediately

و منه نحصل فوريا على:

$$\langle 0|:\varphi^{-}(x)\varphi^{-}(y):|0\rangle=0 \quad (93)$$

It is easy to get the difference between the ordinary product and the normal product using Eqs. (90) and (92) :

من السهل حساب الفرق بين الجداء الطبيعي و العادي:

$$:\varphi(x)\varphi(y):-\varphi(x)\varphi(y)=-\left[\varphi^{+}(x),\varphi^{-}(x)\right] \quad (94)$$

If we replaced the ordinary product with a normal product in the Hamiltonian operator, it is evident to show that the expectations values become zero :

إذا أستبدلنا الجداء العادي بالجداء الطبيعي في عبارة الهاميلتونيان تصبح القيمة المتوقعة للطاقة في الفراغ معدومة:

$$H\to :H:\Rightarrow\langle 0|:H:|0\rangle=0 \quad (93)$$

Thus, the canonical quantization leads to a quantum theory ***renormalizable*** if we used the *normal product* instead of the ordinary product.

و منه ينتج لنا التكميم الثاني نظرية كمية متقاربة إذا استعملنا الجداء الطبيعي بدلا من الجداء العادي,

Now, if we have a physical system composed of many real scalar fields $\{\varphi_l(x)\}$ with $l=1,2,....N$ each type of mass $m$, the Lagrangian density is the sum of all densities as follows :

الان أذا كان لدينا نظام فيزيائ مشكل من عديد الجسيمات السلمية $\{\varphi_l(x)\}$ حيث $l=1,2,....N$ كل نوع بكتلة $m$, كثافة اللاغرنجيان هي مجموع الكثافات لمختلف الأنواع:

$$\Im=\sum_{l=1}^{N}\frac{1}{2}\left(\partial_{\mu}\varphi_l\partial^{\mu}\varphi_l-m_l^2\varphi_l{}^2\right) \quad (94)$$

Each type characterized by conjugate momentum :

كل نوع يميز بعزم قانوني:

$$\Pi_l(x)=\frac{\partial\Im}{\partial\left(\frac{d\varphi_l(x)}{dt}\right)}=\frac{d\varphi_l(x)}{dt} \quad (95)$$

The ***ETCRs*** for each type characterized by the following quantum algebra :

لكل نوع لدينا ***ETCRs*** *تعطي بالجبر:*

$$\begin{aligned}&[\phi_l(t,\vec{r}),\Pi_s(t,\vec{r}')]=i\hbar\delta_{ls}\delta(\vec{r}-\vec{r}')\\&[\phi_l(t,\vec{r}),\phi_s(t,\vec{r}')]=0\\&[\Pi_l(t,\vec{r}),\Pi_s(t,\vec{r}')]=0\end{aligned}\qquad(96)$$

For each type, the commutators $[H,a_l^+(\vec{p})]$ and $[H,a_l(\vec{p})]$ satisfy the algebra:

لكل نوع علاقات التبادل $[H,a_l^+(\vec{p})]$ و $[H,a_l(\vec{p})]$ تحقق الجبر:

$$\begin{aligned}&[H,a_l^+(\vec{p})]=E_{pl}a_l^+(\vec{p})\\&[H,a_l(\vec{p})]=-E_{pl}a_l(\vec{p})\end{aligned}\qquad(97)$$

The mode $\varphi_l(\vec{r},t)$ and its conjugate $\Pi_l(\vec{r},t)$ of each type are given by :

النمط $\varphi_l(\vec{r},t)$ و عزمه القنوني $\Pi_l(\vec{r},t)$ لكل نوع يعطى كمايلي:

$$\begin{cases}\varphi_l(\vec{r},t)=\int\dfrac{d^3p}{2E_p(2\Pi)^3}\left[a_l(\vec{p})\exp\left(-\dfrac{i}{\hbar}p_\mu x^\mu\right)+a_l^+(\vec{p})\exp\left(\dfrac{i}{\hbar}p_\mu x^\mu\right)\right]\\ \Pi_l(\vec{r},t)=\int\dfrac{d^3p}{2E_p(2\Pi)^3}(-iE_p)\left[a_l(\vec{p})\exp\left(-\dfrac{i}{\hbar}p_\mu x^\mu\right)-a_l^+\exp\left(\dfrac{i}{\hbar}p_\mu x^\mu\right)\right]\end{cases}\qquad(98)$$

The occupation number is given by :

مؤثر الاشغال يعطي بالعبارة:

$$\begin{aligned}N(p)&=\sum_{l=1}^{N}N_l\\&=\sum_{l=1}^{N}a_l^+(\vec{p})a_l(\vec{p})\end{aligned}\qquad(99)$$

**Complex scalar fields :**

**الحقول السلمية المركبة:**

If we combined two scalar real fields $\varphi_1(\vec{r},t)$ and $\varphi_2(\vec{r},t)$:

إذا اعتبرنا حقلين سلميين حقيقين $\varphi_1(\vec{r},t)$ و $\varphi_2(\vec{r},t)$:

$$\begin{cases} \varphi_1(\vec{r},t) = \int \frac{d^3p}{2E_p(2\Pi)^3}\left[a_1(\vec{p})\exp\left(-\frac{i}{\hbar}p_\mu x^\mu\right) + a_1^+(\vec{p})\exp\left(\frac{i}{\hbar}p_\mu x^\mu\right)\right] \\ \varphi_2(\vec{r},t) = \int \frac{d^3p}{2E_p(2\Pi)^3}\left[a_2(\vec{p})\exp\left(-\frac{i}{\hbar}p_\mu x^\mu\right) + a_2^+(\vec{p})\exp\left(\frac{i}{\hbar}p_\mu x^\mu\right)\right] \end{cases} \quad (100)$$

With the model :

وفق النموذجك

$$\begin{cases} \varphi(\vec{r},t) = \frac{1}{\sqrt{2}}\left[\varphi_1(\vec{r},t) + i\varphi_2(\vec{r},t)\right] \\ \varphi^+(\vec{r},t) = \frac{1}{\sqrt{2}}\left[\varphi_1(\vec{r},t) - i\varphi_2(\vec{r},t)\right] \end{cases} \quad (101)$$

Gives the following complex scalar field :

نعطي الحقل السلمي المركب:

$$\varphi(\vec{r},t) = \int \frac{d^3p}{2E_p(2\Pi)^3}\left[a(\vec{p})\exp\left(-\frac{i}{\hbar}p_\mu x^\mu\right) + b^+(\vec{p})\exp\left(\frac{i}{\hbar}p_\mu x^\mu\right)\right] \quad (102)$$

With

$$\begin{aligned} a(\vec{p}) &= \frac{1}{\sqrt{2}}\left[a_1(\vec{p}) + ia_2(\vec{p})\right] \\ b^+(\vec{p}) &= \frac{1}{\sqrt{2}}\left[a_1^+(\vec{p}) + ia_2^+(\vec{p})\right] \neq a^+(\vec{p}) = \frac{1}{\sqrt{2}}\left[a_1(\vec{p}) - ia_2(\vec{p})\right] \end{aligned} \quad (103)$$

The Lagrangian density for the complex scalar field is given by :

كثافة اللاغرنجيان للحقل السلمي المركب:

$$\Im = \partial_\mu \varphi^+ \partial^\mu \varphi - m^2 \varphi^+ \varphi \quad (104)$$

The *CCR*s of the creation and annihilation operators which crate the *ETCRs* are given by :

علاقات التبادل *CCR*s لمؤثرات الخفض و التكوين التي تشكل *CCR*s تعطى كمايلي:

$$\begin{aligned}
&\left[a(\vec{p}),a^{+}(\vec{p}')\right]=(2\Pi)^{3}2E_{p}\delta^{(3)}(\vec{p}-\vec{p}')\\
&\left[b(\vec{p}),b^{+}(\vec{p}')\right]=(2\Pi)^{3}2E_{p}\delta^{(3)}(\vec{p}-\vec{p}')\\
&\left[a(\vec{p}),a(\vec{p}')\right]=0\\
&\left[a^{+}(\vec{p}),a^{+}(\vec{p}')\right]=0\\
&\left[b(\vec{p}),b(\vec{p}')\right]=0\\
&\left[b^{+}(\vec{p}),b^{+}(\vec{p}')\right]=0
\end{aligned} \qquad (105)$$

For the complex scalar field, the commutators $\left[H,a^{+}(\vec{p})\right]$ , $\left[H,a(\vec{p})\right]$, $\left[H,b^{+}(\vec{p})\right]$ and $\left[H,b(\vec{p})\right]$ satisfy the algebra:

للحقل السلمي المركب المبدلات $\left[H,b^{+}(\vec{p})\right]$ ,$\left[H,a(\vec{p})\right]$ , $\left[H,a^{+}(\vec{p})\right]$ و $\left[H,b(\vec{p})\right]$ تحقق:

$$\begin{aligned}
&\left[H,a^{+}(\vec{p})\right]=Ea^{+}(\vec{p})\\
&\left[H,a(\vec{p})\right]=-Ea(\vec{p})\\
&\left[H,b^{+}(\vec{p})\right]=Eb^{+}(\vec{p})\\
&\left[H,b(\vec{p})\right]=-Eb(\vec{p})
\end{aligned} \qquad (106)$$

Thus, $a(\vec{p})$ and $b(\vec{p})$ are presents of annihilations of the complex scalar particle with charge positive and the complex scalar particle with charge negative while $a^{+}(\vec{p})$ and $b^{+}(\vec{p})$ are presents of creations of the complex scalar particle with charge positive and the complex scalar particle with charge negative.

هكذا, $a(\vec{p})$ و $b(\vec{p})$ يمثلان مؤثرات الخفض للحقل السلمي المركب بشحنة سالبة او موجبة في حين $a^{+}(\vec{p})$ و $b^{+}(\vec{p})$ يمثلان مؤثرات التكوين لحقل سلمي مركب بشحنة موجبة أو سالبة.

**Charge conservation** :

**انحفاظ الشحنة:**

We have seen in the first part of the first chapter, Eq. (46), the conserved current $j^{\mu}$ is obtained by applying Nother’s theorem :

راينا في القسم الأول من الفصل الأول في المعادلة (46) أن التيار المحفوظ $j^{\mu}$ يمكن الحصول عليه بتطبيق نظرية نوذر:

$$j^{\mu}=\frac{\partial\Im}{\partial(\partial_{\mu}\varphi)}\delta\varphi \quad \text{and} \quad \partial_{\mu}j^{\mu}=0 \qquad (107)$$

If we consider phase transformations :

إذا اعتبرنا التحويل الطوري:

$$\begin{aligned}\varphi(x) &\to \varphi'(x) = \exp(i\beta)\varphi(x) \\ \varphi^{+}(x) &\to \varphi'^{+}(x) = \exp(-i\beta)\varphi^{+}(x)\end{aligned} \tag{108}$$

If the infinitesimal parameter $\beta$ is a constant, this symmetry is known by global gauge transformations, thus the phase transformations reduced to the form :

اذا الوسيط المتناهي في الصغر $\beta$ ثابت’ فأن هذا التناظر يعرف تناظر المعايرة الإجمالي, و منه التحويل الطوري يختصر على النحو:

$$\begin{aligned}\varphi(x) &\to \varphi'(x) \cong (1+i\beta)\varphi(x) \Rightarrow \delta\varphi(x) = \varphi'(x) - \varphi(x) = i\beta\varphi(x) \\ \varphi^{+}(x) &\to \varphi'(x) \cong (1-i\beta)\varphi(x) \Rightarrow \delta\varphi^{+}(x) = \varphi^{+}{}'(x) - \varphi^{+}(x) = -i\beta\varphi^{+}(x)\end{aligned} \tag{109}$$

And

$$\begin{cases}\dfrac{\partial \Im}{\partial(\partial_{\mu}\varphi)} = \partial^{\mu}\varphi^{+} \\ \dfrac{\partial \Im}{\partial(\partial_{\mu}\varphi)} = \partial^{\mu}\varphi\end{cases} \tag{110}$$

Thus, the conserved current $j^{\mu}$ is taking the following form :

و منه يكون التيار المحفوظ $j^{\mu}$ كمايلي:

$$j^{\mu} = i : \varphi^{+}(x)\left(\partial^{\mu}\varphi(x)\right) - \left(\partial^{\mu}\varphi^{+}(x)\right)\varphi(x) : \tag{111}$$

We have replaced the ordinary product with a normal product. The corresponding conserved charge is given by :

قمنا بتعويض الجداء العادي بالجداء الطبيعي. عبارة الشحنة الموافقة تكون:

$$Q = \int d^{3}x j^{0} \qquad \text{(Ch\_1, Part : 1, 47)}$$

**Homework :**

1_In terms of creations and annihilations operators show that the conserved charge is taking the following form :

بدلالة مؤثرات الخفض و التكوين بين ان الشحنة المحفوظة تكون كمايلي:

$$Q = \int \frac{d^{3}p}{2E_{p}(2\Pi)^{3}}\left[a^{+}(\vec{p})a(\vec{p}) - b^{+}(\vec{p})b(\vec{p})\right] \tag{112}$$

2-Show that the charge satisfies the following algebra :

بين ان الشحنة تحقق العلاقات الجبرية:

$$\begin{aligned}&[Q,a^+(\vec{p})]=a^+(\vec{p})\\&[Q,a(\vec{p})]=-a(\vec{p})\\&[Q,b^+(\vec{p})]=-b^+(\vec{p})\\&[Q,a(\vec{p})]=-(-)a(\vec{p})\end{aligned} \qquad (114)$$

We conclude from these results that the operator $a^+(\vec{p})$ can create a particle with charge positive and the operator $a(\vec{p})$ can annihilate a particle with charge positive also, while the operator $b^+(\vec{p})$ can create a particle with charge negative and $b(\vec{p})$ can annihilate a particle with charge negative.

نستنتج من خلال هاته النتائج بان المؤثر $a^+(\vec{p})$ بامكانة تكوين جسيم مشحون موجبا أما المؤثر $a(\vec{p})$ يمكنه الغاء جسما سلميا مشحونا موجبا. في حين أن المؤثر $b^+(\vec{p})$يمكنه تكوين جسيما سلميا مركبا مشحونا سالبا و المؤثر $b(\vec{p})$ بامكانه الغاء جسيم سلمي سالب الشحنة.

**Covariant commutation relations :**

In this section, we want to generalize ***ETCRs*** to any time, which we have seen in Eq. (59). It is easy to show that the general commutator $[\varphi(x),\varphi(y)]$ can be written to the following form :

في هاته الفقرة نريد تعميم *ETCRs لاي لحظة زمنية' التي ريناها في المعادلة (59). من السهل ان نبين ان المبدل العام* $[\varphi(x),\varphi(y)]$ يمكن كتابته على الشكل:

$$[\varphi(x),\varphi(y)]=[\varphi^+(x),\varphi^+(y)]+[\varphi^+(x),\varphi^-(y)]+[\varphi^-(x),\varphi^+(y)]+[\varphi^-(x),\varphi^-(y)] \qquad (115)$$

Since we have :

و لانه لدينا:

$$\begin{aligned}&[\varphi^+(x),\varphi^+(y)]=0\\&[\varphi^-(x),\varphi^-(y)]=0\end{aligned} \qquad (116)$$

The only non-null commutators are :

المبدلات غير المعدومة هي:

$$[\varphi(x),\varphi(y)]=[\varphi^+(x),\varphi^-(y)]+[\varphi^-(x),\varphi^+(y)] \qquad (115)$$

If we use the two modes which we have seen in Eq. (89):

اذا استعملنا النمطين اللذان رايناهما في المعادلة (89) :

$$\varphi^{+}(\vec{r},t)=\int\frac{d^{3}p}{2E_{p}(2\Pi)^{3}}a(\vec{p})\exp(-ipx)$$
$$\varphi^{(-)}(\vec{r},t)=\int\frac{d^{3}p}{2E_{p}(2\Pi)^{3}}a^{+}(\vec{p})\exp(ipx) \qquad (89)$$

The commutator $\left[\varphi^{+}(x),\varphi^{-}(y)\right]$ is then taking the form :

المبدل $\left[\varphi^{+}(x),\varphi^{-}(y)\right]$ يأخذ الشكل:

$$\left[\varphi^{+}(x),\varphi^{-}(y)\right]=\left[\int\frac{d^{3}p}{2E_{p}(2\Pi)^{3}}a(\vec{p})\exp(-ipx),\int\frac{d^{3}p'}{2E_{p'}(2\Pi)^{3}}a^{+}(\vec{p}')\exp(ip'y)\right]$$
$$=\int\frac{d^{3}p}{2E_{p}(2\Pi)^{3}}\int\frac{d^{3}p'}{2E_{p'}(2\Pi)^{3}}\exp(-ipx+ip'y)\left[a(\vec{p}),a^{+}(\vec{p}')\right]$$
$$=\int\frac{d^{3}p}{2E_{p}(2\Pi)^{3}}\int\frac{d^{3}p'}{2E_{p'}(2\Pi)^{3}}\exp(-ipx+ip'y)(2\Pi)^{3}2E_{p}\delta^{(3)}(\vec{p}-\vec{p}') \qquad (116)$$
$$=\int\frac{d^{3}p}{2E_{p}(2\Pi)^{3}}\exp(-ip(x-y))$$

If we introduce the following function :

اذا ادخلنا الترميز التالي:

$$\Delta^{+}(x)=-i\int\frac{d^{3}p}{2E_{p}(2\Pi)^{3}}\exp(-ipx) \qquad (117)$$

Allows us to write Eq. (116) as follows :

هذا يسمح بكتابة المعادلة (116) على النحو:

$$\left[\varphi^{+}(x),\varphi^{-}(y)\right]=i\Delta^{+}(x-y) \qquad (118)$$

Now for the commutator $\left[\varphi^{-}(x),\varphi^{+}(y)\right]$ is then taking the form :

الان المبدل $\left[\varphi^{-}(x),\varphi^{+}(y)\right]$ يأخذ الشكل:

$$\begin{aligned}\left[\varphi^-(x),\varphi^+(y)\right] &= \left[\int \frac{d^3p}{2E_p(2\Pi)^3} a^+(\vec{p})\exp(ipx), \int \frac{d^3p'}{2E_{p'}(2\Pi)^3} a(\vec{p}')\exp(-ip'y)\right] \\ &= -\int \frac{d^3p}{2E_p(2\Pi)^3}\int \frac{d^3p'}{2E_{p'}(2\Pi)^3}\exp(ipx-ip'y)\left[a(\vec{p}'),a^+(\vec{p})\right] \\ &= \int \frac{d^3p}{2E_p(2\Pi)^3}\int \frac{d^3p'}{2E_{p'}(2\Pi)^3}\exp(ipx-ip'y)(2\Pi)^3 2E_p\delta^{(3)}(\vec{p}-\vec{p}') \\ &= \int \frac{d^3p}{2E_p(2\Pi)^3}\exp(ip(x-y))\end{aligned} \tag{119}$$

If we introduce the following function :

اذا ادخلنا الترميز:

$$\Delta^-(x) = i\int \frac{d^3p}{2E_p(2\Pi)^3}\exp(ipx) \tag{120}$$

Allows us to write Eq. (120) as follows :

هذا يسمح بكتابة المعادلة (120) على النحو:

$$\left[\varphi^-(x),\varphi^+(y)\right] = i\Delta^-(x-y) \tag{121}$$

Which give the following result:

مما يسمح بإيجاد النتيجة:

$$\left[\varphi(x),\varphi(y)\right] = i\Delta(x-y) \tag{122}$$

Where $\Delta(x-y)$ is given by :

حيث أن $\Delta(x-y)$ يعطى بالشكل:

$$\begin{aligned}\Delta(x-y) &= \Delta^+(x-y)+\Delta^-(x-y) \\ &= -2\int \frac{d^3p}{2E_p(2\Pi)^3}\sin(p(x-y))\end{aligned} \tag{123}$$

The quantity $[\varphi(x),\varphi(y)]$ is a Lorentz scalar and must be invariant under Lorentz transformations. It is clear, that the function $\Delta(x-y)$ is real and may be written in a covariant form as follows :

الكمية $[\varphi(x),\varphi(y)]$ هي كمية سلمية و التي يجب ان تكون صامدة بتحويلات لورنتز. من الواضح أن $\Delta(x-y)$ حقيقية و التي يمكن كتابتها على الشكل:

$$\Delta(x-y)=-i\int\frac{d^4p}{2E_p(2\Pi)^4}2\Pi\delta(p^2-m^2)\varepsilon(p_0)\exp(-ip(x-y)) \tag{124}$$

Where $p_0\in\left]-\infty,+\infty\right[$ and $d^4p=d^3pdp_0$ while $\varepsilon(p_0)$ is given by :

حيث أن $p_0\in\left]-\infty,+\infty\right[$ و $d^4p=d^3pdp_0$ في حين أن $\varepsilon(p_0)$ تعطى بالعبارة:

$$\varepsilon(p_0)=\frac{p_0}{|p_0|}=\begin{cases}+1 & for\quad p_0\succ 0\\ -1 & for\quad p_0\prec 0\end{cases} \tag{125}$$

The quantity $\Delta(x-y)$ satisfies the Klein-Gordon equation and vanishing when the interval space-time is a space like :

الكمية $\Delta(x-y)$ تحقق معادلة كلين-غوردن و تنعدم من أجل الفترة من النوع الفضائي:

$$(x-y)^2=(t_2-t_1)^2-(x_2-x_1)^2-(y_2-y_1)^2-(z_2-z_1)^2\prec 0\Rightarrow[\varphi(x),\varphi(y)]=0 \tag{126}$$

Thus, the micorocausality condition is satisfied. It is important to notice that the $p^2-m^2$ can be written as follows :

هكذا فان شرط السببية الدقيقة يكون محقق. من المهم حساب $p^2-m^2$ الذي يكون كمايلي:

$$\begin{aligned}p^2-m^2&=p_0^2-\vec{p}^2-m^2\\&=p_0^2-E_p^2=(p_0-E_p)(p_0+E_p)\end{aligned} \tag{127}$$

Allows to consider two poles, the first at $p_0=E_p$ and the second at $p_0=-E_p$, thus the various invariant $\Delta^+(x-y)$ and $\Delta^-(x-y)$as a contour integrals in the complex $p_0$ plane is given by :

مما يسمح بتكوين قطبين الأول $p_0=E_p$ و الثاني $p_0=-E_p$ , و بالتالي مختلف القيم الصامدة $\Delta^+(x-y)$ و $\Delta^-(x-y)$ تكون كونتور تكاملات في المستوى المركب $p_0$ كمايلي:

$$\Delta^{\pm}(x-y)=-\frac{1}{(2\Pi)^4}\int_{C_\pm}d^4p\frac{\exp(-ip(x-y))}{p^2-m^2} \tag{128}$$

The time ordering operation $T$ is defined as :

مؤثر ترتيب المؤثرات زمنيا $T$ يعرف كمايلي:

$$T\left(\varphi\left(x\left(t,\vec{r}\right)\right)\varphi\left(y\left(t',\vec{r}'\right)\right)\right)=\begin{cases}\varphi(x)\varphi(y) & \text{for} \quad t\succ t' \\ \varphi(y)\varphi(x) & \text{for} \quad t'\succ t\end{cases} \tag{129}$$

Using the step function $T\left(\varphi\left(x\left(t,\vec{r}\right)\right)\varphi\left(y\left(t',\vec{r}'\right)\right)\right)$ is given by :

$$T\left(\varphi\left(x\left(t,\vec{r}\right)\right)\varphi\left(y\left(t',\vec{r}'\right)\right)\right)=\theta(t-t')\varphi(x)\varphi(y)+\theta(t'-t)\varphi(y)\varphi(x)$$
$$\text{with} \qquad \theta(t-t')=\begin{cases}+1 & \text{for} \quad t\succ t' \\ 0 & \text{for} \quad t'\succ t\end{cases} \tag{130}$$

The Feynman of the bosonic fields is noted with $\Delta_F(x-y)$ is defined as an expectation value of the quantity :

دالة فاينمان للحقول البوزونية يرمز لها بـ:$\Delta_F(x-y)$ و هي القيمة المتوقعة في الفراغ:

$$\begin{aligned} i\Delta_F(x-y) &= \langle 0|T\left(\varphi\left(x\left(t,\vec{r}\right)\right)\varphi\left(y\left(t',\vec{r}'\right)\right)\right)|0\rangle \\ &= \theta(t-t')\langle 0|\varphi(x)\varphi(y)|0\rangle+\theta(t'-t)\langle 0|\varphi(y)\varphi(x)|0\rangle \\ &= \theta(t-t')\langle 0|\left[\varphi^+(x),\varphi^-(y)\right]|0\rangle-\theta(t'-t)\langle 0|\left[\varphi^-(x),\varphi^+(y)\right]|0\rangle \\ &= \theta(t-t')\langle 0|i\Delta^+(x-y)|0\rangle-\theta(t'-t)\langle 0|i\Delta^-(x-y)|0\rangle \\ &= \theta(t-t')i\Delta^+(x-y)\langle 0\|0\rangle-\theta(t'-t)i\Delta^-(x-y)\langle 0\|0\rangle \\ &= \theta(t-t')i\Delta^+(x-y)-\theta(t'-t)i\Delta^-(x-y) \end{aligned} \tag{131}$$

Thus, the Feynman function for the real scalar field $\Delta_F(x-y)$ is given by :

و بالتالي دالة فاينمان $\Delta_F(x-y)$ تعطى كمايلي:

$$\Delta_F(x-y)=\theta(t-t')\Delta^+(x-y)-\theta(t'-t)\Delta^-(x-y) \tag{132}$$

For $t'\succ t$, the Feynman function for the real scalar field $\Delta_F(x-y)$ reduces to the quantity $\langle 0|\varphi(y)\varphi(x)|0\rangle$, we interpreted as a propagation of a virtual particle from the coordinate $x(t,\vec{r})$ (which is created with creation operator $a^+(\vec{p})$)to the coordinate $y(t',\vec{r}')$ which annihilates with annihilation operator $a(\vec{p})$.

من أجل $t'\succ t$, دالة فاينمان للحقول السلمية الحقيقية $\Delta_F(x-y)$ تختصر للكمية $\langle 0|\varphi(y)\varphi(x)|0\rangle$, نفسرها على أنها تمثل انتشار جسيم افتراضي ينشأ في الموضع $x(t,\vec{r})$ بمؤثر التكوين $a^+(\vec{p})$ لينتشر للموضع $y(t',\vec{r}')$حيث يتم إعدامه بمؤثر الخفض.

For $t \succ t'$, the Feynman function for the real scalar field $\Delta_F(x-y)$ reduces to the quantity $\langle 0|\varphi(x)\varphi(y)|0\rangle$, we interpreted as a propagation of a virtual particle from the coordinate $y(t',\vec{r}')$ (which is created with creation operator $a^+(\vec{p})$)to the coordinate $x(t,\vec{r})$ which annihilates with annihilation operator $a(\vec{p})$.

من أجل $t \succ t'$, دالة فاينمان للحقول السلمية الحقيقية $\Delta_F(x-y)$ تختصر للكمية $\langle 0|\varphi(x)\varphi(y)|0\rangle$, نفسرها على أنها تمثل انتشار جسيم افتراضي ينشأ في الموضع $y(t',\vec{r}')$ بمؤثر التكوين $a^+(\vec{p})$ لينتشر للموضع $x(t,\vec{r})$ حيث يتم إعدامه بمؤثر الخفض.

Finally, the Feynman function for the complex scalar field $\Delta_F(x-y)$ is given by :

في الأخير, دالة فاينمان للحقول السلمية المركبة تعرف كمايلي:

$$i\Delta_F(x-y) = \langle 0|T\left(\varphi(x(t,\vec{r}))\varphi^+(y(t',\vec{r}'))\right)|0\rangle \tag{133}$$

و تكون معالجة المسالة بنفس الأسلوب المتبع في حالة الحقل السلمي الحقيقي.

The physical treatment must be similar to the case of the scalar real field.

**Wait for the second chapter soon**

**أنتظرو الجزء الثاني**

**Wait for the second chapter soon**

**أنتظرو الجزء الثاني**

**References**

1- Jack Gunion and U.C. Davis, *Class notes for quantum field theory: Section* I, 230A, U.C. Davis, Fall Quarter

2-W. Greiner, *Relativistic quantum mechanics*, ISBN: 3-540-67457-8, 3rd Edition Springer-Verlage Berlin Heidelberg New York (**2000**).

3-Hendrik van Hees, *Introduction to relativistic quantum field theory*, Texas AM University Cyclotron Institute MS-3366. USA (**2006**).

4-John W. Norbury, Quantum field theory, Physics department, University of Wisconsin-Milwaukee (**2000**).

5-C. Grossetete, *Relativité restreinte et structure atomique de la matiére*, ellipses, ISBN 2-7298-8554-4 (1**985**).

6-Abdelmadjid Maireche, *A physical view of the theory of special relativity with a general review of classical mechanics* (Arabic version) OPU, ISBN 978.9961.0.485.1 (**2014**).

7-Abdelmadjid Maireche, *Classical theory of physical fields* (Arabic version) OPU, ISBN 978.9961.0.1793.7 (**2015**).

8- D. Bailin and A. Love, Introduction to Gauge Field Theory, IOP Publishing Limited 1993. ISBN 0-85274-817-5. England.

9-Mikkail Shaposhnikov, Seven Bachmann, Champs quantiques relativistes, Ecole Polytechnique Fédérale de Lausanne (2005).

10- J. D. Drell, S. D. Bjorken, Relativistic Quantum Mechanics (International Series in Pure & Applied Physics), Mcgraw-Hill College, 1965.

**Prepared by professor Abdelmadjid MAIRECHE**

*Laboratory of Physics and Material Chemistry, Physics department, Sciences Faculty, University of M'sila- Algeria.*

Researchgate: https://www.researchgate.net/profile/Abdelmadjid_Maireche
Google Scholar: https://scholar.google.fr/scholar?hl=fr&as_sdt=0%2C5&q=Abdelmadjid+maireche&oq=

ORCIDO: https://orcid.org/0000-0002-8743-9926