TP03: Backstepping Control of Permanent Magnet Synchronous Motors

PMSM MODEL

The setting in the state form of the PMSM model allows the simulation of this latter. In the rotor rotating ($d-q$) reference frame, the PMSM stator current model is described as follows [18], [19]:

$$\begin{cases} \dot{x} = f(x) + Bu + DT_L \\ x = \begin{bmatrix} x_1 & x_2 & x_3 \end{bmatrix}^T = \begin{bmatrix} i_d & i_q & \omega_r \end{bmatrix}^T \\ u = \begin{bmatrix} u_d = V_d \\ u_q = V_q \end{bmatrix};\ B = \begin{bmatrix} b_1 & 0 & 0 \\ 0 & b_2 & 0 \end{bmatrix}^T \\ D = \begin{bmatrix} 0 & 0 & d \end{bmatrix}^T \end{cases} \tag{1}$$

With the following expression of field vector $f(x)$:

$$\begin{cases} f_1(x) = a_1 x_1 + a_2 x_2 x_3 \\ f_2(x) = a_3 x_2 + a_4 x_3 + a_5 x_1 x_3 \\ f_3(x) = a_6 x_2 + a_7 x_3 + a_8 x_1 x_2 \end{cases} \tag{2}$$

The components of this vector are expressed according to the PMSM parameters as follows:

$$\begin{cases} a_1 = -\dfrac{R_s}{L_d};\ a_2 = \dfrac{L_q}{L_d};\ a_3 = -\dfrac{R_s}{L_q}\ a_4 = -\dfrac{\varphi_f}{L_d};\ a_5 = \dfrac{L_d}{L_q} \\ a_6 = -\dfrac{n_p^2 \varphi_f}{J};\ a_7 = -\dfrac{f}{J};\ a_8 = \dfrac{n_p^2 \varphi_f}{J}(L_d - L_q) \\ d = -\dfrac{n_p}{J};\ b_1 = \dfrac{1}{L_d};\ b_2 = -\dfrac{1}{L_q} \end{cases}$$

Where : i_d , i_q : *d*, *q* axis stator current;

V_d , V_q : *d*, *q* axis stator voltage;

L_d , L_q : *d*, *q* axis stator inductance;

R_s : Stator resistance;

φ_f : Rotor permanent magnet flux.

ω_r : Mechanical rotor speed ($\omega_r = n_p \Omega$)

f : Viscous friction coefficient

L_T : Load torque

J : Moment of Inertia

As presented in the appendix we take in this paper in PMSM with smooth poles $L_d = L_q = L$ in this case $(a_8 = 0)$

The use of the classical controllers such as the proportional and integral controller (PI) is insufficient to provide good speed tracking performance. To overcome these problems, a robust controller based on backstepping control approach is proposed.

III. BACKSTEPPING CONTROL TECHNIQUE

The Backstepping is a systematic and recursive design methodology for nonlinear feedback control. This approach is based upon a systematic procedure for the design of feedback control strategies suitable for the design of a large class of feedback linearisable nonlinear systems exhibiting constant uncertainty, and it guarantees global regulation and tracking for the class of nonlinear systems transformable into the parametric-strict feedback form. The backstepping design alleviates some limitations of other approaches [18, 20, 21]. It offers a choice of design tools to accommodate uncertainties and nonlinearities and can avoid wasteful cancellations.

The idea of backstepping design is to select recursively some appropriate functions of state variables as pseudo-control inputs for lower dimension subsystems of the overall system. Each backstepping stage results into a new pseudo-control design, expressed in terms of the pseudo-control designs from the preceding design stages. When the procedure terminates, a feedback design for the true control input results and achieves the original design objective by virtue of a Lyapunov function, which is formed by summing up the Lyapunov functions associated with each individual design stage [21, 17, 22].

The control objective in this case is to force the PMSM speed ($\omega_r = x_3$) to follow its reference x_3^* and maintain in the same time the direct current ($i_d = x_1$) to zero under load torque disturbance. The application of the backstepping control strategy to the PMSM in this case is divided into two steps (see [18, 20]).

1. Speed regulator:

This first step consists to identify the error e_ω which represents the error between real speed $\omega_r = x_3$ and reference $\omega_r^* = x_3^*$. In this case we control x_3 by x_2 .

Let the Lyapunov function:

$$V_1 = \frac{1}{2} e_\omega^2 = \frac{1}{2}(x_3 - x_3^*)^2 \tag{3}$$

Whose derivative is:

$$\dot{V}_1 = e_\omega \dot{e}_\omega = (x_3 - x_3^*)(\dot{x}_3 - \dot{x}_3^*) \tag{4}$$

The error derivative is given by:

$$\dot{e}_\omega = a_6 x_2 + a_7 x_3 + dT_L - \dot{x}_3^* \tag{5}$$

If we selected stabilizing functions as follows:

$$x_2^* = \frac{1}{a_6}(-a_7 x_3 - K_1 e_\omega - dT_L + \dot{x}_3^*) \tag{6}$$

Where $k_1 > 0$

Then the derivative of Lyapunov function $\dot{V}_1$ is written as:

$$\dot{V}_1 < -K_1 e_\omega^2 < 0 \tag{8}$$

This guarantees convergence of the speed ω_r to its reference x_3^* with robustness respect to load torque disturbance.

2. Direct and Quadrature currents regulator:

The second step consists to control the currents $i_d = x_1$ and $i_q = x_2$ by the voltages $u_d = V_d$ and $u_q = V_q$; where $x_1 \to x_1^* = 0$ and $x_2 \to x_2^*$

Consider the following Lyapunov function:

$$V = \frac{1}{2}e_\omega^2 + \frac{1}{2}e_q^2 + \frac{1}{2}x_1^2 \quad (9)$$

Where $e_\omega = (x_3 - x_3^*)$ and $e_q = (x_2 - x_2^*)$

The derivative of V with respect to time is:

$$\dot{V} = e_\omega \dot{e}_\omega + e_q \dot{e}_q + x_1 \dot{x}_1 \quad (10)$$

From (10) and in order to control x_2 by u_d. The term of the derivative V can be written as:

$$x_1 \dot{x}_1 = x_1(f_1(x) + b_1 u_d) \quad (11)$$

If we take the first control low as:

$$u_d = \frac{1}{b_1}(-f_1(x) - K_3 x_1) \quad (12)$$

Then this term is written:

$$x_1 \dot{x}_1 = -K_3 x_1^2 < 0 \quad (13)$$

Where $K_3 > 0$ then the convergence of x_1 to 0 is ensuring.

The remaining terms of (10) Let $e = e_\omega \dot{e}_\omega + e_q \dot{e}_q$ as:

$$e = (x_3 - x_3^*)(\dot{x}_3 - \dot{x}_3^*) + (x_2 - x_2^*)(\dot{x}_2 - \dot{x}_2^*) \quad (14)$$

The first term can be written as:

$$e_\omega \dot{e}_\omega = (x_3 - x_3^*)(a_6 x_2 + a_7 x_3 + dT_L - \dot{x}_3^*) \quad (15)$$

or by adding and subtracting the term $a_6 x_2^*$ we get:

$$e_\omega \dot{e}_\omega = (x_3 - x_3^*)(a_6(x_2 - x_2^*) + a_6 x_2 + a_7 x_3 + dT_L - \dot{x}_3^*) \quad (16)$$

By simplification :

$$\begin{aligned} e_\omega \dot{e}_\omega = {} & a_6(x_2 - x_2^*)(x_3 - x_3^*) + \\ & (x_3 - x_3^*)(a_6 x_2^* + a_7 x_3 + dT_L - \dot{x}_3^*) \end{aligned} \quad (17)$$

By replacing the term $a_6 x_2^*$ presented in (6) in this last equation we get:

$$\begin{aligned} e_\omega \dot{e}_\omega = {} & a_6(x_2 - x_2^*)(x_3 - x_3^*) + \\ & (x_3 - x_3^*)(-K_1 e_\omega + dT_L) \end{aligned} \quad (18)$$

From (6) and (18) we get:

$$e_\omega \dot{e}_\omega = a_6(x_2 - x_2^*)(x_3 - x_3^*) + \dot{V}_1 \quad (19)$$

Otherwise if one chooses:

$$u_q = \frac{1}{b_2}(-f_2(x) + x_2^* - K_4(x_2 - x_2^*) - a_6(x_3 - x_3^*)) \quad (20)$$

From the second term of (14)

$$e_q \dot{e}_q = (x_2 - x_2^*)(f_1(x) + b_2 u_q - \dot{x}_2^*) \quad (21)$$

By replacing (20) in (21) we get:

$$e_q \dot{e}_q = -K_4(x_2 - x_2^*)^2 - a_6(x_3 - x_3^*)(x_2 - x_2^*) \quad (22)$$

Where $K_4 > 0$. Finally, by grouping terms (19) and (22) we obtain:

$$\begin{aligned} \dot{V} = {} & -K_4(x_2 - x_2^*)^2 - a_6(x_3 - x_3^*)(x_2 - x_2^*) + \\ & a_6(x_2 - x_2^*)(x_3 - x_3^*) + \dot{V}_1 \end{aligned} \quad (23)$$

By simplification:

$$\dot{V} = -K_4(x_2 - x_2^*)^2 + \dot{V}_1 \quad (24)$$

From (8) and (24) we get:

$$\dot{V} = -K_4(x_2 - x_2^*)^2 - K_1(x_3 - x_3^*)^2 \quad (25)$$

Finely, from the foregoing, it is clear that it suffices to properly select the different gains K_i ($i = 1,2,3,4$) for the set-negativity of the derivative of the complete Lyapunov function ($\dot{V} \le 0$) overall V defined by (25). This implies that all the error variables are globally uniformly bounded and maintain the system closed loop performance in presence of load torque disturbances.

Rated Values	Power	22	W
	Frequency	50	Hz
	n_p	2	
Rated parameters	R_s	3.4	Ω
	L_d	0.0121	H
	L_q	0.0121	H
	φ_f	0.4212	H
	J	0.0001	Kg.m^2
	f	0.0005	IS

a1	a2	a3	a4	a5	a6	a7	a8	a9	b1	b2	d
-297.52	1	-297.52	-1.074	-1	520	-0.5	0	0.026	82.644	82.644	-20000