2) <u>calcul de A^n :</u> on a $D = P^{-1}AP \Rightarrow A = PDP^{-1}$

et donc $A^n = PD^nP^{-1}$

$$A^n = \begin{pmatrix} 1 & 1 & 0 \\ -2 & 0 & 1 \\ 1 & -1 & 0 \end{pmatrix} \begin{pmatrix} 2^n & 0 & 0 \\ 0 & 4^n & 0 \\ 0 & 0 & 4^n \end{pmatrix} \begin{pmatrix} \frac{1}{2} & 0 & \frac{1}{2} \\ \frac{1}{2} & 0 & -\frac{1}{2} \\ 1 & 1 & 1 \end{pmatrix}$$ 1

<u>finalement :</u>

$$A^n = \begin{pmatrix} 2^{n-1} + \frac{4^n}{2} & 0 & 2^{n-1} - \frac{4^n}{2} \\ -2^n + 4^n & 4^n & -2^n + 4^n \\ 2^{n-1} - \frac{4^n}{2} & 0 & 2^{n-1} - \frac{4^n}{2} \end{pmatrix}$$ 1

3) <u>Résoudre $X_{n+1} = AX_n$:</u> $A = PDP^{-1}$

$X_{n+1} = AX_n = PD\underbrace{P^{-1}X_n}_{Y_n}$: on pose $Y_n = P^{-1}X_n \Rightarrow$

$\boxed{X_n = PY_n} \longrightarrow (1)$ 1 et $\boxed{Y_{n+1} = DY_n} \longrightarrow (2)$

De (2) : $\begin{pmatrix} u_{n+1} \\ v_{n+1} \\ w_{n+1} \end{pmatrix} = \begin{pmatrix} 2 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 4 \end{pmatrix} \begin{pmatrix} u_n \\ v_n \\ w_n \end{pmatrix} \Rightarrow \begin{cases} u_{n+1} = 2u_1 \\ v_{n+1} = 4v_n \\ w_{n+1} = 4w_n. \end{cases}$

$\Rightarrow \begin{cases} u_n = \alpha 2^n \\ v_n = \beta 4^n \\ w_n = \gamma 4^n. \end{cases}$ et De (1) : $\begin{pmatrix} x_n \\ y_n \\ z_n \end{pmatrix} = \begin{pmatrix} 1 & 1 & 0 \\ -2 & 0 & 1 \\ 1 & -1 & 0 \end{pmatrix} \begin{pmatrix} \alpha 2^n \\ \beta 4^n \\ \gamma 4^n \end{pmatrix}$

$\Rightarrow \begin{cases} x_n = \alpha 2^n + \beta 4^n \\ y_n = -2\alpha 2^n + \gamma 4^n \\ z_n = \alpha 2^n - \beta 4^n \end{cases}$ 1