Solutions TD N°3

Exercice 1

1. Les deux antennes dipôles sont alimentées par des courants de mêmes amplitudes et déphasés entre eux de α. Si on prend l'antenne à l'origine comme antenne de référence :

$$E_{tot} = E_{\lambda/2} + E_{\lambda/2}\, e^{+jk2d\cos\gamma} \cdot e^{+j\alpha}$$

avec $\cos\gamma = |\hat{a}_r \cdot \hat{a}_x| = \sin\theta \cdot \cos\phi$

$$E_{\lambda/2} = \frac{\cos(\pi/2 \cdot \cos\psi)}{\sin\psi} \;;\quad \sin\psi = \sqrt{1-\sin^2\theta \cdot \sin^2\phi}$$

$$\cos\psi = |\hat{a}_r \cdot \hat{a}_y| = \sin\theta \cdot \sin\phi$$

$$E_{tot} = 2E_{\lambda/2}\, e^{j\Phi}\left(\frac{e^{j\Phi}+e^{-j\Phi}}{2}\right) \;;\quad \Phi = kd\cdot\cos\gamma + \frac{\alpha}{2}$$

soit,

$$|E_{tot}|_n = \frac{\cos(\pi/2\,\sin\theta\cdot\sin\phi)}{\sqrt{1-\sin^2\theta\cdot\sin^2\phi}} \cdot \cos\left(\frac{\pi}{2}\sin\theta\cdot\cos\phi + \frac{\alpha}{2}\right)$$

2. Dans le plan $xoz \rightarrow \phi = 0° \rightarrow f(\theta) = \cos\left(\frac{\pi}{2}\sin\theta + \frac{\alpha}{2}\right)$

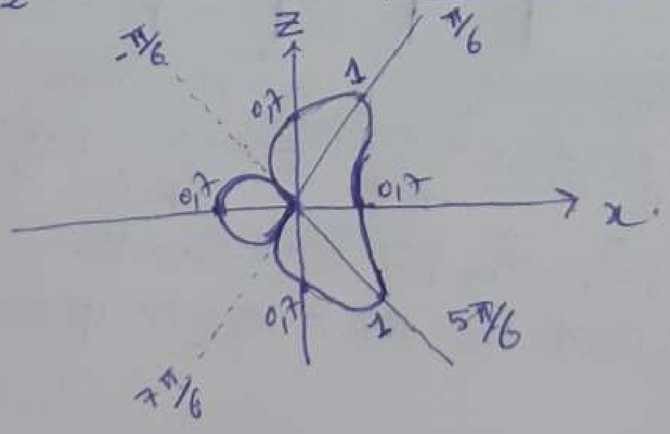
a- $\alpha = -\frac{\pi}{2} \rightarrow f(\theta) = \cos\left[\frac{\pi}{2}\sin\theta - \frac{\pi}{4}\right]$

$f(\theta) = 0 \rightarrow \cos\left(\frac{\pi}{2}\sin\theta - \frac{\pi}{4}\right) = 0 \rightarrow \frac{\pi}{2}\sin\theta - \frac{\pi}{4} = \pm\frac{\pi}{2} + 2n\pi \rightarrow$

$\sin\theta = \pm 1 + (4n + \frac{1}{2}),\ n = 0, 1, 2, \dots \rightarrow \boxed{\theta_n = -\frac{\pi}{6}, \frac{7\pi}{6}}$

$f(\theta) \rightarrow \max \rightarrow \cos\left(\frac{\pi}{2}\sin\theta - \frac{\pi}{4}\right) = \pm 1 \rightarrow \frac{\pi}{2}\sin\theta - \frac{\pi}{4} = m\pi,\ m = 0, 1, 2, \dots$

$\sin\theta = 2m + \frac{1}{2},\ m = 0, 1, 2, \dots \rightarrow \boxed{\theta_m = \frac{\pi}{6}, \frac{5\pi}{6}}$



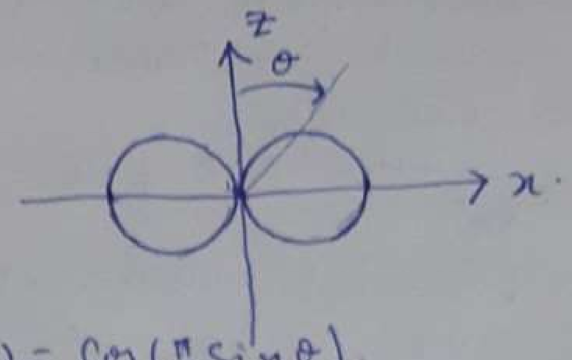
b. $\alpha = \pi \rightarrow f(\theta) = \cos\left[\frac{\pi}{2}(\sin\theta + 1)\right]$

$f(\theta) = 0 \rightarrow \sin\theta = \pm 1 + (4n-1)$, $n = 0, 1, 2, \ldots$

$\rightarrow \boxed{\theta_n = 0, \pi}$

$f(\theta) \rightarrow \text{Max} \rightarrow \frac{\pi}{2}(\sin\theta + 1) = m\pi \rightarrow \sin\theta = 2m - 1, m = 0, 1, 2, \ldots$

$\rightarrow \boxed{\theta_m = \pm\frac{\pi}{2}}$



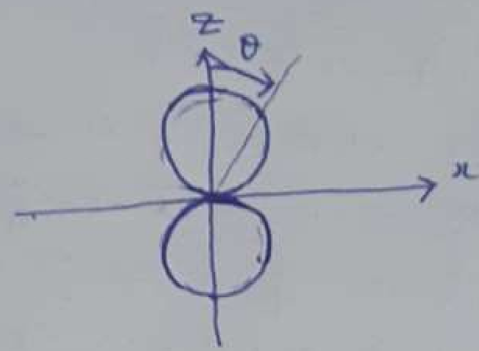
c. $\alpha = 0 \rightarrow f(\theta) = \cos\left(\frac{\pi}{2}\sin\theta\right)$

$f(\theta) = 0 \rightarrow \sin\theta = \pm 1 + 4n, \quad n = 0, 1, 2, \ldots$

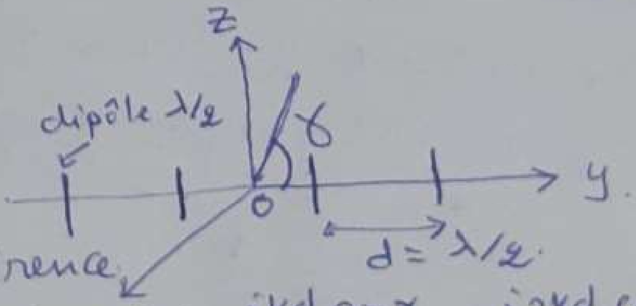
$\rightarrow \boxed{\theta = \pm\frac{\pi}{2}}$

$f(\theta) \rightarrow \text{Max} \rightarrow \cos\left(\frac{\pi}{2}\sin\theta\right) = \pm 1 \rightarrow \sin\theta = 2m, m = 0, 1, 2, \ldots$

$\rightarrow \boxed{\theta = 0, \pi}$



Exercice 2



$E_{\lambda/2} = \dfrac{\cos\left(\frac{\pi}{2}\cos\theta\right)}{\sin\theta}$

on prend comme référence l'origine O.

$AF = e^{jk\frac{d}{2}\cos\gamma} + e^{jk\frac{3d}{2}\cos\gamma} + e^{-jk\frac{d}{2}\cos\gamma} + e^{-j\frac{3kd}{2}\cos\gamma}$

avec : $\cos\gamma = |\hat{a}_r \cdot \hat{a}_y| = \sin\theta \cdot \sin\phi$.

posons $\psi = \frac{kd}{2}\cos\gamma \rightarrow AF = 2\left[\cos\psi + \cos 3\psi\right]$

Sachant que $\cos 3\psi = 4\cos^3\psi - 3\cos\psi \rightarrow$

$AF = 2(4\cdot\cos^3\psi - 2\cos\psi) = 4\cdot\cos\psi\,(2\cos^2\psi - 1) \rightarrow$

$\boxed{AF = 4\cdot\cos\psi\cdot\cos 2\psi}$; soit le champ total normalisé:

$$E_n = \frac{\cos(\pi/2\cdot\cos\theta)}{\sin\theta}\cdot\cos\psi\cdot\cos 2\psi \quad ; \quad \psi = \frac{\pi}{2}\sin\theta\cdot\sin\phi.$$

Dans le plan horizontal $\rightarrow \theta = 90° \rightarrow$

$E_n = \cos\psi\cdot\cos 2\psi$; $\psi = \frac{\pi}{2}\sin\theta$.

* $E_n = 0 \rightarrow \cos\psi = 0 \vee \cos 2\psi = 0$

$\cos\psi = 0 \rightarrow \psi = \pm n\frac{\pi}{2}$, $n = 1, 3, 5, \cdots \rightarrow \sin\phi = \pm n \rightarrow$

$\boxed{\phi_n = \pm\pi/2}$

$\cos 2\psi = 0 \rightarrow 2\psi = \pm n\pi/2 \rightarrow \sin\phi = \pm\frac{n}{2}$; $n = 1, 3, 5, \cdots$

$\rightarrow \boxed{\phi_n = \mp\frac{\pi}{6}, -\frac{5\pi}{6}, \frac{7\pi}{6}}$

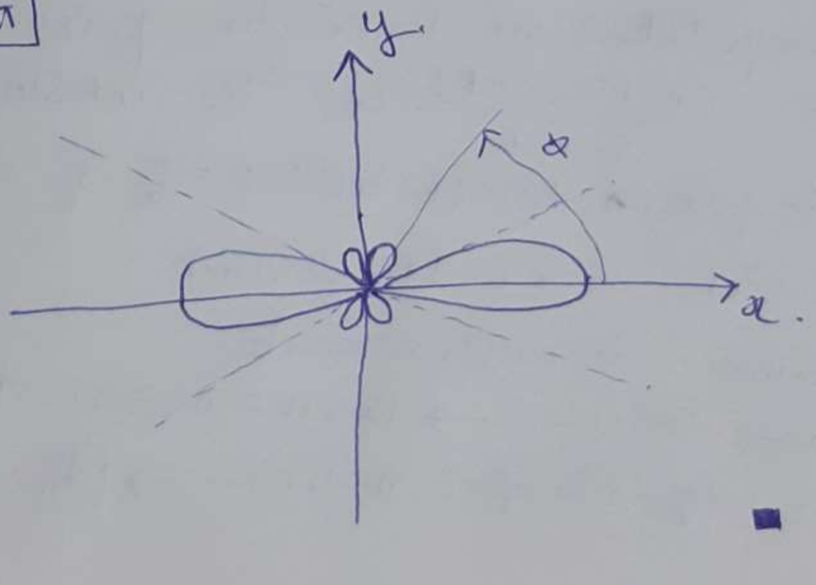
* $E_n \rightarrow Max \rightarrow \cos\psi = \pm1$ et $\cos 2\psi = \pm1$.

$\cos\psi = \pm1 \rightarrow \psi = \pm m\pi$, $m = 0, 1, 2, \cdots \rightarrow \boxed{\phi_m = 0, \pi}$

$\cos 2\psi = \pm1 \rightarrow 2\psi = \pm m\pi$, $m = 0, 1, 2 \rightarrow \boxed{\phi_m = 0, \pi, \pm\pi/2}$

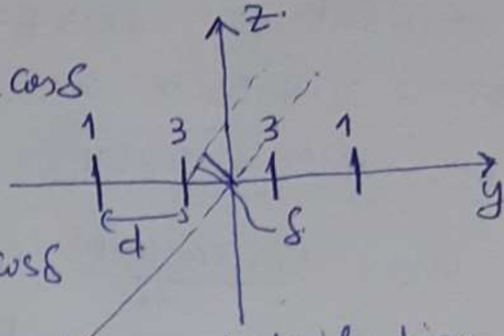
L'intersection des deux solutions donne:

$\boxed{\phi_m = 0, \pi}$



Exercice 3

$$(AF)_{2M}=\sum_{n=1}^{M} a_n \cdot \cos\left[(2n-1)u\right], \quad u=\frac{kd}{2}\cos\delta$$



distribution binômiale

$2M=4 \rightarrow M=2.$

$$(AF)_4=\sum_{n=1}^{2} a_n \cdot \cos\left[(2n-1)u\right], \quad u=\frac{\pi}{2}\cos\delta$$

$$(AF)_4=a_1\cdot\cos u+a_2\cdot\cos 3u$$

avec $a_1=3,\ a_2=1 \rightarrow (AF)_4=3\cos u+\cos 3u\ ;\ u=\frac{\pi}{2}\cos\delta$

Autrement:

$$AF=3e^{j\frac{kd}{2}\cos\delta}+3e^{-j\frac{kd}{2}\cos\delta}+e^{jk\frac{3d}{2}\cos\delta}+e^{-jk\frac{3d}{2}\cos\delta}$$

$AF=3\cos u+\cos 3u$, avec $u=\frac{kd}{2}\cos\delta=\frac{\pi}{2}\cos\delta.$

$\cos\delta=|\hat{a}_r\cdot\hat{a}_y|=\sin\theta\cdot\sin\phi \rightarrow u=\frac{\pi}{2}\sin\theta\cdot\sin\phi.$

Simplifions le AF;

$$AF=3\cos u+\cos 3u=3\cos u+\cos(u+2u)=3\cos u-\cos 2u\cos u-\sin u\cdot\sin 2u$$

$$AF=\cos u(1+2+\cos 2u)-2\sin u\cdot\sin u\cdot\cos u$$

$$AF=\cos u\left[2+2\cos^2 u-2\sin^2 u\right]=2\cos u\left[1+\cos 2u\right]$$

$$AF=4\cos^3 u \rightarrow \underline{(AF)_n=\cos^3 u}\ ;\ \underline{u=\frac{\pi}{2}\cdot\sin\theta\cdot\sin\phi}$$

Le champ total est donné donc par:

$$E_{tn}=\frac{\cos\left(\pi/2\cos\theta\right)}{\sin\theta}\cdot\cos^3\left(\pi/2\cdot\sin\theta\cdot\sin\phi\right)$$

Dans le plan horizontal: $\theta=\frac{\pi}{2}\rightarrow$

$E_{tn}=\cos^3 u$, avec $u=\frac{\pi}{2}\sin\phi$

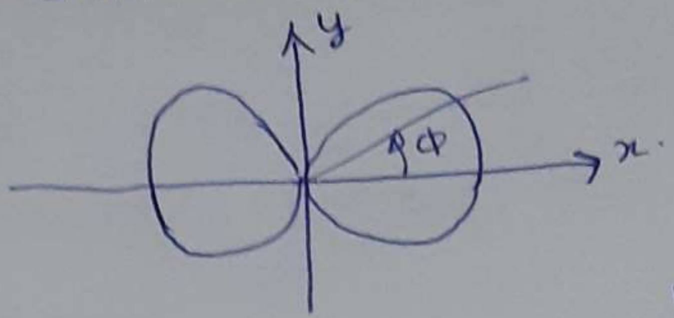
Diagramme de rayonnement:

$E_{tn}=0 \rightarrow \cos^3 u=0 \rightarrow \cos u=0 \rightarrow u=\pm\frac{n\pi}{2},\ n=1,3,5\ldots$

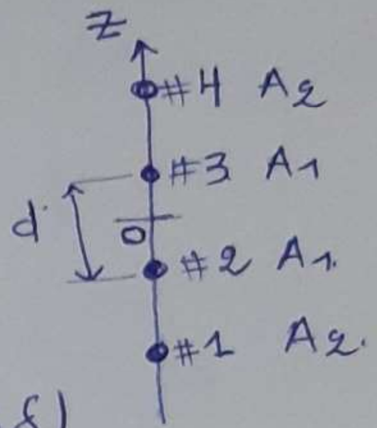
$\frac{\pi}{2}\sin\phi=\pm\frac{n\pi}{2} \rightarrow \sin\phi=\pm,\ n=1,3,5\ldots \rightarrow \boxed{\phi_n=\pm\pi/2}$

$E_{tn} \rightarrow Max \rightarrow \cos^3 u = \pm 1 \rightarrow \cos u = \pm 1 \rightarrow u = \pm m\pi \rightarrow$

$\frac{\pi}{2} \sin\phi = \pm m\pi \rightarrow \sin\phi = \pm 2m \; ; \; m = 0, 1, 2, \dots \rightarrow \boxed{\phi_m = 0, \pi}$



Exercice 4



$E_3 = A_1 e^{+j\delta} \; , \; E_2 = A_1 e^{-j\delta}$

$E_1 = A_2 e^{-j3\delta} \; ; \; E_4 = A_2 e^{+j3\delta}$

avec $\delta = \frac{kd}{2} \cos\theta = \frac{\pi}{2} \cos\theta$

$E_{23} = 2A_1 \cos\delta$ et $E_{14} = 2A_2 \cos 3\delta$

$E_T = E_{23} + E_{14} = 2(A_1 \cos\delta + A_2 \cos 3\delta)$

$\rightarrow \boxed{E_{Tn} = A_1 \cos\left(\frac{\pi}{2}\cos\theta\right) + A_2 \cos\left(3\frac{\pi}{2}\cos\theta\right)}$