Chapter 1: Generalities and basic definitions (Force vectors)

2.1 Concept of force

The force may be defined as an action that changes or tends to change the state of rest or motion of a body to which it is applied.
A typical example of force is the push or pull we can apply with our hands to an object. A force is a vector quantity since its effect dependents on both its magnitude and direction.
For a complete description of a force, three parameters must be stated: (1) magnitude, (2) direction, and (3) point of application.
To represent a force vector when writing, an arrow could be placed over the letter, such as $\vec{F}$, and its magnitude as $|\vec{F}|$ or simply *F*.

In physics, forces are categorized into two main types: external and internal.
External forces: Forces that act on an object from outside its system such as gravitational force, frictional force and air resistance. They can change the motion of the object.
Internal forces: Forces that act within a system and do not change the overall motion of the center of mass of the system. For example, the normal force: The support force exerted upon an object that is in contact with another stable object (like a book on a table). Internal forces usually come from external forces.

2.1. Graphical representation of force

Graphically, a force is represented by a straight-line segment, called the line of action. The length of the segment represents, to a suitable scale, the magnitude of the force. The direction of the force is indicated by putting an arrowhead at one end of the segment. Either the beginning (or tail) or the end (or head) of the arrow may be used to indicate the point of application of the force.
When a force acts over a very small area compared with the other dimensions of the body, it may be regarded as a concentrated force of a point load with negligible loss of accuracy. A distributed force can act over an area as in the case of mechanical contact; over a volume as the weight of a body; or over a line as the weight of a suspended cable (see Fig. 1.1).

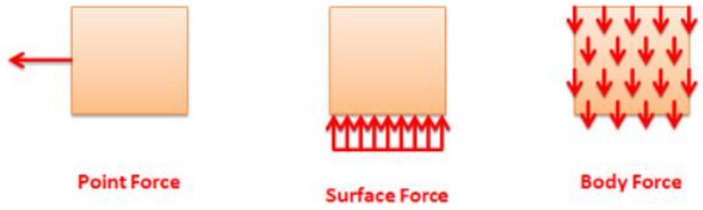

Fig. 1.1: The box being pulled along a frictionless surface has three types of forces acting on it: the tension in the cable is best represented by a point force, the normal force supporting the box is best represented by a surface force, and the gravitational force on the box is best represented as a body force.

2.2. Kinds of vectors

A vector can be one of the following three kinds:

1. Bound or Fixed Vector: It is a vector that requires the specification of a distinct point of application. For example, if a force is applied on a deformable body, the reaction forces and the deformations produced within the body depend on the point of application of the force, as well as on its magnitude and line of action.

2. Sliding or Transmissible Vector: This kind of a vector may be applied at any point along its line of action. As a result, the external effects do not change.

3. Free Vector: It may be moved anywhere in space provided it maintains the same direction and magnitude. For example, if a body has rectilinear motion, then the displacement of any point may be taken as a vector. This vector describes equally well the magnitude and direction of displacement of every point of the body. Therefore, we may represent the displacement of such a body as a free vector.

2.3. System of forces

The classification of forces systems is presented in Fig. 1.2.

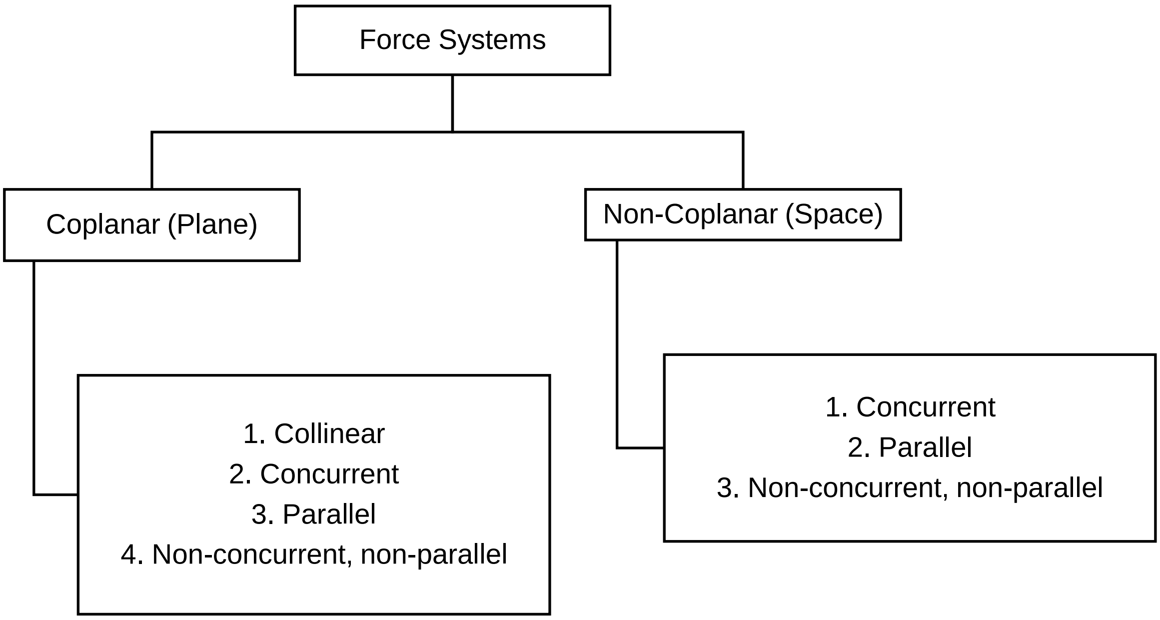

Fig. 1.2: Classification of force systems.

2.3.1. Characteristics of force systems

1. Coplanar forces: As shown in Fig. 1.3, the lines of action of all forces lie in the same plane.

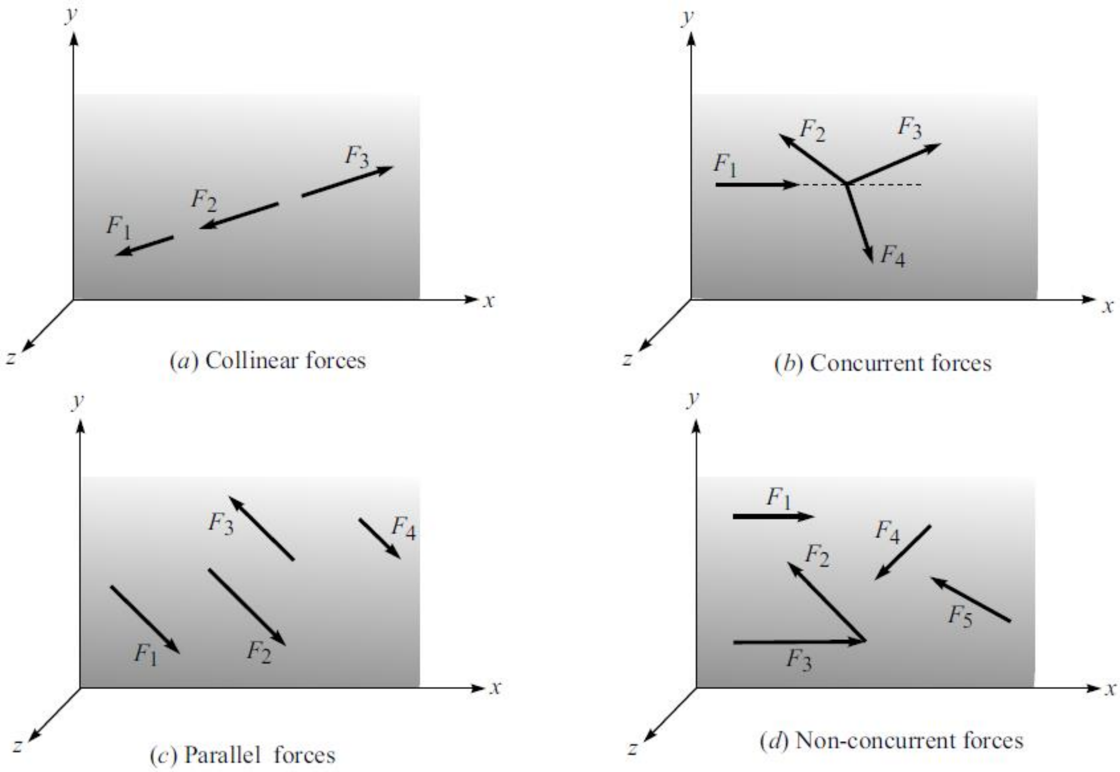

Fig. 1.3: Coplanar force systems.

2. Non-coplanar forces: As Fig. 1.4 shows, not all forces have their lines of action in the same plane.

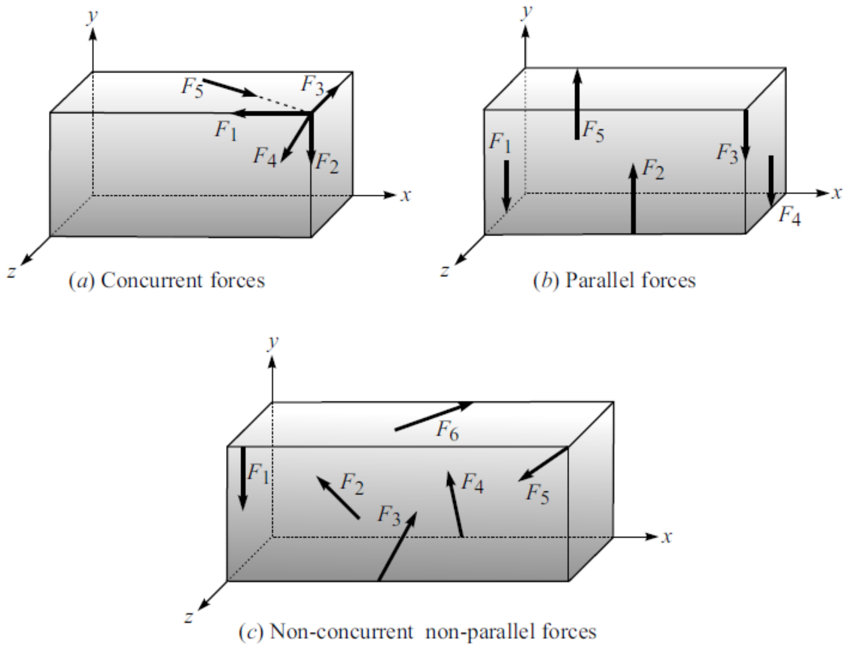

Fig. 1.4: Non-coplanar force systems.

3. Collinear forces: The lines of action of all forces lie along the same line (Fig. 1.4a). Collinear forces have to be coplanar as well.

4. Concurrent forces: The lines of action of all forces meet at one point. Concurrent forces may be coplanar (Fig. 1.3b), or non-coplanar (Fig. 1.4a).

5. Non-concurrent forces: The lines of action of all forces do not meet at one point. Non-concurrent forces may be coplanar (Fig. 1.3d), or non-coplanar (Fig. 1.4c).

6. Parallel forces: The lines of action of all forces are parallel to each other. Parallel forces may be coplanar (Fig. 1.3c), or non-coplanar (Fig. 1.4b).

7. Non-parallel forces: The lines of action of all forces are not parallel to each other. Non-parallel forces may be coplanar (Fig. 1.3d) or non-coplanar (Fig. 1.4c).

2.4. Angle between two vectors

The angle between two vectors is the smaller of the two angles formed between their directions when their arrows either point away from (or point toward) a common point. For example, in Fig. 1.5, the lines of action of two vectors P and Q meet at point B; and the angle between the two vectors is wrongly seen as φ. In fact, the correct angle between the vectors P and Q is θ, as seen in Fig. 1.5.

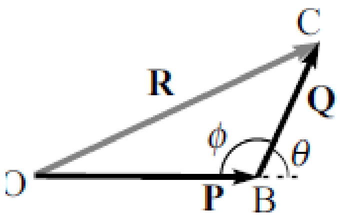

Fig. 1.5: Angle between two vectors.

2.5. Addition of vectors

The resultant of the addition of two forces depends not only on their magnitudes but also on their directions.

Resultant Force: If several forces, P, Q, R, S,…. are simultaneously acting on a particle, it is possible to find a single force that could replace all these forces without any change in the effect produced. This single force is called the resultant force, and the given forces are called the component forces.

2.5.1 Composition of forces

The composition of forces, also known as compounding of forces, is the process of determining the resultant of several forces. In the following only techniques of finding the resultant of two forces will be considered.

Two vectors can be added in two distinct manners: graphical and analytical.

2.5.1.a Graphical Method: Graphically, we can find the resultant of two vectors by either parallelogram law or triangle law.

(i) Parallelogram low: It states that the resultant R of two coplanar vectors P and Q is the diagonal of the parallelogram for which P and Q are the adjacent sides. All three vectors P, Q and R are concurrent (i.e., pass through the same point), as shown in Fig. 1.6. The addition of vectors P and Q by the parallelogram law requires that:

1. Vectors P and Q are placed together to point away from (as in Fig. 1.6a) or to point towards (as in Fig. 1.6b) the point O.
2. A parallelogram is made with P and Q as adjacent sides.
3. The resultant R is given by the diagonal passing through the point O.

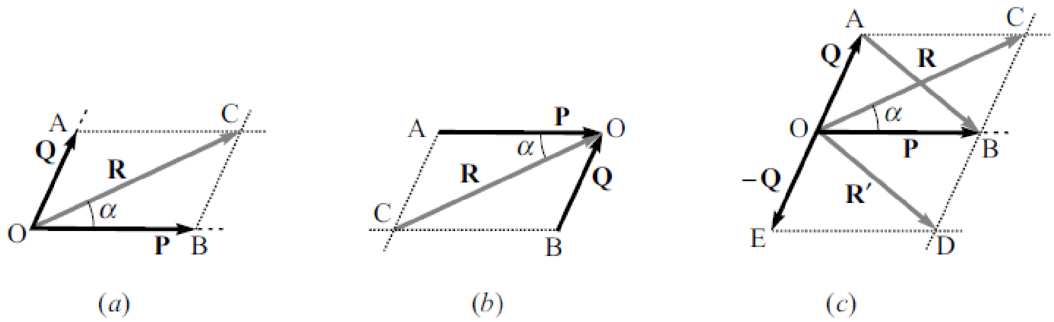

Fig. 1.6: Addition and subtraction of vectors by the parallelogram law.

Subtraction of Vectors: For subtracting the vector Q from the vector P, we add the vector P to the negative of the vector Q, as shown in Fig. 1.6c. That is, P - Q = P+ (-Q) = R'. Thus, the diagonal OD of the parallelogram OEDB gives the resultant R'. Note that vector $\overrightarrow{OD} = \overrightarrow{AB}$. Thus, the diagonal OC of the parallelogram OACB gives the addition P + Q; and the diagonal AB gives the subtraction P-Q.

(ii) Triangle law: It is a corollary of the parallelogram law. In Fig. 1.6a, the side BC is equal and parallel to the side OA. Hence, BC also represents the vector Q. Separating the triangle OBC from the parallelogram of Fig. 1.6a, we get a triangle shown in Fig. 1.7a. Here, the tail of the vector Q is put at the head of the vector P. The resultant R is then drawn from the tail of P to the head of Q. Here, vectors P and Q are placed one after the other in the same sense to make two sides of the triangle; the third side is drawn from the tail of P to the head of Q gives the resultant R. Thus, we get P + Q = R.

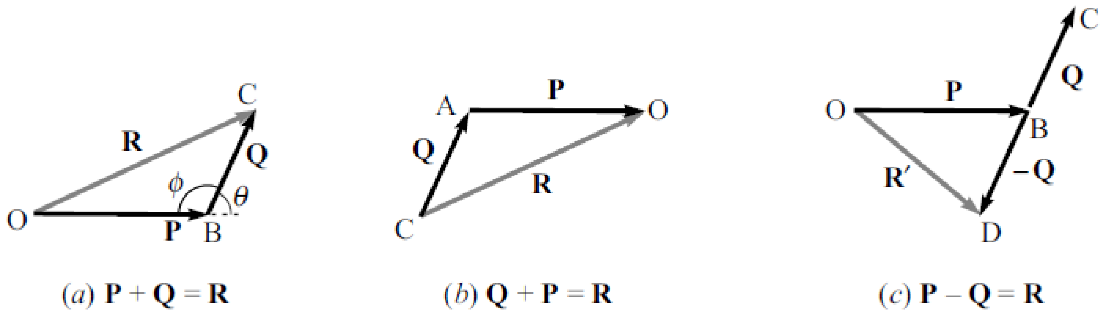

Fig. 1.7: Triangle law.

Alternatively, we could separate the triangle CAO from the parallelogram of Fig. 1.6b to get the triangle of Fig. 1.7b. The side CA represents the vector Q. This triangle shows that Q+P=R This also shows that vector addition is commutative, i.e., P+Q=Q+P.

Subtraction of vectors: For subtracting the vector Q from the vector P, the vector P is added to the negative of the vector Q, as shown in Fig. 1.7c. That is, P- Q = P+(-Q) = R.

2.5.1.b Analytical method: Sometimes the graphical methods become inconvenient. Alternatively, the analytical method can be used to get accurate results by just making simple calculations.

Fig. 1.8 shows two forces P and Q inclined at an angle θ and acting at a common point O. The parallelogram OACB can be used to find the resultant R given by the diagonal OC. The resultant R makes an angle α with the vector P, and an angle β with the vector Q. From point C, let us drop a perpendicular CM on the line of action of the vector P.

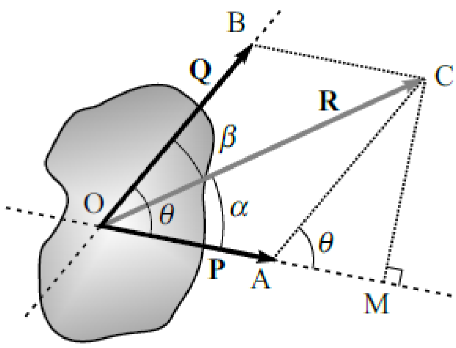

Fig. 1.8: Addition of vectors using the analytical method.

The magnitude of the resultant R can be found by applying Pythagoras theorem to the triangle OMC, as follows:

$$OC^2 = OM^2 + MC^2 = (OA + AM)^2 + MC^2 = (OA + AC\cos\theta)^2 + (AC\sin\theta)^2$$

$$OC^2 = (OA^2 + AC^2 cos^2\theta + 2OAAC\cos\theta) + AC^2 sin^2\theta$$

$$R^2 = (P^2 + Q^2 cos^2\theta + 2PQ\cos\theta) + Q^2 sin^2\theta$$

$$R^2 = P^2 + Q^2 + 2PQ\cos\theta$$

$$R = \sqrt{P^2 + Q^2 + 2PQ\cos\theta} \qquad (1.1)$$

The orientation of the resultant R can be defined by the angle α made by the resultant R and the component R. So, the angle α can be obtained from the triangle OMC:

$$\tan\alpha = \frac{CM}{OM} = \frac{CM}{OA+AM} = \frac{AC\sin\theta}{OA+AC\cos\theta}$$

$$\tan\alpha = \frac{Q\sin\theta}{P+Q\cos\theta} \qquad (1.2)$$

Remark: On a rigid body, even if two forces are not acting at a common point, as in Fig. 1.9, they can be slid along their lines of action to intersect at a point O'. The parallelogram O'A'C'B' can then be made to find their resultant O'C'. Furthermore, even if the intersection point O' falls outside the boundary of the body, we can proceed by assuming this point to be rigidly fixed to the body by an imaginary extension, as shown by the dotted outline in Fig. 1.9.

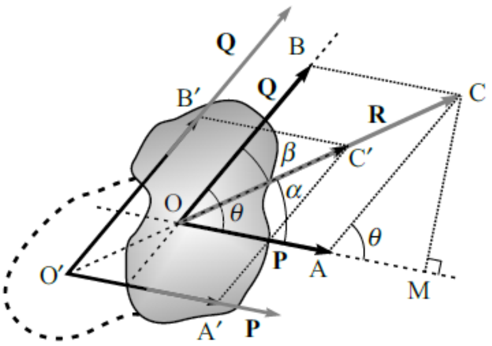

Fig. 1.9: Addition of vectors two forces not acting at a common point.

Suitable trigonometric formulate (Fig. 1.10):

Cosine law: $a^2 = b^2 + c^2 - 2bc \cos\alpha$

$b^2 = c^2 + a^2 - 2ca \cos\beta$

$c^2 = a^2 + b^2 - 2ab \cos\gamma$

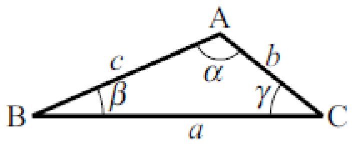

Fig. 1.10

Sine law: $\frac{a}{\sin\alpha} = \frac{b}{\sin\beta} = \frac{c}{\sin\gamma}$

Example 1: Determine the magnitude and direction of the resultant of two forces of 50 N and 80 N acting at an angle of 30°.

Solution: Given: P = 50 N, Q = 80 N, θ = 30°.

The magnitude of the resultant:

$R = \sqrt{P^2 + Q^2 + 2PQ\cos\theta} = \sqrt{50^2 + 80^2 + 2.50.80\cos 30} = 142.6\ N$

The angle α between the resultant and the force P = 50 N is given by:

$\tan\alpha = \frac{Q\sin\theta}{P+Q\cos\theta} = \frac{80\sin 30}{50+80\cos 30}$, $\alpha = \tan^{-1} 0.33 = 18.26°$

2.6. Representation of vectors using rectangular components

In engineering applications, it is customary to describe vectors using their rectangular components and then to perform vector operations, such as addition, in terms of these components.

The reference frame frequently used is the rectangular Cartesian coordinate system. If a vector A is resolved into its rectangular components, as illustrated in Fig. 1.11, it can be written as:

$$\vec{A} = A_x\vec{\imath} + A_y\vec{\jmath} + A_z\vec{k} \quad (1.3)$$

Where A_x, A_y and A_z are the scalar components of vector A. They can be obtained:

$$A_x = A\cos\theta_x \qquad A_y = A\cos\theta_y \qquad A_z = A\cos\theta_z \tag{2.4}$$

Where θ_x, θ_y and θ_z are the angles between A and the positive coordinate axes.

The scalar components can be positive or negative, depending upon whether the corresponding vector component points in the positive or negative coordinate direction.

The magnitude of A is related to its scalar components by:

$$A = \sqrt{A_x^2 + A_y^2 + A_z^2} \tag{1.5}$$

The direction of A is specified by its direction cosines defined as:

$$cos\ \theta_x = \frac{A_x}{A} \qquad \cos\theta_y = \frac{A_y}{A} \qquad \cos\theta_z = \frac{A_z}{A} \tag{1.6}$$

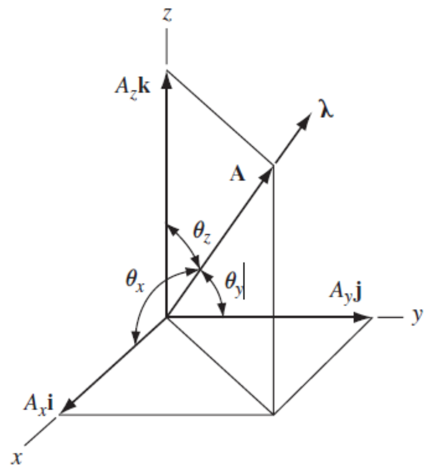

Fig. 1.11

The unit vector along the vector A is given as:

$$\overrightarrow{\mu_A} = \frac{\vec{A}}{A} = co\,s\,\theta_x\vec{\imath} + co\,s\,\theta_y\vec{\jmath} + cos\ \theta_z\vec{k}\ ,\ \mu_A = 1 \tag{1.7}$$

For example, the unit vectors along the three axes of Cartesian coordinate system are given as: $\vec{\imath}$ is the unit vector along the x-axis, $\vec{\jmath}$ is the unit vector along the y-axis and $\vec{k}$ is the unit vector along the z-axis.

Coplanar vector can be represented in terms of its three rectangular components (x, y and z). In addition, it can also be presented by its magnitude and the angle made with positive x-axis. The former is called rectangular form and the latter polar form. For example:

Rectangular form: $\vec{A} = A_x\vec{\imath} + A_y\vec{\jmath} + A_z\vec{k}$

Polar form: $\vec{A} = A\angle\theta_x$

2.6.1. Vector addition using rectangular components

Consider the two vectors $\vec{A} = A_x\vec{\imath} + A_y\vec{\jmath} + A_z\vec{k}$ and $\vec{B} = B_x\vec{\imath} + B_y\vec{\jmath} + B_z\vec{k}$. Letting C be the sum of A and B, we have:

$$\vec{C} = \vec{A} + \vec{B} = A_x\vec{\imath} + A_y\vec{\jmath} + A_z\vec{k} + B_x\vec{\imath} + B_y\vec{\jmath} + B_z\vec{k} \tag{1.8}$$

$$\vec{C} = (A_x + B_x)\vec{\imath} + (A_y + +B_y)\vec{\jmath} + (A_z + B_z)\vec{k} \tag{1.9}$$

Hence, the rectangular components of *C* are:

$$C_x = A_x + B_x,\ C_y = A_y + B_y,\ C_z = A_z + B_z \tag{1.10}$$

Equations (2.10) indicates that each component of the sum equals the sum of the components.

The subtraction of two vectors can be gone in the similar manner than the addition as follow:

$$\vec{C} = \vec{A} - \vec{B} = (A_x - B_x)\vec{\imath} + (A_y - B_y)\vec{\jmath} + (A_z - B_z)\vec{k} \tag{1.11}$$

2.7. Vector Multiplication

2.7.1. Dot or scalar product

Fig. 1.12 shows two vectors A and B, with θ being the angle between their positive directions.

The dot product of A and B yield in scalar as defined below:

$\vec{A} \cdot \vec{B} = AB\cos\theta$ (1.12)

When A and B are expressed in rectangular form, their scalar product becomes:

$\vec{A} \cdot \vec{B} = A_x B_x + A_y B_y + A_z B_z$ (1.13)

Knowing that: $\vec{\imath} \cdot \vec{\imath} = \vec{\jmath} \cdot \vec{\jmath} = \vec{k} \cdot \vec{k} = 1$ and $\vec{\imath} \cdot \vec{\jmath} = \vec{\imath} \cdot \vec{k} = \vec{\jmath} \cdot \vec{k} = 0$

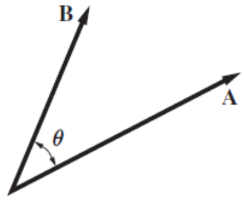

Fig. 1.12

Two properties of the scalar product:

- The dot product is commutative: $\vec{A} \cdot \vec{B} = \vec{B} \cdot \vec{A}$
- The dot product is distributive: $\vec{A} \cdot (\vec{B} + \vec{C}) = \vec{A} \cdot \vec{B} + \vec{A} \cdot \vec{C}$

2.7.2. Cross or vector product

The cross product of two coplanar vectors A and B is a vector C such as:

$$\vec{C} = \vec{A} \times \vec{B} = (AB\sin\theta)\overrightarrow{e_n} \quad (1.14)$$

Where $\overrightarrow{e_n}$ is a unit vector in the direction normal to the plane of vectors A and B.

The magnitude of vector C equals the area of the parallelogram bounded by the vectors A and B as the adjacent sides, as shown in Fig. 1.13. and its direction is perpendicular to the plane of vectors A and B, and is given by the right-hand screw rule.

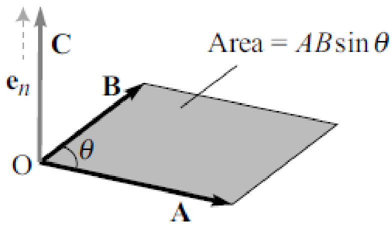

Fig. 1.13: Cross product of vectors A and B.

The cross product is distributive: $\vec{A} \times (\vec{B} + \vec{C}) = \vec{A} \times \vec{B} + \vec{A} \times \vec{C}$

The cross product is not commutative: $\vec{A} \times \vec{B} = -\vec{B} \times \vec{A}$

From the properties of the cross product, we deduce that the base vectors of a rectangular coordinate system satisfies the following identities:

$$\vec{\imath} \cdot \vec{\imath} = \vec{\jmath} \cdot \vec{\jmath} = \vec{k} \cdot \vec{k} = \vec{0}$$

$$\vec{\imath} \cdot \vec{\jmath} = \vec{k}, \vec{\jmath} \cdot \vec{k} = \vec{\imath} \text{ and } \vec{k} \cdot \vec{\imath} = \vec{\jmath}$$

When $\vec{A}$ and $\vec{B}$ are expressed in rectangular form, their cross product becomes:

$$\vec{A} \times \vec{B} = (A_x\vec{\imath} + A_y\vec{\jmath} + A_z\vec{k}) \times (B_x\vec{\imath} + B_y\vec{\jmath} + B_z\vec{k}) \quad (1.15)$$

Using the rules for expanding a 3 × 3 determinant:

$$\vec{A} \times \vec{B} = \begin{bmatrix} \vec{\imath} & \vec{\jmath} & \vec{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{bmatrix}$$

$$\vec{A} \times \vec{B} = (A_y B_z - A_z B_y)\vec{\imath} - (A_x B_z - A_z B_x)\vec{\jmath} + (A_x B_y - A_y B_x)\vec{k} \quad (1.16)$$

Example: $\vec{A} = 8\vec{\imath} + 4\vec{\jmath} - 2\vec{k}$, $\vec{B} = 2\vec{\jmath} + 6\vec{k}$ and $\vec{C} = 3\vec{\imath} - 2\vec{\jmath} + 4\vec{k}$, calculate the following:

(1) $\vec{A} \cdot \vec{B}$

(2) the orthogonal component of $\vec{B}$ in the direction of $\vec{C}$

(3) the angle between $\vec{A}$ and $\vec{C}$

(4) $\vec{A} \times \vec{B}$

(5) the unit vector μ that is perpendicular to both $\vec{A}$ and $\vec{B}$

(6) $\vec{A} \times (\vec{B} \cdot \vec{C})$

2.8. Moment of a force

In general, a force acting on a body has two kinds of effects. In addition to the tendency to move the body in the direction of its application, the force causes or tends to rotate the body about a point or axis. The rotational tendency is called **the moment (M) of a force** about a point or axis. This rotational effect depends on the magnitude of the force and the distance between the point and the line of action of the force.

2.8.1. Definition

Consider a force of magnitude F acting at point A and O a point that is not on the line of action of F, as shown in Fig. 1.14. Note that the force F and the point *O* determine a unique plane. r is distance from point O to point A. The moment of the force F about point O is defined as:

$$\overrightarrow{M_o} = \overrightarrow{OA} \times \vec{F} \qquad (1.17)$$

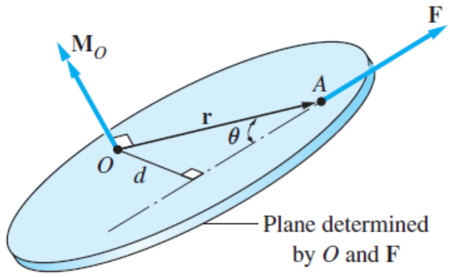

Fig. 1.14: Concept of moment of force.

The moment of F about point O is a vector by definition. From the properties of the cross product of two vectors, M_O is perpendicular to the plane containing r and F, with its sense determined by the right-hand rule, as shown in Fig. 1.14 a scalar computation of the magnitude of the moment can be obtained from the geometric interpretation of Eq. (1.17). The magnitude of M_O is given by:

$$M_o = \|\overrightarrow{M_o}\| = \|\overrightarrow{OA} \times \vec{F}\| = rF\sin\theta \qquad (1.18)$$

where θ is the angle between r and F. It is visible from Fig. 2.12 that:

$$rF\sin\theta = d \qquad (1.19)$$

where *d* is the perpendicular distance from the *moment centre* (point O) to the line of action of the force F called the *moment arm* of the force. Thus, the magnitude of M_O is:

$$M_o = Fd \qquad (1.20)$$

In SI units, moment is measured in newton-meters (N·m).

According to equation (1.20), the magnitude of the moment M_O depends only on the magnitude of the force and the perpendicular distance d, a force may be moved anywhere along its line of action without changing its moment about a point, i.e., the point A can be any point on the line of action of F.

A double arrow is normally used to represent moment of force M to differentiate it from the force F, see Fig.2.12. The direction of moment M of the force F about the point O is along the O-O axis and its sense is upward as given by the right-hand rule, see Fig.1.14.

2.8.2. Graphical presentation and sign convention of moment of a force

Representing moments of coplanar forces about a point by curved arrows permits us to treat the moment vector M_o as an algebraic scalar with positive and negative signs. The moment of any coplanar force about the point O is along the axis O-z (Fig. 1.15); and it may be directed either upward or downward. As can be seen from Fig. 1.15a, the upward direction is marked by a counterclockwise curved arrow, since this moment tends to cause counterclockwise rotation of the body. The magnitude of the moment about point O for the 100 N force in Fig. 1.15c is also 200 N·m, but in this case its direction is clockwise, as viewed from the positive z-axis. For this force, $M_O = -200$ kN·m, as shown in Fig. 1.15d. Although the vector description for both forces is $-100\vec{\imath}$ N, their moments about point O are oppositely directed.

Through the present course, the followed sign convention is, *to take counterclockwise (CCW) moment as positive and clockwise (CW) moment as negative*, although the opposite convention works equally well.

There are two reasons for choosing this convention. First, it matches the sign convention for angles. The second reason, the moment M of a force F is given by the cross-product

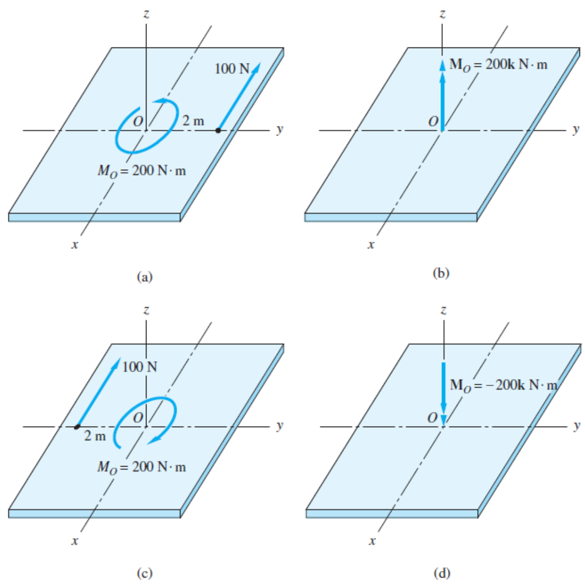

Fig. 1.15: Representation of moment.

Example: Find the moment of 12 N force applied at the edge A about the point O, see Fig. 1.16.

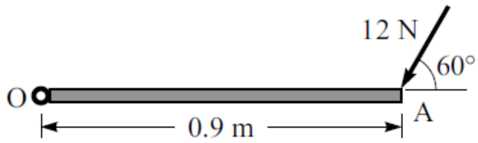

Fig. 1.16

2.8.3. Principle of moments: Varignon's theorem

Varignon's Theorem is a method to calculate moments developed in 1687 by French mathematician Pierre Varignon (1654 – 1722). It states that that *the algebraic sum of moments of a system of concurrent forces about a moment centre is equal to the moment of their resultant force about the same moment centre*.

Alternately, the moment of a force about a point equals the sum of the moments of its components.

Proof: The theorem will be proved by taking an example of a system of two concurrent forces F_1 and F_2, whose resultant is *R*, as shown in Fig. 1.17. Let us take point O as the moment centre. The moment arms for force F_1, F_2 and resultant *R* are d_1, d_2 and d, respectively.

$$\overrightarrow{M_0(\vec{R})} = \overrightarrow{OA} \times \vec{R} = Rd$$

$$\overrightarrow{M_0(\vec{R})} = \overrightarrow{M_0(\vec{F_1})} + \overrightarrow{M_0(\vec{F_1})}$$

$$\overrightarrow{M_0(\vec{R})} = \overrightarrow{OA} \times \vec{F_1} + \overrightarrow{OA} \times \vec{F_2}$$

$$\overrightarrow{M_0(\vec{R})} = \overrightarrow{OA} \times (\vec{F_1} + \vec{F_2}) = \overrightarrow{OA} \times \vec{R}$$

$$Rd = F_1 d_1 + F_2 d_2 \qquad (1.21)$$

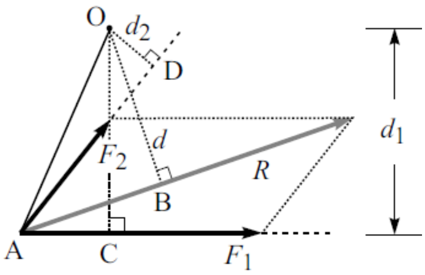

Fig. 1.17: Varignon's theorem.

Example: Use Varignon's Theorem to find the moment that the forces in Fig.2.16 below exert about point A.

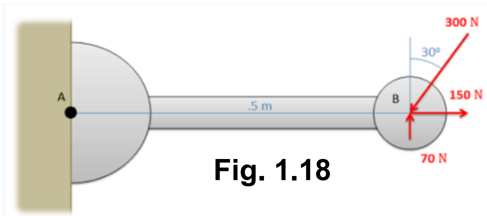

Fig. 1.18

2.8.4. Couple

As pointed out before, a force has two effects on a rigid body: translation due to the force itself and rotation due to the moment of the force. A couple, on the other hand, is a purely rotational effect—it has a moment but no resultant force. Couples play an important role in the analysis of force systems.

2.8.4.1. Definition

Two parallel, noncollinear forces that are equal in magnitude and opposite in direction are known as a couple.

Fig. 1.19 presents a typical couple. The two forces of equal magnitude F are oppositely directed along lines of action that are separated by the perpendicular distance d. (In a vector description of the forces, one of the forces would be labelled F and the other −F.) The lines of action of the two forces determine a plane that we call the plane of the couple. The two forces that form a couple have some interesting properties, which will become apparent when we calculate their combined moment about a point.

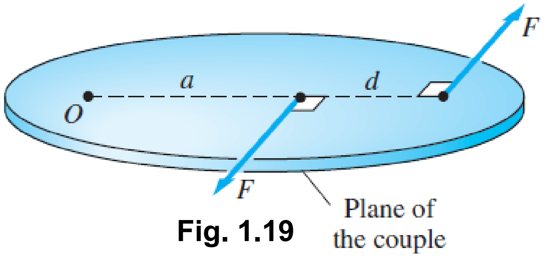

Fig. 1.19

2.8.4.2. Moment of a couple about a point

Let us calculate the moment of the couple shown in Fig. 1.19 about the point O. Note that O is an arbitrary point in the plane of the couple and that it is located a distance a from the force on the left. The sum of the moments about point O for the two forces is:

$$M_o = F(a + d) - Fa = Fd \quad (1.22)$$

It can be observed that the moment of the couple about point O is independent of the location of O, because the result is independent of the distance a.

So, a couple possesses two important characteristics:

1. A couple has no resultant force ($\sum \vec{F} = \vec{0}$)
2. The moment of a couple is the same about any point in the plane of the couple.

2.8.4.3. Equivalent couples

Because a couple has no resultant force, its only effect on a rigid body is its moment. For this reason, two couples that have the same moment are said to be equivalent (have the same effect on a rigid body). Figure 1.20 illustrates the four operations that may be performed on a couple without changing its moment; all couples shown in the figure are equivalent.

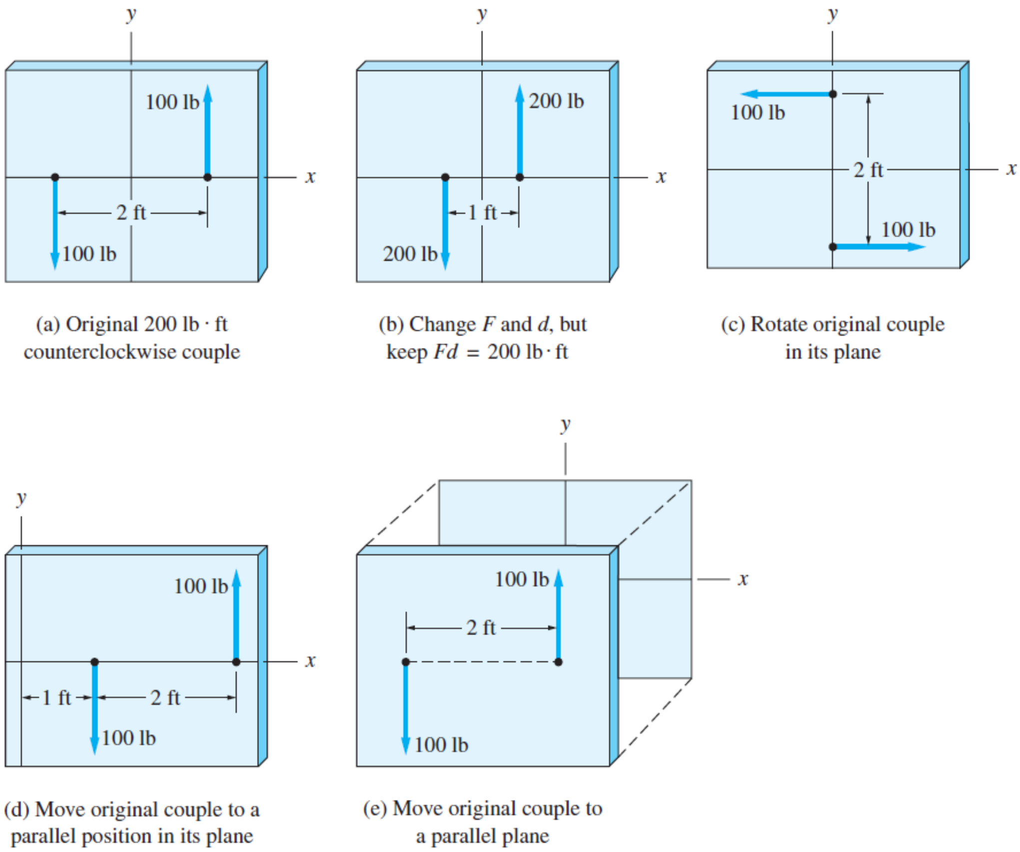

Fig. 1.20: Equivalent couples.

The operations are:

1. Changing the magnitude F of each force and the perpendicular distance d while keeping the product *F.d* constant.
2. Rotating the couple in its plane
3. Moving the couple to a parallel position in its plane
4. Moving the couple to a parallel plane

2.8.4.4. Notation and terminology

The moment of the couple presented in Fig. 1.21a has a magnitude of C = 1800N·m and is directed counterclockwise in the xy-plane (see Fig. 1.21b). Because the only rigid-body effect of a couple is its moment, the representations in Figs. 1.21a and 1.21b are equivalent. In other words, it is possible to replace a couple that acts on a rigid body by its moment without changing the external effect on the body. This equivalence also applies to the terminology—rather than referring to *C* as the moment of the couple, it usually is called simply the couple.

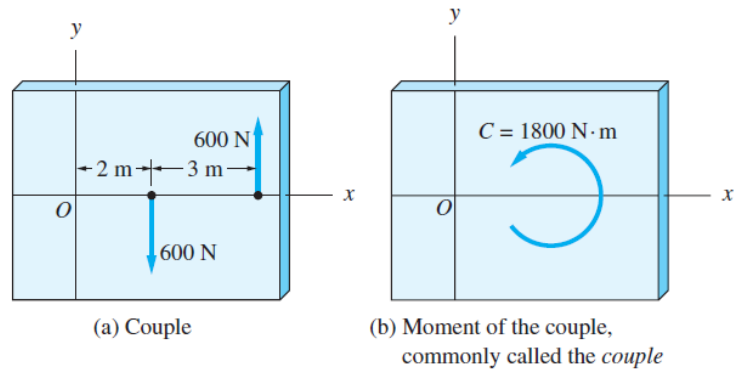

Fig. 1.21: Notation and terminology of the moment of a couple.

Example 1: Which of the systems are equivalent to the couple in Fig. 2.22a?

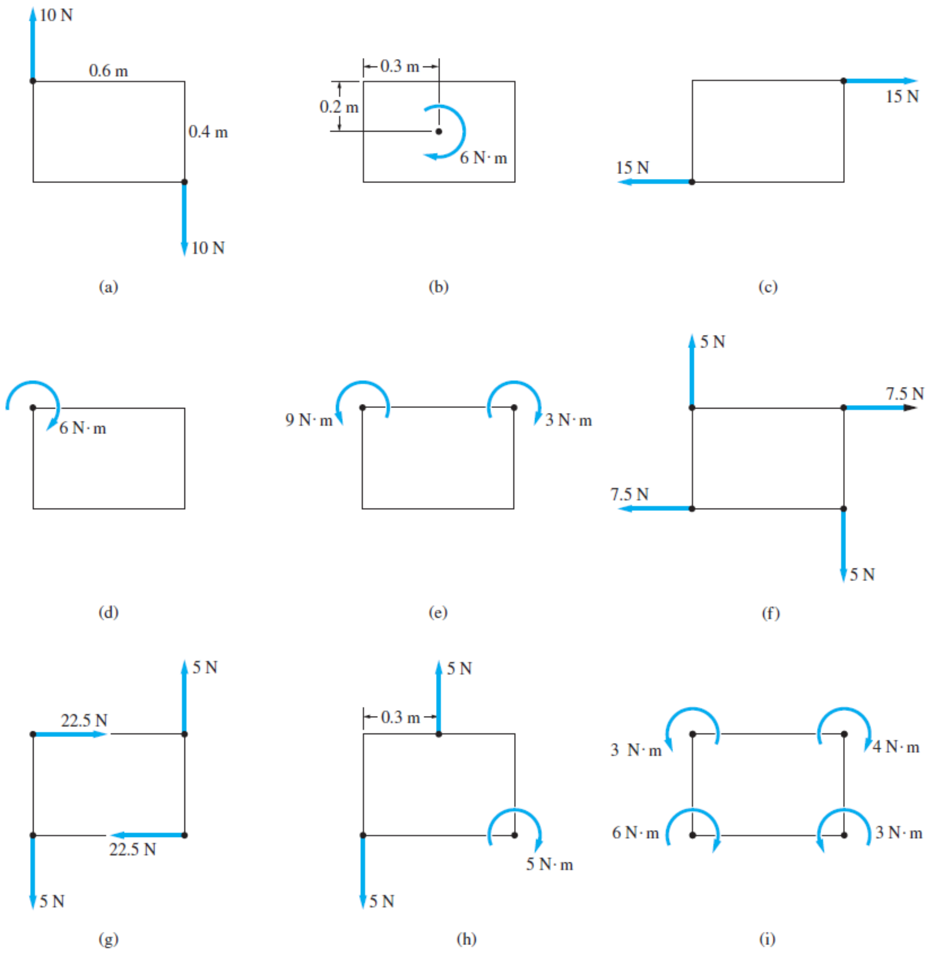

Fig. 1.22.

Example 2: The three forces shown in Fig. 1.23 are equivalent to a 50-kN upward force at A and a 170-kN ·m counterclockwise couple. Determine P and Q.

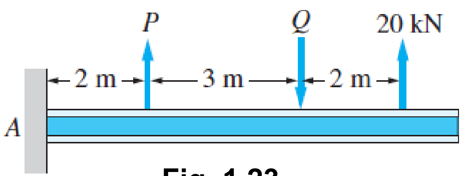

Fig. 1.23