Exercise 1

Consider a population of simple creatures, with a single chromosome of length $n = 1000$. Each entry in the chromosome can take four values (A, C, G, T). Assume that the population size is equal to M.

1. How many possible chromosomes are there?
2. Assuming that the chromosome length and the population size remain constant, what is the upper limit of the number of different chromosomes evaluated in the course of G generations?
3. If the population size is constant and equal to 10^{12}, how large a fraction q of the total number of chromosomes will be evaluated during 10^9 generations, assuming that all evaluated chromosomes are different?

Exercise 2

Consider the simple genetic algorithm applied on a population of 8 integer numbers. Suppose that at time t of the evolution the population has the following composition:

- $x=1$: 2 copies
- $x=2$: 3 copies
- $x=3$: 3 copies

Assuming that the fitness function is $f(x)=x^2$, calculate the probability of selecting the individuals $x=1$, $x=2$, and $x=3$ using roulette wheel selection.

Exercise 3

Consider a population consisting of five individuals with the fitness values (before ranking) $f_1 = 5, f_2 = 7, f_3 = 8, f_4 = 10, f_5 = 15$. Compute the probability that individual 4 will be selected (in a single selection step) with

(a) roulette wheel selection
(b) tournament selection, with tournament size equal to 2, and the probability of selecting the best individual (in a given tournament) equal to 0.75
(c) roulette wheel selection, based on linearly ranked fitness values, where the lowest fitness value is set to 1 and the highest fitness value set to 10.

Exercise 4

Calculate the probability that a binary chromosome of length L will not be changed by applying the usual bit-flip mutation with probability $p_m = 1/L$

Exercise 5

Write an evolutionary algorithm that searches for the shortest route between N cities. Use an encoding method such that the chromosomes consist of lists of integers determining the indices of the cities. Examples of five-city paths starting in city 4 are e.g. (4,3,1,2,5),(4,1,5,2,3),(4,5,1,2,3) etc. The first chromosome thus encodes the path 4 → 3 → 1 → 2 → 5 → 4. The fitness should be taken as the inverse of the route length (calculated using the ordinary cartesian distance measure, i.e. not the city-block distance measure). The program should always generate syntactically correct routes, i.e. routes in which each city is visited once and only once until, in the final step, the tour ends by a return to the starting city. Specialized operators for crossover and mutation are needed in order to ensure that the paths are syntactically correct.

(a) Define a mutation operator for the TSP that maps valid chromosomes (i.e. paths) onto other valid chromosomes.
(b) Define a crossover operator for the TSP that maps valid chromosomes onto other valid chromosomes.

(c) Using the specialized crossover and mutation operators, write an Evolutionary Algorithm that solves the Travelling Salesman Problem.

Exercise 6

Implement an Evolutionary Algorithm that solves the 8-queens problem.

Exercise 7

Consider a generational GA that has a population size of 100 individuals and uses roulette–wheel selection. Suppose that after running for *t* generations, the mean population fitness is 76.0 and that in the population there is only one copy of the best member, which has fitness 157.0. Also suppose that parents parent selection is performed on the population

- What is the expected number of copies of the best individual in the set of selected parents?
- What it the probability that there will be no copies of the best individual in the selected parents?

Exercise 8

Consider a population containing four individuals with chromosomes 101010, 000111 , 010101, and 011011, and fitness values 1,2,3 and 4 respectively. In a given selection step, assume that individual 1 (with chromosome 101010) has been selected (using roulettewheel selection) as the first parent. What is the probability that the schema 10xxxx will be represented in either of the two individuals resulting from the selection of a second parent, followed by crossover? (Crossover may occur, with equal probability, at any of the five available crossover points).

Exercise 9

Write a GA to find four integer numbers a,b,c,d such as : a +2 b +3 c +4 d = 30.
Run manually two iterations of this GA using parameters value of your choice.

<u>Exercise 10</u>

We want to look for the integer number of [0, 15] that maximize the function $f(x) = \frac{1}{4}\left|15x^2 - x^3\right| + 4$

Solve this problem using a GA with binary encoding and a population of 6 individuals.

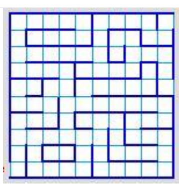
<u>Exercise 11</u>

You have 10 cards numbered 1 to 10. You have to divide them into two piles so that: the sum of the first pile is as close as possible to 36 and the product of all in the second pile is as close as possible to 360.

1. Determine individuals encoding for this problem.
2. Define a fitness function to evaluate solutions.

Exercise 12

We propose to find the path to the exit of a labyrinth.



Labyrinth

1. Determine individuals encoding for this problem.
2. Define a fitness function to evaluate solutions.

Exercise 13

We wish to realize a 2D tower with N bricks of width: 2,3,5 and 7 units. We want to build the tower, which is at the same time the highest, and the most solid. To evaluate this last criterion, we have a simulator capable of applying to the tower the combined effect of gravity and jolts of a given amplitude. After the calculation, the simulator returns a tower, potentially different, resulting from the possible displacement of the bricks during the simulation as well as the number of bricks having moved. The simulator also allows knowing the height of the tower (height of the highest element).

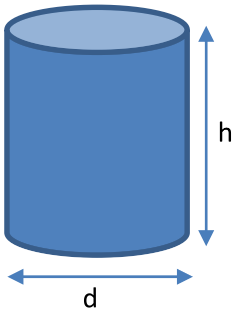
1. Determine a fitness function and the associated assessment procedure.

2. It is assumed that a tower cannot be more than 32 units wide. We propose you the following coding:
- A solution is a sequence of N 7-bit words.
- A word represents the type of brick (coded on 2 bits) and a horizontal position (on 5 bits) to which one deposits a brick. The brick is then deposited as low as possible on the chosen column. Example: 01 00100 corresponds to a brick of size 3 arranged at 4 units of the extreme left point. What are the qualities and defects of this coding?
3. Propose another way to represent a tower by seeking to remove the flaws you identified in the previous question.

Exercise 14

Solve the following problem using a GA:

$$\begin{aligned} \text{Minimize} \quad & f(d,h) = c((\pi d^2/2) + \pi d h), \\ \text{Subject to} \quad & g_1(d,h) \equiv (\pi d^2 h/4) \geq 300, \\ \text{Variable bounds} \quad & d_{\min} \leq d \leq d_{\max}, \\ & h_{\min} \leq h \leq h_{\max}. \end{aligned}$$



Take c= 0.654 ; $d_{min} = h_{min} = 0$; $d_{max} = h_{max} = 20$;

Exercise 15

Maxf(x)=sin(x)

$0 \leq x \leq \pi$

Consider 6 bit string to represent the solution, then 000000 = 0 and 111111 = π.

Assume population size of 4.

Exercise 16

$$Minimize\ f = (x_1^2 + x_2 - 11)^2 + (x_1 + x_2^2 - 7)^2$$

$0 \leq x \leq 10$; $0 \leq y \leq 10$

Références bibliographiques :

1. J. Hertz, A. Krogh & R. G. Palmer. An introduction to the theory of Neural Computation. (Addison-Wesley)
2. D. Michie, D.J. Spiegelhalter & C.C. Taylor. Machine Learning, Neural and Statistical Classification. (Ellis Horwood)
3. P. Naïm, P.H. Wuillemin, Ph. Leray, O.Pourret, A. Becker. Réseaux Bayésiens (Eyrolles)
4. http://www.librecours.org/cgi-bin/domain?callback=info&elt=190
5. http://asi.insa-rouen.fr/enseignement/siteUV/rna/