pour $\lambda = \lambda_2 = 4$: $(I) \Rightarrow \begin{cases} -x - z = 0 \rightarrow \boxed{z = -x} \\ 2x + 2z = 0 \\ -x + (-z) = 0 \end{cases}$

Donc $X = \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} x \\ y \\ -x \end{pmatrix}$

$= \begin{pmatrix} x \\ 0 \\ -x \end{pmatrix} + \begin{pmatrix} 0 \\ y \\ 0 \end{pmatrix} = x \underbrace{\begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}}_{V_2} + y \underbrace{\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}}_{V_3}$

$E(\lambda_2) = E(4) = \langle V_2 ; V_3 \rangle$ et $\boxed{\dim E(\lambda_2) = 2 = m_2}$

A est diag.

$D = P^{-1} A P \longrightarrow \begin{pmatrix} 2 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 4 \end{pmatrix} = P^{-1} A P$ 1

où $P = (V_1 V_2 V_3) = \begin{pmatrix} 1 & 1 & 0 \\ -2 & 0 & 1 \\ 1 & -1 & 0 \end{pmatrix}$

calcul de P^{-1} : $X' = PX \iff X = P^{-1} X'$

$\begin{pmatrix} x' \\ y' \\ z' \end{pmatrix} = \begin{pmatrix} 1 & 1 & 0 \\ -2 & 0 & 1 \\ 1 & -1 & 0 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} \Rightarrow \begin{cases} x' = x + y \longrightarrow (1) \\ y' = -2x + z \longrightarrow (2) \\ z' = x - y \longrightarrow (3) \end{cases}$

De (1) + (3) : $2x = x' + z' \longrightarrow x = \frac{1}{2}x' + \frac{1}{2}z'$ 1

De (2) : $z = 2x + y' = x' + y' + z'$

De (3) : $y = x - z' = \frac{1}{2}x' + \frac{1}{2}z' - z' = \frac{1}{2}x' - \frac{1}{2}z'$

Donc $P^{-1} = \begin{pmatrix} \frac{1}{2} & 0 & \frac{1}{2} \\ \frac{1}{2} & 0 & -\frac{1}{2} \\ 1 & 1 & 1 \end{pmatrix}$