Solution de la partie II de l'exercice 3 de la série 1:

$J = \int_{-\pi}^{\pi} \cos(px)\cos(qx)\,dx.$ et $K = \int_{-\pi}^{\pi} \sin(px)\sin(qx)\,dx.$

$J + K = \int_{-\pi}^{\pi} \cos(p-q)\,x dx.$ et $J - K = \int_{-\pi}^{\pi} \sin(p-q)\,x dx.$

On discute deux cas:

1) $p = q$:

$$\begin{cases} J + K = \int_{-\pi}^{\pi} dx \\ J - K = 0 \end{cases} \Longrightarrow \begin{cases} J + K = x]_{-\pi}^{\pi} \\ J - K = 0 \end{cases} \Longrightarrow \begin{cases} J + K = 2\pi \\ J = K \end{cases}$$

Alors $J = K = \pi$.

2) $p \neq q$:

$$\begin{cases} J + K = 2\int_{0}^{\pi} \cos(p-q)\,x dx \\ J - K = 2\int_{0}^{\pi} \sin(p-q)\,x dx \end{cases} \Longrightarrow \begin{cases} J + K = 2\left.\dfrac{1}{p-q}\sin(p-q)\,x\right]_{0}^{\pi} \\ J - K = -2\left.\dfrac{1}{p-q}\cos(p-q)\,x\right]_{0}^{\pi} \end{cases}$$

$$\Longrightarrow \begin{cases} J + K = 0 \\ J - K = \begin{cases} \dfrac{-2}{p-q} & \text{si } p-q \text{ est pair} \\ \dfrac{2}{p-q} & \text{si } p-q \text{ est impair} \end{cases} \end{cases}$$

a) Si $p - q$ est pair:

$$\begin{cases} J + K = 0 \\ J - K = \dfrac{-2}{p-q} \end{cases} \Longrightarrow J = \frac{-1}{p-q} \quad \text{et} \quad K = \frac{1}{p-q}$$

b) Si $p - q$ est impair:

$$\begin{cases} J + K = 0 \\ J - K = \dfrac{2}{p-q} \end{cases} \Longrightarrow J = \frac{1}{p-q} \quad \text{et} \quad K = \frac{-1}{p-q}$$