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جامعة محمد بوضياف - المسيلة
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2019-2020, Bachelor 2nd Year (L2)
Faculty of Technology
Depart: Electronic Engineering

Class: Numerical Methods Lab

Lab 3 : The Fixed-point method

The objectives of this lesson:

- **Understand the bisection method.**
- **Write an algorithm/flowchart for this method.**
- **Write a Matlab script for this method.**
- **Be able to apply this method to solve a non-linear equation $f(x) = 0$.**
- **Be able to use various stopping criteria to exit the algorithm of the bisection method.**

Basic ideas and fundamental concepts:

It consists of re-writing eq $f(x)=0$ as follows

$$x = g(x).$$

Now, the solution is a number α which satisfies $\alpha = g(\alpha)$, and we say that α is a fixed point of $g(x)$.

$$\left.\begin{array}{l} f(x) = 0 \quad \text{—— (1)} \\ \text{search } \alpha \text{ s.t} \\ \quad f(\alpha) = 0 . \end{array}\right\} \iff \left\{\begin{array}{l} x = g(x) \quad \text{—— (2)} \\ \text{Search } \alpha \text{ s.t} \\ \quad \alpha = g(\alpha) . \end{array}\right.$$

Using Eq. (2), we can formulate an iterative method to solve Eq (1), as follows

$$x_{k+1} = g(x_k), \quad k = 0, 1, 2, \ldots$$

By starting an initial approx. x_0 of the solution, one can calculate the other (next) approximations as follows

$$x_1 = g(x_0), \quad x_2 = g(x_1), \ \ldots, \ x_k = g(x_{k-1}).$$

Stopping Criteria :

Criteria #1. The length of the intrvl. [a,b] << → $|a-b| < \varepsilon$

→ Stop if abs(a-b) < epsilon → while (abs(a-b) > eps) ↴

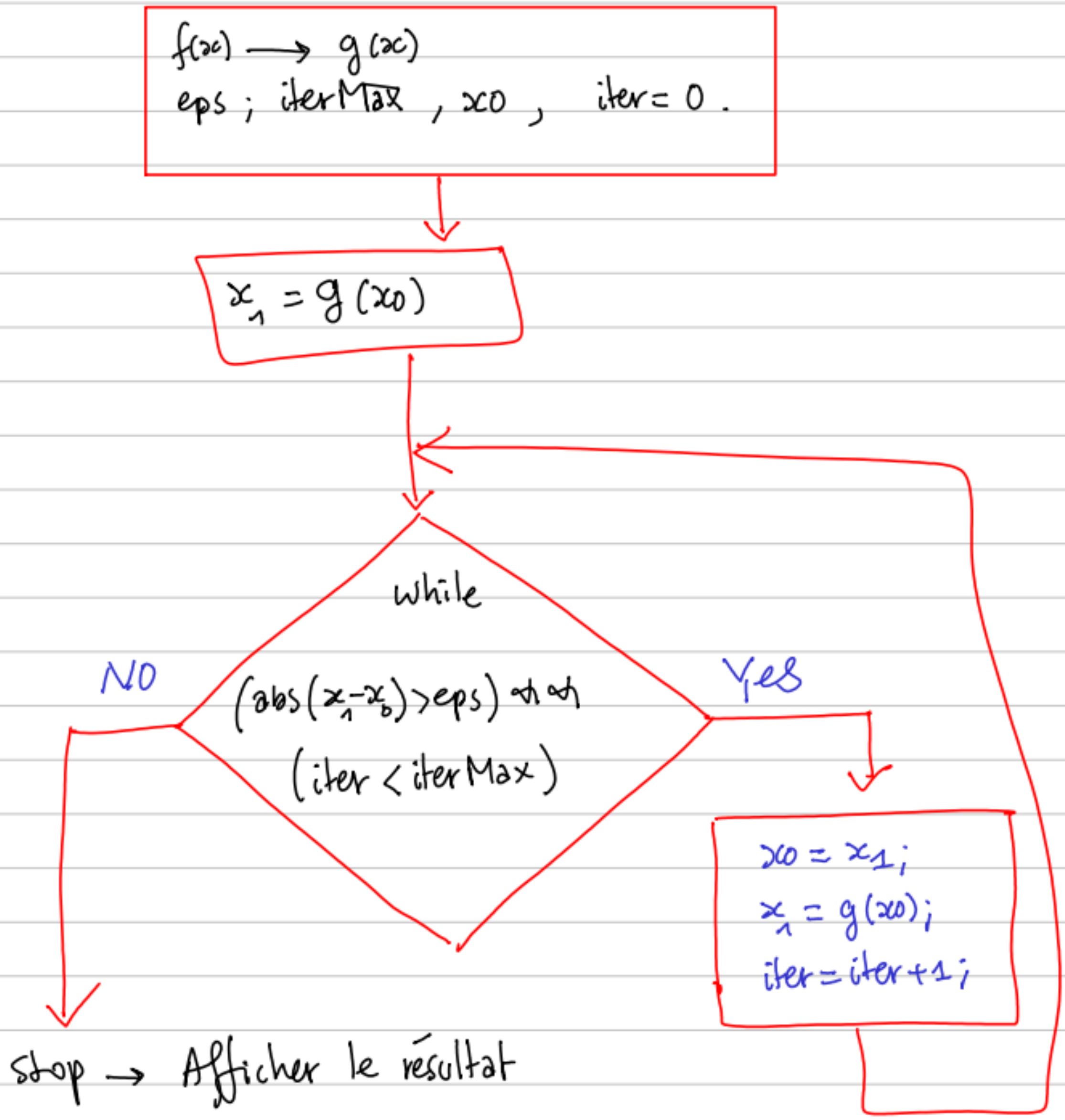
Criteria #2. The number of iterations (MaxIter)

→ for iter = 1 : MaxIter ↴

Criteria #3. Both criteria combined →

while ((abs(a-b) > eps) && (iter < MaxIter)) ↴

A flowchart for the method



$$f(x) = x^3 + 4x^2 - 10 .$$

$$\rightarrow \quad x = \left(10 - 4x^2\right)^{1/3} = g_1(x)$$

$$\rightarrow \quad x = \left(\left(10 - x^3\right)/4\right)^{1/2} = \sqrt{\left(10 - x^3\right)}/2 = g_2(x)$$

$$\rightarrow \quad x = \left(10 - x^3\right)/4x = g_3(x)$$

Matlab script(s) for the method:

```
% Plot the two functions f & g
figure
f = inline('x.^3 + 4*x.^2 - 10');
x=0.5:0.01:5;
plot(x,f(x)), grid on
figure
g= inline('sqrt(10-x.^3)/2');
x=0.5:0.01:5;
plot(x,g(x),x,x), grid on
%--------------------------
a=1;
b=2;
x0 =1.5;
x1 = g(x0);
eps=1.0e-6;
iter=0;
if ((f(a)*f(b))< 0)
while (abs(x1-x0) > eps)
x0=x1;
x1 = g(x0);
iter=iter+1;

fprintf('For iteration =%d \t , the solution is x0=%f\n',iter,x0)
end
fprintf('The final solution is x0 = %f \n',x0) ;
else
disp('There is no solution in [a,b]')
end
```

```
% Plot the two functions f & g
figure
f = inline('x.^3 + 4*x.^2 - 10');
x=0.5:0.01:5;
plot(x,f(x)), grid on
figure
g= inline('sqrt(10-x.^3)/2');
x=0.5:0.01:5;
plot(x,g(x),x,x), grid on
%--------------------------
a=1;
b=2;
x0 =1.5;
x1 = g(x0);

iterMax=10;
if ((f(a)*f(b))< 0)
for iter=1:iterMax
x0=x1;
x1 = g(x0);

fprintf('For iteration =%d \t , the solution is x0=%f\n',iter,x0)
end
fprintf('The final solution is x0 = %f \n',x0) ;
else
disp('There is no solution in [a,b]')
end
```