

Composé type ALG-3

EXAMEN 2023

Exo 1. $A = \begin{pmatrix} 3 & 0 & -1 \\ 2 & 4 & 2 \\ -1 & 0 & 3 \end{pmatrix}$

1) Diag. A.

$$* p(\lambda) = \det(A - \lambda I_3) = \begin{vmatrix} 3-\lambda & 0 & -1 \\ 2 & 4-\lambda & 2 \\ -1 & 0 & 3-\lambda \end{vmatrix}$$

$$= (3-\lambda)^2(4-\lambda) - (4-\lambda) = (4-\lambda)[(3-\lambda)^2 - 1]$$

$$= (4-\lambda)(3-\lambda-1)(3-\lambda+1) \quad \text{Dmc}$$

$$\boxed{p(\lambda) = (2-\lambda)(4-\lambda)^2}$$

$$* p(\lambda) = 0 \Rightarrow \begin{cases} \lambda_1 = 2 & ; m_1 = 1 \\ \lambda_2 = 4 & ; m_2 = 2 \end{cases}$$

$$* E(\lambda) = \text{Ker}(A - \lambda I_3) : (A - \lambda I_3)X = 0 \Rightarrow$$

$$\begin{cases} (3-\lambda)x - z = 0 \\ 2x + (4-\lambda)y + 2z = 0 \\ -x + (3-\lambda)z = 0 \end{cases} \rightarrow (I)$$

Pour $\lambda = \lambda_1 = 2$: (I) \Rightarrow

$$\begin{cases} x - z = 0 \rightarrow \boxed{x = z} \\ 2x + 2y + 2z = 0 \\ -x + z = 0 \rightarrow \boxed{y = -2z} \end{cases}$$

$$X = \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} z \\ -2z \\ z \end{pmatrix} = z \begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix}$$

$$E(\lambda_1) = E(2) = \langle v_1 \rangle \quad \text{et} \quad \boxed{\dim E(\lambda_1) = 1 = m_1}$$