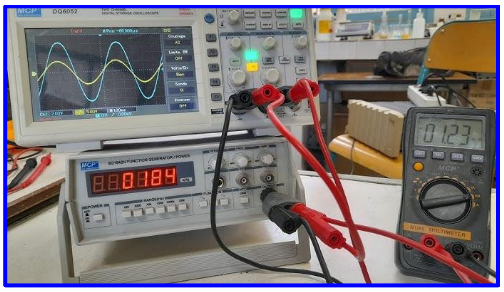
Experiment 02:
Resonance in a Series RLC Circuit



Experiment progression:

This report is prepared by:

Full name		Remarks	Group
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Resonance in a
Series RLC Circuit

Objectives:

- **To study** a series RLC circuit and its comportment with sinusoidal voltage supply
- After this lab experiment, the student will be able **to define** resonance frequency in RLC circuit

Background:

Fig. 1 shows a series RLC circuit contains a resistor (R), an inductor (L) and a capacitor (C) connected in series with sinusoidal voltage source v_S of **frequency** $\frac{\omega_s}{2\pi}$.

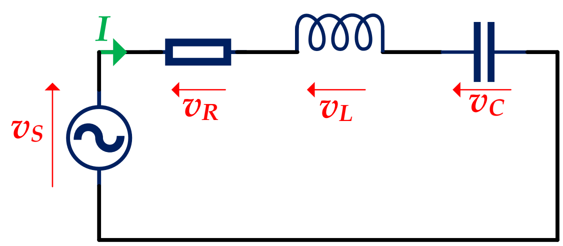

Fig. 1 RLC circuit

Applying Kirkoff's law we get:

$$v_R + v_L + v_c = V_S \sin(\omega_s t)$$

$$Ri + L\frac{di}{dt} + \frac{1}{C}\int i\,dt = V_S \sin(\omega_s t)$$

after derivation it we get the equation of motion for the voltage

$$\frac{d^2i}{dt^2} + \frac{R}{L}\frac{di}{dt} + \frac{1}{LC}i = \frac{\omega_s v_{s_{max}}}{L}\cos(\omega_s t)$$

This differential equation is a second order and written the following form

$$\frac{d^2i_c}{dt^2} + \gamma\frac{di_c}{dt} + \omega_n^2 i_c = \frac{\omega_s v_{s_{max}}}{L}\cos(\omega_s t)$$

Therefore, the **natural angular frequency ω_n** of the series RLC circuit is given as

$$\omega_n = \frac{1}{\sqrt{LC}} = 2\pi f_n$$

Note: we should differentiate between **frequency of the voltage source** $\frac{\omega_s}{2\pi}$ and **natural frequency** $\frac{\omega_n}{2\pi}$.

We can also write the electrical equation in form of complex quantities.

$$\underline{V_S} = \underline{V_R} + \underline{V_L} + \underline{V_C} = \underline{I}(R + j(L\boldsymbol{\omega_s} - \frac{1}{C\boldsymbol{\omega_s}}))$$

The total impedance is

$$\underline{Z} = R + j(L\boldsymbol{\omega_s} - \frac{1}{C\boldsymbol{\omega_s}})$$

The phase between current and voltage is given as

$$\varphi = \tan^{-1}(\frac{\left(L\boldsymbol{\omega_s} - \frac{1}{C\boldsymbol{\omega_s}}\right)}{R})$$

And the magnitude of Z is

$$Z = \sqrt{R^2 + (L\boldsymbol{\omega_s} - \frac{1}{C\boldsymbol{\omega_s}})^2}$$

The magnitude of the current I is

$$I = \frac{V_S}{\sqrt{R^2 + (L\boldsymbol{\omega_s} - \frac{1}{C\boldsymbol{\omega_s}})^2}}$$

Resonance frequency

Resonance frequency occurs when the frequency of the voltage source $\frac{\omega_s}{2\pi}$ mouches the natural frequency $\frac{\omega_n}{2\pi}$ of the circuit. Plus, at resonance frequency, the inductive impedance effect cancels the capacitive impedance effect because at $\boldsymbol{\omega_s = \omega_n}$ they have same impedances but **in opposite direction.**

So, at resonance frequency we have:

- *The value of resonance frequency f_{res}*

$$f_{res} = \frac{\omega_{res}}{2\pi} = \frac{\omega_s}{2\pi} = \frac{\omega_n}{2\pi} = \frac{1}{2\pi\sqrt{LC}}$$

- *Total impedance takes the min value, and it is purely resistive* $Z_{min} = R$

$$Z = \sqrt{R^2 + \left(L\boldsymbol{\omega_{res}} - \frac{1}{C\,\boldsymbol{\omega_{res}}}\right)^2}\Bigg|_{\omega_{res}=\frac{1}{\sqrt{LC}}} = R$$

- *The current delivered to the load is at its maximum amplitude while the impedance is at its minimum* $Z_{min} = R$

$$I = \frac{V_S}{\sqrt{R^2 + (L\boldsymbol{\omega_{res}} - \frac{1}{C\,\boldsymbol{\omega_{res}}})^2}\Big|_{\omega_{res}=\frac{1}{\sqrt{LC}}}} = \frac{V_S}{Z_{min}} = \frac{V_S}{R} = \frac{\sqrt{2}V_{S,RMS}}{R}$$

- *This current magnitude changes according to the variation of* ω_s, *can be represented in function of* ω_s, *see Fig. 2*

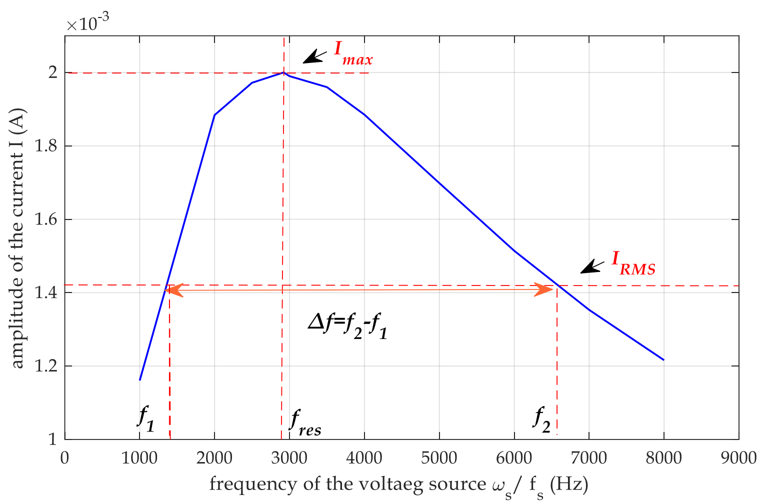

Fig. 2 Current magnitude variation in function of source voltage frequency

Three cases are possible when the variation of the frequency of the source according to the sign of $(L\omega_s - \frac{1}{C\omega_s})$

- $\left(L\omega_s - \frac{1}{C\omega_s}\right) > 0$ this means that the inductive reactance is greater than the capacitive reactance; therefore, the circuit behaves as an inductive with the current **lagging** the voltage of the source.

- $\left(L\omega_s - \frac{1}{C\omega_s}\right) = 0$ this means that the inductive reactance is **equal** to the capacitive reactance but have **opposite** sign. (read the paragraph of the resonance frequency) the current is **in phase with** the voltage
- $\left(L\omega_s - \frac{1}{C\omega_s}\right) < 0$ the capacitive reactance is greater than the inductive reactance with the current **leading** the voltage.

The summary of the above is illustrated in the Fig. 3

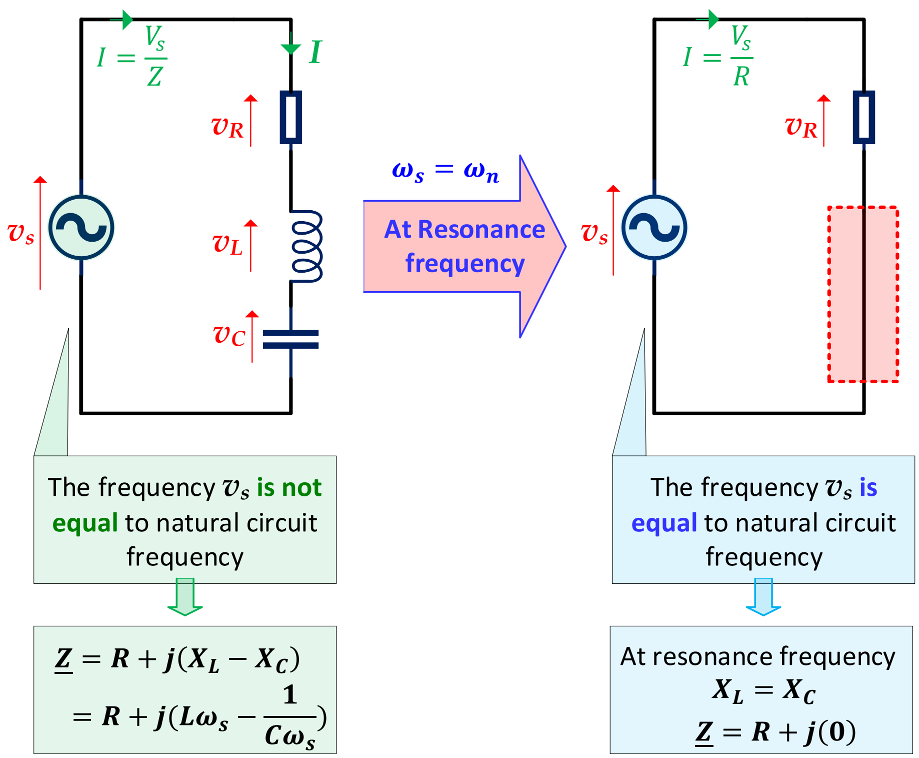

Fig. 3 RLC circuit behavior at resonance circuit

1. Calculate resonance frequency $f_{res} = \frac{\omega_{res}}{2\pi}$ ((R = 1 $k\Omega$, L = 25.92 mH, C = 0.1 μF))

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2. When occurs resonance frequency in RLC circuit?

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3. What happens to inductive and capacitive impedances at resonance?

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4. Why does resonance lead to maximum current flow in the RLC circuit?

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Experimental procedure

To well understand resonance frequency phenomenon in series **RLC** circuit we need to implement the **RLC** circuit shown in Fig. 4. To check the resonance frequency, we have to verify the phase between source voltage and the current through the resistor, and these are some tools to be used:

- Oscilloscope
- Function Generator (Alternating source where pick-to-pick is 4v)
- Resistor R = 1 kΩ,
- Coil inductor L = 25.92 mH and
- Capacitor C = 0.1 μF.

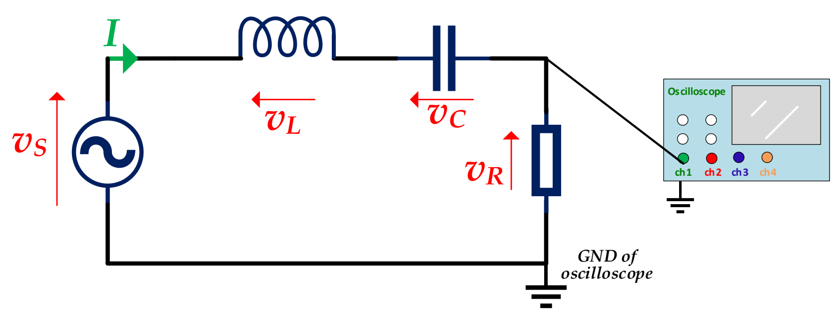

Fig. 4 Circuit to be implemented

1- Implement the circuit and then complete this table

f(kHz)	1	2	3	4	5	6	7	8
U_R(V)								
I(A)								

2- Sketch the curve $I = f(f_s)$

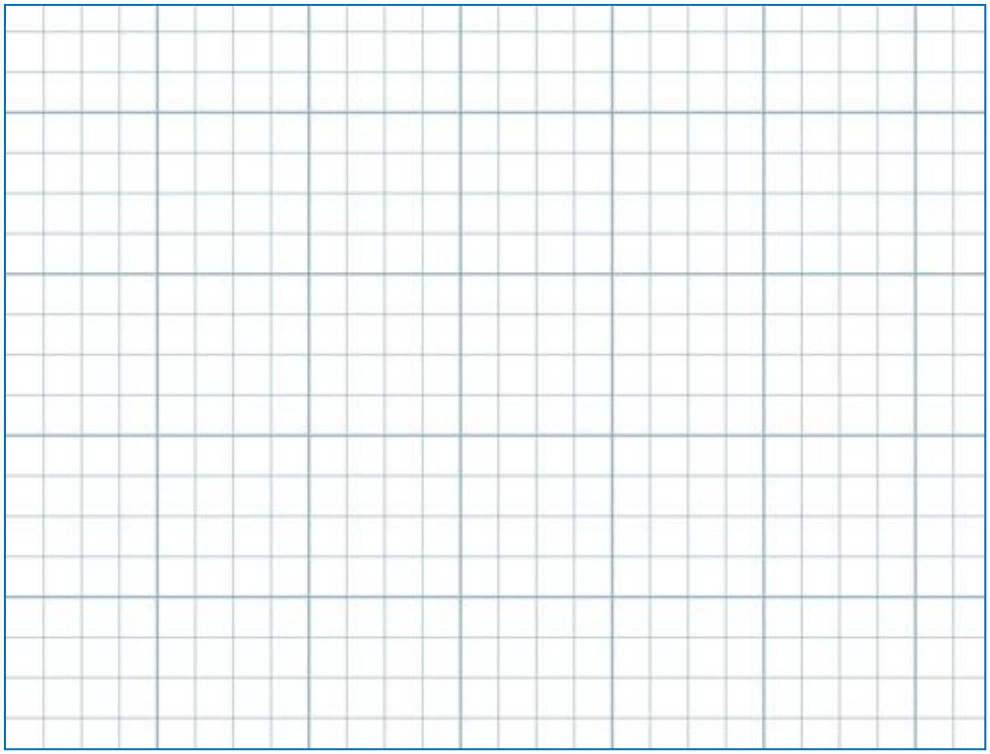

Fig. 5 Paper graph to draw $I = f(f_s)$

3- Deduce the resonance frequency and compare this value with theoretical value

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4- Determine the bandwidth $\Delta f = f_2 - f_1$ where the frequency f is corresponding to RMS value of the current at resonance frequency $I_{\text{RMS}} = \frac{I_{max}}{\sqrt{2}}$

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5- Deduce the quality factor $Q = \frac{f_{res}}{\Delta f}$

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