Ex2. $B = \begin{pmatrix} -1 & 2 & -2 \\ 1 & -1 & 0 \\ 1 & 0 & -1 \end{pmatrix}$.

1) $P(\lambda) = \det(B - \lambda I_3) = \begin{vmatrix} -1-\lambda & 2 & -2 \\ 1 & -1-\lambda & 0 \\ 1 & 0 & -1-\lambda \end{vmatrix} =$

$= (-1-\lambda)^3 - \cancel{2(-1-\lambda)} - \cancel{2(1+\lambda)}$

$\Rightarrow \boxed{P(\lambda) = (-1-\lambda)^3 = -(\lambda+1)^3}$ 1

2) $Q(\lambda) = \lambda + 1 \rightarrow Q(B) = \begin{pmatrix} 0 & 2 & -2 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix} \neq 0_3$

ou $Q(\lambda) = (\lambda+1)^2 \rightarrow Q(B) = \begin{pmatrix} 0 & 2 & -2 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}^2 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 2 & -2 \\ 0 & 2 & -2 \end{pmatrix} \neq 0_3$.

Donc $\boxed{\text{ou } Q(\lambda) = (\lambda+1)^3}$ 1

Ce qui prouve que B n'est pas diag. mais trig.

3) $P(\lambda) = 0 \Rightarrow \lambda_1 = -1$; $m_1 = 3$.

$E(\lambda) = \ker(B - \lambda I_3)$: $(B + I_3)X = 0$

$\Rightarrow \begin{pmatrix} 0 & 2 & -2 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \Rightarrow \begin{cases} 2y - 2z = 0 \\ x = 0 \\ x = 0 \end{cases} \rightarrow \boxed{y = z}$

$X = \begin{pmatrix} 0 \\ z \\ z \end{pmatrix} = z \underbrace{\begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}}_{V_1}$, $E(\lambda_1) = E(-1) = \langle V_1 \rangle$

1 et $\boxed{\dim E(\lambda_1) = 1 \neq m_1}$