La nouvelle base $\mathcal{B}' = (v_1, v_2, v_3)$ avec :

$$\begin{cases} v_1 = e_2 + e_3 \\ v_2 = e_1 \\ v_3 = e_2 \end{cases} \quad \begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{vmatrix} = 1 \neq 0 \quad \text{et} \quad \begin{cases} e_1 = v_2 \\ e_2 = v_3 \\ e_3 = v_1 - e_2 = v_1 - v_3 \end{cases}$$

$$\begin{cases} f(v_1) = \lambda_1 v_1 = -v_1 \\ f(v_2) = f(e_1) = -e_1 + e_2 + e_3 = -v_2 + \cancel{v_3} + v_1 - \cancel{v_3} = v_1 - v_2 \\ f(v_3) = f(e_2) = 2e_1 - e_2 = 2v_2 - v_3 \end{cases}$$

<u>Finalement :</u> $T = \begin{pmatrix} -1 & 1 & 0 \\ 0 & -1 & 2 \\ 0 & 0 & -1 \end{pmatrix} = P^{-1} B P$

où $P = (v_1 v_2 v_3) = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}$ et $P^{-1} = \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & -1 \end{pmatrix}$.

3) On a $N = B + I_3 = \begin{pmatrix} 0 & 2 & -2 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}$

$\Rightarrow \boxed{N^3 = O_3}$, N est nilpotente d'indice 3.

* $e^{tB} = e^{t(N - I_3)} = e^{tN} \cdot e^{-tI_3}$

or $e^{-tI_3} = e^{-t} I_3$, donc $e^{tB} = e^{-t} \cdot e^{tN}$.

* $X' = BX \Rightarrow X = e^{tB} X_0 = e^{-t} \cdot e^{tN} X_0$

* $e^{tN} = I_3 + tN + \frac{(tN)^2}{2} = I_3 + tN + \frac{t^2}{2} N^2$

$\Rightarrow \begin{pmatrix} x \\ y \\ \ldots \end{pmatrix} = e^{-t} \begin{pmatrix} 1 & 2t & -2t \\ t & 1 + \frac{t^2}{2} & -\frac{t^2}{2} \\ \ldots \end{pmatrix} \begin{pmatrix} c_1 \\ c_2 \\ \ldots \end{pmatrix} \Rightarrow \begin{cases} x = e^{-t}(c_1 + 2c_2 t - 2c_3 t) \\ y = e^{-t}(t c_1 + (1+t^2) c_2 - t^2 c_3) \\ z = e^{-t}(t c_1 t + c_2 t^2 + (1 - t^2) c_3) \end{cases}$