; **University of Mohamed Boudiaf-Msila**

Faculty of Sciences and Technologies **Academic Year 2023-2024.**

Module :Mathematics 01 **Renewable energy+License ST 1^{st} year**

جامعة محمد بوضياف - المسيلة
Université Mohamed Boudiaf - M'sila

SERIES OF EXERCISES N°1

جامعة محمد بوضياف - المسيلة
Université Mohamed Boudiaf - M'sila

Exercise 01

Complete the ellipses with the appropriate logical connector $\Leftrightarrow$; $\Leftarrow$; $\Rightarrow$.

1. $x \in \mathbb{R}, \quad x^2 = 4..........x = 2.$
2. $z \in \mathbb{C}, \quad \bar{z} = z..........z \in \mathbb{R}.$
3. $x \in \mathbb{R}, \quad x = \pi..........e^{2ix} = 1$

Exercise 02

Among the following statements, which ones are true, which ones are false? Provide their negation."

1. $(2+2=4) \wedge (1+1=3)$
2. $(2+2=4) \vee (1+1=3)$
3. $(2+2=4) \implies (1+1=3)$
4. $(1+1=3) \implies (2+2=4)$
5. $\forall x \in [1; +\infty[; x^2 \geq 1$
6. $\forall x \in \mathbb{R}; x^2 \geq 1$
7. $\forall x \in \mathbb{R}, x^2 = 1.$
8. $\exists x \in \mathbb{R}, x^2 = 1.$

9. $\forall x \in \mathbb{R}, \exists y \in \mathbb{R}, y = x^2$.
10. $\forall x \in \mathbb{R}, \exists y \in \mathbb{R}, x = y^2$.
11. $\exists y \in \mathbb{R}, \forall x \in \mathbb{R}, y = x^2$.

Exercise 03

Let P be "For every real number x, there exists at least one natural number n greater than or equal to x."

1. Write the proposition P using quantifiers.
2. Write the negation of P.

Exercise 04

Let's consider the implication $(a = b) \implies a^2 = b^2$ where $(a, b) \in \mathbb{R}^2$.

• Determine if the contrapositive, the converse, and the biconditional(the equivalent) statements are true. If not, determine a subset of $\mathbb{R}$ on which the statements are true.

Exercise 05

1. Give the contrapositive of the following statement: $-1 \leq x \leq 1 \implies |x| \leq 1$
2. Give the converse of the following statement: $-1 \leq x \leq 1 \implies |x| \leq 1$
3. Give the negation of the following statement: $-1 \leq x \leq 1 \implies |x| \leq 1$

Exercise 06

"Consider the following four statements:

(a) $\exists x \in \mathbb{R}, \forall y \in \mathbb{R} : x + y > 0$

(b) $\forall x \in \mathbb{R}, \exists y \in \mathbb{R} : x + y > 0$

(c) $\forall x \in \mathbb{R}, \forall y \in \mathbb{R} : x + y > 0$

(d) $\exists x \in \mathbb{R}, \forall y \in \mathbb{R} : y^2 > x$

1. Are statements the a, b, c, and d true or false?

2. Provide their negation."

Exercise 07 (Direct proof)

1. Show that if $a, b \in \mathbb{Q}$, then $a + b \in \mathbb{Q}$.
2. Let $a, b \in \mathbb{R}$. Prove that if $a \leq b$, then $a \leq \frac{a+b}{2} \leq b$.
3. Let $n \in \mathbb{N}$. Prove that if n is a multiple of 3, then n^2 is divisible by 9".

Exercise 08 (Proof by Contrapositive)

1. Let $n \in \mathbb{N}$. Show that if n^2 is even then n is even
2. Let $x \in \mathbb{R}$. Show that if $x^5 + x < 2$ then, $x < 1$.
3. Let $x, y \in \mathbb{R}$. Show that
$$x + y > 1 \implies y > \frac{4}{5} \quad \text{or} \quad x > \frac{1}{5}.$$

Exercise 09 (Proof by induction)

1. Prove by induction that, $\forall n \in \mathbb{N}^*$
$$P(n): \qquad \sum_{k=1}^{n} 3k^2 + k = n(n+1)^2.$$
2. Prove by induction that, $\forall n \in \mathbb{N}^*$
$$P(n): \qquad \sum_{k=1}^{n} k^2 + k = \frac{n(n+1)(n+2)}{3}.$$
3. Prove by induction that, $\forall n \in \mathbb{N}, \quad n(n+1)$ est pair.
4. Prove by induction that, $\forall n \in \mathbb{N}^*$
$$P(n): \qquad \sum_{k=1}^{n} k^3 = \frac{n^2(n+1)^2}{4}.$$

Exercise 10 (Proof by contradiction)

1. Prove by contradiction that, $\sqrt{2} \notin \mathbb{Q}$.
2. Let $a, b \geq 0$. Prove by contradiction that, $\left(\frac{a}{1+b} = \frac{b}{1+a}\right) \implies a = b$.

Exercise 11 (Proof by cases)

1. Show that, $\forall n \in \mathbb{N}, \quad n(n+1)$ is even.

2. Show that, $\forall x \in \mathbb{R}, \quad g(x) = x^2 - x + 1 - |x-1| \geq 0.$

3. Show that, $\forall x \in \mathbb{R}, \quad \sqrt{x^2+1} - x \geq 0.$

Exercise 12 (Proof by counterexample)

1. Give a counterexample to disprove that following statement $\forall x, y \in \mathbb{R}_+, \sqrt{x+y} \neq \sqrt{x} + \sqrt{y}$ is false.